produced by the addition of the impurity X. From a measurement of the heat capacity of phenol-water near the critical temperature,[12] we roughly estimate α to be less than 0.1. Using the fact that $f \simeq 0.1$ in our system, one estimates $\gamma_X - \gamma \lesssim 0.02$. This result is consistent with the data in Table I when experimental uncertainties are taken into account.

To summarize, it is found that impurities added to a critical system have a small effect on at least some of its thermodynamic and transport properties. Aside from the theoretical implications of this work, it suggests that one need not use extremely high-purity samples to obtain meaningful values of γ and γ^* in fluid systems.[13]

We are grateful to Professor M. E. Fisher and Mr. P. E. Scesney for permission to quote their work prior to its publication. Also we have profited by correspondence and conversations with Professor B. Widom and Professor Fisher. One of us (C.S.B.) would like to thank Dr. P. N. Pusey for assistance in sample preparation and linewidth measurements.

[1]The critical point in a ternary system is often referred to as the plait point. See, for example, J. S. Rowlinson, Liquids and Liquid Mixtures (Butterworths Scientific Publications, Ltd., London, England 1959), Chap. 6.

[2]P. N. Pusey and W. I. Goldburg, Phys. Rev. Letters 23, 67 (1969).

[3]L. D. Landau and E. M. Lifshitz, Statistical physics (Addison-Wesley Publishing Company, Inc., Reading, Mass., 1958), Chap. 12.

[4]B. Chu and F. J. Schoenes, Phys. Rev. Letters 21, 6 (1968).

[5]L. D. Landau and E. M. Lifshitz, Fluid Mechanics (Addison-Wesley Publishing Company, Inc., Reading, Mass., 1959), Chap. 6.

[6]M. E. Fisher, Phys. Rev. 176, 257 (1968).

[7]B. Widom, J. Chem. Phys. 46, 3324 (1967). See also J. C. Wheeler, thesis, Cornell University, 1968 (unpublished).

[8]M. E. Fisher and P. E. Scesney, to be published.

[9]P. N. Pusey and W. I. Goldburg, Appl. Phys. Letters 13, 321 (1968).

[10]L. P. Kadanoff et al., Rev. Mod. Phys. 39, 395 (1967).

[11]See Ref. 8. This equation holds when $10^{-6} \lesssim |\Delta T|/T \lesssim 10^{-4}$. It was actually obtained for the exponent β, which characterizes the shape of the coexistence curve. It seems reasonable to assume that the same model would yield a similar result for γ.

[12]Kh. Amirkhanov, I. G. Gurvich, and E. M. Matizen, Dokl. Akad. Nauk SSSR 100, 735 (1955).

[13]Though only the critical exponents γ and γ^* are discussed here, we call attention to recent experimental work by Zollweg and Widom in which the critical exponent β has been measured in a ternary system. The results are in agreement with the prediction of Ref. 7. See J. A. Zollweg, thesis, Cornell University, 1969 (unpublished).

OBSERVATION OF SUPERFLUID-HELIUM PERSISTENT CURRENT BY DOPPLER-SHIFTED SPLITTING OF FOURTH-SOUND RESONANCE*

I. Rudnick, H. Kojima, W. Veith,† and R. S. Kagiwada
Department of Physics, University of California, Los Angeles, California 90024
(Received 13 October 1969)

Persistent currents in superfluid helium contained in the pores of a packed powder have been observed by means of the Doppler-shifted splitting of an azimuthal resonant fourth-sound mode of the cylindrical resonator containing the helium.

A direct method to determine the velocity of a fluid is to measure the Doppler shift of a sound wave. The purpose of this Letter is to describe the first such determination of the velocity of a circulating persistent current in superfluid helium. Previous measurements of persistent currents depend on gyroscopic effects associated with their angular momentum.[1] In an imaginative series of experiments Reppy and his coworkers have exploited such measurements to study a variety of phenomena.[1] The momentum measurements yield the product $\rho_s(T)\omega_s(T)$ and the relative temperature dependence of ρ_s (ρ_s and ω_s are the superfluid component density and angular velocity, respectively). If ρ_s is known at one temperature then $\rho_s(T)$ and $\omega_s(T)$ are determined. The only circumstance in which there is an uncertainty in ρ_s occurs where size effects are important[2] and this is possible since in such experiments the normal fluid is locked by the small pores of a porous medium which fills the test chamber. We determine the velocity of persistent currents by the measurement of the Doppler shift experienced by fourth sound, the sound wave which propagates in the helium-filled pores of such systems.[2] Since the fundamental condition on the quantized vortices which comprise the velocity field establishes their velocity rather than their momentum, it would appear that such a measurement is preferable to a momentum de-

termination. This may well be; however, it will be seen [see Eq. (5)] that this method, which is based on the measurement of the splitting of a resonant mode, yields a value of $\rho_s(T)\omega_s(T)$, just as do the momentum experiments. The value of $\rho_s(T)$ is obtained simultaneously from the mean frequency of the doublet resonance, once an empirically determined scattering correction for the grains of the porous medium is used.[2]

The velocity v of He II (density ρ) is given by $v=(\rho_s/\rho)v_s+(\rho_n/\rho)v_n$, where the subscripts s and n refer to the superfluid and normal-fluid components, respectively. If $v_n=0$, as it is in this case,

$$v=(\rho_s/\rho)v_s. \tag{1}$$

Accordingly the Doppler-shifted velocity C_4 of a fourth-sound wave traveling along the line of flow will be given by

$$C_4=C_{4\,\mathrm{stat}}\pm(\rho_s/\rho)v_s, \tag{2}$$

where $C_{4\,\mathrm{stat}}$ is the velocity of fourth sound in a stationary medium.[3] An approximate value, good to better than 3%, is $C_{4\,\mathrm{stat}}=(\rho_s/\rho)^{1/2}C_1$, where C_1 is the velocity of first sound.[4]

In a stationary rigid-walled cavity (filled with a superleak which locks the normal fluid), the lowest mode, whose motion is entirely in the azimuthal direction around the axis of the cylinder, will have the pressure distribution

$$p=\cos\varphi J_1(kr)\cos\omega_a t, \tag{3}$$

where ω_a is the resonant frequency, φ is the azimuthal angle, p is the acoustic pressure, J_1 is the first-order Bessel function, r is the radial coordinate, and $k=1.84/\mathrm{radius}$. If the superfluid component rotates with an angular velocity ω_s, then (3) is the pressure distribution in a frame which rotates with the angular velocity of He II, which according to (1) is $(\rho_s/\rho)\omega_s$. Thus in a stationary frame

$$p=\cos[\varphi+(\rho_s/\rho)\omega_s t]J_1(kr)\cos\omega_a t,$$

$$p=\tfrac{1}{2}\left\{\cos\left[\left(\omega_a+\frac{\rho_s}{\rho}\omega_s\right)t+\varphi\right]+\cos\left[\left(\omega_a-\frac{\rho_s}{\rho}\omega_s\right)t-\varphi\right]\right\}J_1(kr). \tag{4}$$

The original mode has its degeneracy lifted, and the resulting doublet has a separation of $\Delta\omega$ given by

$$\Delta\omega=2(\rho_s/\rho)\omega_s. \tag{5}$$

It is evident from inspection of Eq. (4) that the higher frequency mode is a running wave traveling in the sense of the rotation, and the lower frequency mode is a running wave traveling in the opposite sense.[5] It is now clear that the original degeneracy arises from the fact that waves travel with the same velocity for either sense of rotation when the medium is stationary.

Figure 1 is a schematic of the fourth-sound cell. The source and receiver transducers are identical condenser transducers whose active elements are stretched 0.0006-cm-thick aluminized Mylar films. The other plates of the condensers rigidly terminate the cylinder. They are segmented and for the purposes of this discussion these segments are shown as semicircles. The source is driven as a dipole at a frequency $\frac{1}{2}\omega$, the frequency being doubled when the sound is produced because the force on the diaphragm is proportional to the square of the voltage. Similarly the receiver transducer operates as a dipole receiver. Because of the dipole character of the transducers, they are equally effective in exciting and responding to both modes of Eq. (4).

The cylinder is packed with Al_2O_3 powder (grain diameter 170 to 325 Å) at a sufficiently high pressure so that it cakes. The effective pore diameter is estimated to be about equal to the grain di-

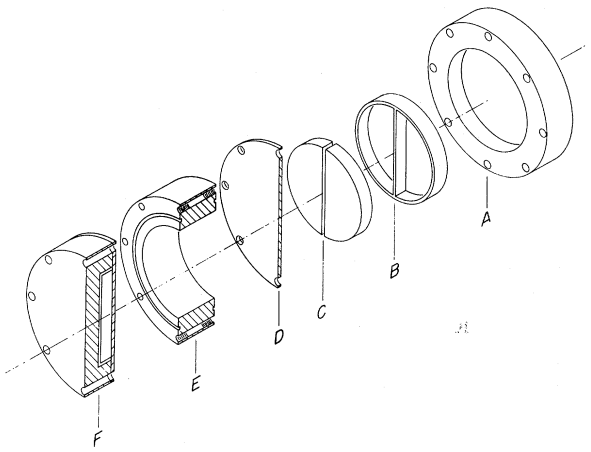

FIG. 1. Partially exploded view of fourth-sound cylindrical resonator and capacitative transducers. *A*, *B*, *C*, and *D* are the elements of the source transducer. *A* is the case. *C* are the electrodes driven out of phase so that the transducer is a dipole sound source, *B* is epoxy-resin insulation. *D* is 0.0006-cm-thick Mylar, aluminized on the left-hand side; it is the active element of the transducer. *E* is the cylindrical resonator 3.81 cm in diameter and 2.13 cm long. It is packed with powder (grain size 170 to 325 Å). *F* is the receiver transducer which is exactly the same as the source and is shown assembled.

ameter.

The cylinder was rotated about its axis starting at $T > T_\lambda$ and the temperature was gradually lowered. The rotation is halted at some low temperature (e.g., 1.5°K). The response of the cylindrical resonator is then determined by a slow frequency sweep using a General Radio Model No. 1900A wave analyzer in its tracking generator mode. Figure 2 shows the recording of the responses. The top curve is the mode in question after a cool-down without rotation. The frequency is 3240 cps and the half-power bandwith is approximately 4 cps. The middle curve is the response after a cool-down with a driven rotation rate of 5.4 cps. The splitting is 8.7 cps. In the bottom curve the driven rotation rate was 13.5 cps and the splitting is 15.2 cps. The temperature in all cases is approximately 1.2°K. The cylinder inner diameter being 3.81 cm, the persistent velocity at the rim for the bottom curve of Fig. 2 is calculated to be 100 cm/sec.[6] Many such splittings have been observed, and in all cases the original unsplit line is regained when the resonator is heated above T_λ and then cooled down without rotation.

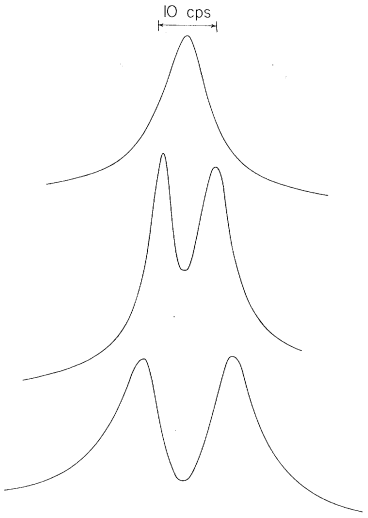

FIG. 2. Recordings of the response of the azimuthal mode of the resonator. Temperature = 1.2°K. Top curve: cool-down without rotation; resonant frequency 3240 cps. Middle curve: response after cool-down with driven rotation rate of 5.4 cps; the splitting is 8.7 cps. Bottom curve: response after cool-down with driven rotation rate of 13.5 cps; the splitting is 15.2 cps. The angular velocity of the persistent current can be obtained from Eq. (5).

According to Eq. (2) the splitting should decrease as the temperature is increased because of the decrease in ρ_s/ρ and this is observed. Moreover, when the temperature is increased high enough so that the critical velocity is decreased to less than the rim velocity of the original persistent current, it is expected that there should be an added decrease in splitting and, further, that the line shapes should change since there should no longer be a unique angular velocity. This, too, has been observed. We have also found that, provided the temperature is not increased too much, all these effects are reversible and the original persistent velocity is regained when the temperature is reduced. Within the accuracy of our preliminary observations (several percent) there was insignificant decay of the current in intervals up to 6 h, after an initial stabilizing period.

The spirt of the investigation (and this Letter) is one of establishing the existence of the splitting and the feasibility of studying persistent current phenomena by this new method. There has been no attempt to obtain accurate measurements of these effects. A careful examination of these and other effects will be the object of further measurements.

With the methods described above, the minimum observable splitting is about equal to the bandwidth. The minimum half-power bandwidth we have obtained is 2 cps. However, there is a way to achieve greater resolution. Consider a source transducer with its back plate divided into n equal pie-shaped sectors. If, now, signals are applied to the sectors with a phase difference of $2\pi/n$ between neighboring sectors and with the phase difference advancing with a given sense of rotation, then only one of the two split lines will be excited. With the opposite sense of advance of phase, the other line of the pair is excited alone. If the receiver transducer is exactly similar and phased in the same way, the selectivity should be further increased. We hope by such methods to increase the resolution by an order of magnitude.

Since the preparation of this Letter some experiments on rotating fourth sound resonators have been reported[7] which yield effects different from those reported here.

E. Guyon, J. C. Fraser, and G. G. Natale contributed to the experimental effort.

---

*Research supported in part by Office of Naval Research, Acoustics Branch.

†North Atlantic Treaty Organization Postdoctoral Fellow.

[1]For a bibliography on persistent angular-momentum measurements see J. S. Langer and J. D. Reppy, Pro-

gress in Low Temperature Physics, edited by C. J. Gorter, (North-Holland Publishing Company, Amsterdam, The Netherlands, to be published), Vol. VI; J. B. Mehl and W. Zimmerman, Jr., Phys. Rev. 167, 214 (1968).

[2]I. Rudnick, E. Guyon, K. A. Shapiro, and S. A. Scott, Phys. Rev. Letters 19, 488 (1967). E. Guyon and I. Rudnick, J. Phys. (Paris) 29, 1081 (1968).

[3]Equation (2) has also been rigorously obtained from the equations of two-fluid hydrodynamics, as given by I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. 30, 617 (1956) [translation: Soviet Phys.-JETP 3, 649 (1956)].

[4]An exact value for $C_{4\ \mathrm{stat}}$ is

$$\left[\frac{\rho_s}{\rho}C_1^2+\frac{\rho_n}{\rho}C_2^2\left(1-\frac{2\beta_1^2}{\gamma s}\right)\right]^{1/2},$$

where C_2 is the velocity of second sound, β is the isobaric expansion coefficient, γ is the ratio of specific heats, and s is the entropy.

[5]We are indebted to S. Alexander and C. G. Kuper for pointing this out to us.

[6]A value of 0.9 for ρ_s/ρ was used in this calculation.

[7]J. M. Hubert, E. V. Larson, and C. F. Squire, J. Low Temp. Phys. 1, 375 (1969).

HALL EFFECT IN DIRTY TYPE-II SUPERCONDUCTORS

Kazumi Maki
Department of Physics, Tôhoku University, Sendai, Japan
(Received 18 July 1969)

The Hall effect in the vortex state of a dirty type-II superconductor is re-examined microscopically. The inclusion of the kinetic-energy term of the moving pair (which was completely neglected by Caroli and Maki) in the chemical potential results in a large Hall effect in the vortex state consistent with experiments on Nb-Ta alloys by Niessen et al. Furthermore it is shown that for a dirty superconductor the Hall angle of the vortex state in the zero induction limit reduces to a constant value independent of temperature.

A rapid increase in the Hall angle observed in the vortex state of a number of superconducting alloys[1-5] has perplexed theoreticians for a long time. In a local model given by Bardeen and Stephen,[6] in which they replaced a vortex line by a cylinder of the normal-state electrons with a radius of the order of the coherence distance, the Hall angle was unaffected by the transition into the vortex state (i.e., the Hall angle is just the Hall angle of the normal metal in a magnetic field equivalent to one at the center of the vortex core). With a slightly different assumption as to the nature of forces acting on a vortex line, Nozières and Vinen[7] arrived at the same conclusion as Bardeen and Stephen for a dirty material.

In a recent microscopic calculation by Caroli and the author[8] it was even concluded that the Hall angle rather decreases rapidly upon entrance into the vortex state, which is in a flat contradiction to the above experiments.[1-5] In this note we would like to point out that the kinetic-energy term of the electron pair in the chemical potential was neglected in I. In fact the inclusion of this term gives rise to a large Hall effect in the vortex state of a dirty type-II superconductor, which is consistent with the above-mentioned experiments.

We have shown in I that the order parameter of a dirty superconductor in a magnetic field H slightly below the upper critical field H_{c2} obeys

$$\{\partial/\partial t+2i\mu(\vec{r})-D[\nabla+2ie\vec{A}(\vec{r})]^2-\epsilon_0(T)\}\Delta=0 \quad (1)$$

and

$$-\ln\frac{T}{T_{c0}}=\psi\left(\frac{1}{2}+\frac{\epsilon_0(T)}{4\pi T}\right)-\psi(\tfrac{1}{2}), \quad (2)$$

where μ is the chemical potential, $\vec{A}$ is the vector potential, $D=lv/3$ is the diffusion constant, and $\psi(z)$ is the digamma function.

In the presence of a uniform electric field which is described by the scalar potential $\varphi(\vec{r})$, the chemical potential $\mu(\vec{r})$ is given by

$$\mu(\vec{r})=e\varphi(\vec{r})-(1/8m)(\nabla+2ie\vec{A})^2. \quad (3)$$

The second term in Eq. (3) represents the change in the local chemical potential due to the motion of the condensed pair (in analogy to the kinetic-energy term in a classical fluid) and is always present independently of whether the material is pure or dirty. This term was mentioned previously by Abrahams and Tsuneto[9] in connection with the Galilean invariance of the order parameter but the significance of this term remained unnoticed. In the previous treatment[8] the existence of this term was completely ignored.

Let us consider the situation where a magnetic field H slightly smaller than H_{c2} and an electric field E are applied in the z direction and in the