larger than that which can arise from a background three-pion configuration with $C = +1$ lying within the experimental cut taken as defining the ω. See H. Yuta and S. Okubo, Phys. Rev. Letters 21, 781 (1968).

[23]Possibility (II) is rather unlikely since the bosons with $C = +1$ would have opposite parity and possibly different spins, thus making their near degeneracy in mass implausible.

A_2 SPLITTING AND A NEW DUALITY FOR BROKEN INTERNAL SYMMETRIES?

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We propose that both SU(3) and broken nonchiral SU(2) ⊗ SU(2) internal symmetries are relevant for classification of hadronic states. The theory explains the A_2 splitting while predicting the existence of at least one abnormal meson with $(I^G, J^P) = (2^-, 2^+)$ degenerate in mass with the A_2.

Sometime ago, Lee[1] made the interesting conjecture that when hadronic mass levels become sufficiently clustered and overlapping, level classification by one scheme can by a recoupling procedure be made to look like another scheme—thus giving rise to a new duality[2] for approximate symmetry schemes of hadrons. Actually in nuclear physics we have both collective- and shell-model descriptions. In atomic physics we are familiar with electron levels associated with the approximate LS and jj coupling schemes where the general case, when both approximate "symmetries" are in evidence, belongs to the intermediate-coupling domain. A relevant question is to search for the "intermediate couplings" of particle physics[3,4] and their relevance to hadronic mass-level degeneracies.

We are aware that the internal-symmetry group SU(3) is a good broken symmetry for the hadrons; in particular the more specific quark version $(q\bar{q})$ with L excitations[5] appears to reproduce the gross features of meson and baryon levels rather well. Careful analysis of the empirical data[6] (with special emphasis on the η'-δ degeneracy problem) suggests that of all internal symmetry groups which commute with the Lorentz transformation, the nonchiral SU(2) ⊗ SU(2) group [which is not contained in SU(3)] represents the optimum candidate as the dual broken symmetry shared by hadron mass spectra. In the present note we wish to show that application of nonchiral SU(2) ⊗ SU(2) to the A_2 meson-splitting puzzle[7] can give, for reasonable multiplet assignments, an adequate explanation of this phenomenon while predicting at the same time the existence of two additional abnormal mesons with $(I^G, J^P) = (2^-, 2^+)$ and $(0^-, 2^+)$ mass degenerate with the A_2. Of particular current interest is that it provides a theoretical framework for understanding the possible discovery of an $I^G = 2^-$ resonance.[8] Some other examples taken from meson and baryon systems where traces of this broken symmetry may be in evidence are briefly discussed.

A_2 splitting.—A highly imaginative solution to the observed double peaking of the A_2 meson resonance has recently been proposed by Arnold and Uretsky.[9] They conjecture that this phenomenon arises from a destructive interference in the decays of two different resonances, coherently produced, into the same final state. The interfering resonance can have different isospin and/or G parity from the A_2 $[(I^G, J^P) = (1^-, 2^+)]$ and mixes via virtual electromagnetic transitions.[10] Hence the physics is entirely analogous to the experimentally known isospin- and G-nonconserving decay $\omega \to 2\pi$ which manifests itself as a negative interference "notch" in a $\pi^+\pi^-$ mass histogram in the ρ^0 peak.[11]

The same authors[9] also point out that the A_2 has a width of about 90 MeV and is seen to split into two 30-MeV-wide peaks separated by a 30-MeV-wide valley. Hence they conclude that the interfering resonance has a width less than 30 MeV. From the symmetric appearance of the splitting, the mass of that resonance must be very nearly degenerate with that of the A_2 (hence enhanced virtual electromagnetic transitions are expected[10]). The spin and parity is 2^+. Since the splitting was observed in the CERN $\pi^- + p \to X^- + p$ (missing X^- mass) experiment,[7] the only possible isospins here are 1 and 2. Presuming that either the isospin or G parity differs from that of the customary A_2 $(I^G = 1^-)$, the allowable I^G quantum numbers for the interfering resonance can be only 1^+, 2^-, and 2^+. From the theoretical standpoint of understanding the near mass degeneracy [in terms of a $(q\bar{q}q\bar{q})$ system with no orbital angular momentum], Arnold and Uretsky prefer the assignment $(I^G, J^P) = (2^+, 2^+)$ for their postulated interfering resonance.

We adopt the physics of the above analysis. However, as the underlying theoretical principle behind the mass degeneracy required, we believe nonchiral SU(2)⊗SU(2) classification of these resonant states (hereafter labelled by $A_{2,I}$, where $I=1$ corresponds to the usual A_2) to be the relevant one. It has been speculated[6] that, for mesonic systems, SU(2)⊕SU(2) can lead to explanations of degeneracy between members of different isospin multiplets—up to electromagnetic mass differences.

Following the very useful Pais doublet scheme[12] for SU(2)⊗SU(2), if $\vec{I}_1$ and $\vec{I}_2$ are the generators of the two SU(2) groups, then we identify isospin $\vec{I}$ with $\vec{I}_1+\vec{I}_2$; in other words, the isospin subgroup consists of elements of the form (I_1, I_2). The assignments η', δ, $(\frac{1}{2}, \frac{1}{2})$; Λ, Σ, $(\frac{1}{2}, \frac{1}{2})$; N, $(\frac{1}{2}, 0)$; K, $(\frac{1}{2}, 0)$; π, $(1, 0)$; η, $(0, 0)$; and d, $(0, 0)$ have been shown to give a reasonable description of data.[6] For nonstrange mesonic states we need to extend SU(2)⊗SU(2) to include G parity.[6,13] The relevant result of physical interest here is that, for our choice of assignments, G parity must be the same for each member of an SU(2)⊗SU(2) meson multiplet—in order to preserve crossing symmetry.[6,14] Of course members of such a multiplet must possess the same spin and parity since an internal symmetry is involved.

Application of SU(2)⊗SU(2) to the A_2 complex suggests naturally that the (1, 1) multiplet assignment be used. Actually a $(\frac{1}{2}, \frac{3}{2})$ or $(\frac{3}{2}, \frac{1}{2})$ representation would be more economical in that the isospin classification would involve just $I^G = 1^-$, 2^-; $J^P = 2^+$ states (but no $I^G = 0^-$; $J^P = 2^+$ state). However these (nontrivial) representations [unlike $(\frac{1}{2}, \frac{1}{2})$, (1, 1)] are not self-conjugate under exchange of the two SU(2)'s, and corresponding isospin states do not yield useful symmetry and antisymmetry properties under such an exchange. This is (at least) esthetically less than satisfactory in terms of the theoretical extension of SU(2)⊗SU(2) by G parity for nonstrange mesonic states—though perhaps we should keep an open mind on this question.[15,16] For defineteness the (1, 1) assignment will be assumed in the rest of the discussion. In terms of isospin classification, three mass-degenerate resonances $A_{2,I}$ $(I^G = 0^-, 1^-, 2^-$; $J^P = 2^+)$ are present (assuming that $A_{2,1}$ has $G = -1$). The $I^G = 2^-, 0^-$ members are abnormal mesons not coupled to the $q\bar{q}$ (or $N\bar{N}$) system, nor with the S-wave states formed out of the $(q\bar{q}q\bar{q})$ configuration.[9] They are connected, for instance, with the $(q\bar{q}q\bar{q})$ description via two $L=0$ $(q\bar{q})$ pairs in a relative D wave! Conserved quantum numbers of strong interactions allow all three resonances $A_{2,I}$ $(I = 0, 1, 2)$ to have strong $(\pi\rho)$ decays while forbidding $(K\bar{K})$, $(\eta\pi)$ strong decays for $A_{2,0}$ and $A_{2,2}$. However, virtual electromagnetic transitions between $(A_{2,2})^\pm$ and $(A_{2,1})^\pm$ states can give rise to splittings (via destructive interference) in the mass destributions of $(\rho\pi)^\pm$, $(K_1^0K^\pm)$, and $(\eta^0\pi^\pm)$. An intriguing possibility is the examination of notch (spike) structure in the $K_1^0K_1^0$ mass spectrum, since evidence of such effect will be indicative of breakdown of C invariance in electromagnetism as implied by a recent experiment.[17] Likewise the experiment $p+d \to He^3 + (MM)$ is of interest since structure in the A_2 region of missing mass (MM) tests specifically C-nonconserving electromagnetic transition between $A_{2,1}^0$ and $A_{2,0}^0$.

Further properties of $A_{2,2}$ can be inferred from the very fact that interference is observable in charged decay modes. We expect that (i) $A_{2,2}$ is produced fairly copiously when compared with $A_{2,1}$ in meson-baryon collisions, and (ii) it must have small decay width into the well-explored $(\pi\rho)$ mode (<30 MeV) to account for the splitting property and yet escape easy detection. Applying broken SU(2)⊗SU(2) to strong decays by assigning ρ to $(1, 0)$[17] forbids $(\pi\rho)$ decay of $A_{2,2}$ (and $A_{2,0}$). However, the dominant $(\rho\pi)$ mode of $A_{2,1}$ is also forbidden![18] We invoke a rule which is somewhat reminescent of the heuristic notion that systems coupled to $q\bar{q}$ manifest at least their "on mass shell" SU(3) properties [as opposed to the complementary SU(2)⊗SU(2)] more prominently. The rule states (analogous to Ref. 9) that decays of resonances not coupled to $q\bar{q}$ (like $A_{2,2}$ and $A_{2,0}$) into a pair of members (e.g., π and ρ), each belonging to the low-lying states reached by $q\bar{q}$ $(L=0)$, are severely inhibited. Note that $A_{2,1}$ is coupled to the $(q\bar{q})$ system.[16] Production of $A_{2,2}$ (and $A_{2,0}$) is expected to remain copious relative to $A_{2,1}$ by t-channel exchange of Regge-recurrence mesons (e.g., $J^P = 3^-, 5^-, 7^-, \cdots$ partners of ρ in πN collision).

The crucial test of our theory remains the search for $A_{2,0}$ and (especially) $A_{2,2}$ resonances of narrow widths in the 1300 MeV region. Assuming C invariance, identification of the $(\rho^0\pi^0)$ mode of $A_{2,0}$ from $K^- + p \to \Lambda^0 + A_{2,0}$, $p+d \to He^3 + A_{2,0}$, $p+p \to p+p+A_{2,0}$, and $K^+ + p \to K^+ + p + A_{2,0}$ are of interest. Likewise, the search for $A_{2,2}$

should be continued by identifying the $(\rho^{\pm}\pi^{\pm})$ doubly charged decay modes from $\pi^- d \to pp + (\pi^-\pi^-\pi^0)$ and $\pi^+ p \to n + (\pi^+\pi^+\pi^0)$. Note that these reactions violate SU(2) ⊗ SU(2) for our choice of assignments [with He^3: $(\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$]. This need not be alarming[18] since we expect $\pi N \to NA_{2,1}$ [also SU(2) ⊗ SU(2) forbidden[19]] to set the scale for the type of production cross section expected where suitable t-channel Regge meson exchanges are allowed. In any case SU(2) ⊗ SU(2) allowed reactions are available for $p + \bar{p} \to 2A_{2,I}$ where enormous resonance-production cross sections and the existence of exotic and abnormal states have been suggested[20]; another allowed reaction is $\pi N \to 2A_{2,I}N$.

$K_{1/2}^*(1420)$ splitting?—Experience dictates that, when several open channels are available for decay into low-lying mesonic states (as is true with the tensor nonet $J^P = 2^+$), the opportunity for observing resonance structure is enhanced. It is tempting to speculate that a similar sharp notch (or spike) in $K_{1/2}^*(1420)$ may be present due to destructive (or constructive) interference with narrow $I = \frac{3}{2} K_{3/2}^*(1420)$ of the same $J^P = 2^+$. The SU(2) ⊗ SU(2) classification can be $(1, \frac{1}{2})$ or $(\frac{1}{2}, 1)$. Search for $I = \frac{3}{2}$ member from $K^+p \to (K^+\pi^+)n$ and $K^+p \to (K\pi\pi)n$ remains a relevant task.

Baryon system.—No G-parity problem is involved for the baryons, hence more examples of broken nonchiral SU(2) ⊗ SU(2) multiplets should be present here than for the mesons.[21] The $S = -1$ system is replete with examples belonging to the $(\frac{1}{2}, \frac{1}{2})$ representation, e.g., the pair $\Sigma(1680)$ and $\Lambda(1670)$ with $J^P = \frac{1}{2}^-$ and the pair $\Sigma(1660)$ and $\Lambda(1690)$ with $J^P = \frac{3}{2}^-$. It is amusing to note that even the gross features of their branching ratios[22] can be understood with our choice of SU(2) ⊗ SU(2) assignments. Systems like (KN), (NN), where normal resonance activity is suppressed (for instance K^+p and K^+n are not reached by reonances associated with the L-excitation quark model), may offer more clear-cut cases of broken SU(2) ⊗ SU(2). The $Z_1^*(1900)$ and $Z_0^*(1865)$ are particularly interesting examples which can be accommodated by the $(\frac{1}{2}, \frac{1}{2})$ representation if they have the same spin and parity. Belief in the existence of the once doubted Z_1^* has been recently revived by Harari[5] in connection with the Regge-resonance duality postulate of Dolen et al.[2]

L'Envoi.—In conclusion, we emphasize that the hypothesis that hadron mass levels share dual broken internal symmetries is only a theoretical speculation—borrowed by analogy from other areas of physics. However, nonchiral SU(2) ⊗ SU(2) offers an economical interpretation of such major puzzles as the A_2 splitting and is amenable to immediate experimental tests. To the extent that SU(2) ⊗ SU(2) is a subgroup of special global symmetry,[23] we are perhaps justified in asserting that the report of inutility of the Gell-Mann–Feynman–Schwinger global symmetry[24] is premature.

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[1]T. D. Lee, private communication.

[2]This is distinct, of course, from the Regge duality of R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).

[3]I owe this remark to Professor J. R. Oppenheimer.

[4]G. F. Chew, Comments Nucl. Particle Phys. 2, 107 (1968), has proposed that, in fact, nuclear and particle physics share identical underlying physical principles, thus transcending even the analogy we draw between similarity of level structures for the two situations.

[5]H. Harari, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 195.

[6]S. F. Tuan and T. T. Wu, Phys. Rev. Letters 18, 349 (1967).

[7]M. N. Focacci, et al., Phys. Rev. Letters 17, 890 (1966); H. Benz, et al., Phys. Letters 28B, 233 (1968); B. French, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 91.

[8]R. Vanderhagen, et al., Phys. Letters 24B, 493 (1967) claimed to see an $I^G = 2^-$ $(\rho^-\pi^-)$ enhancement at 1320 MeV.

[9]R. C. Arnold and J. L. Uretsky, Phys. Rev. Letters 23, 444 (1969).

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[12]A. Pais, Phys. Rev. 110, 574, 1480 (1958), and 112, 624 (1958).

[13]T. D. Lee and G. C. Wick, Phys. Rev. 148, 1385 (1966).

[14]See also S. K. Bose and E. C. G. Sudarshan, Phys. Rev. 162, 1396 (1967).

[15]Note that the mixed representations $(\frac{1}{2},\frac{3}{2}) \pm (\frac{3}{2},\frac{1}{2})$ are self-conjugate but reducible [under SU(2) ⊗ SU(2)]. They lead to the same experimental consequences (absence of $I^G = 0^-$, $J^P = 2^+$ state) as the individual $(\frac{1}{2},\frac{3}{2})$

or $(\frac{3}{2},\frac{1}{2})$ representations. We shall not discuss further here the consequences of assuming mixed representations for the A_2 structure.

[16]Alternatively, by assigning ρ to (0,1), $(\pi\rho)$ decays of all three $A_{2,I}$ $(I=0,1,2)$ are SU(2) ⊗ SU(2) allowed. We then invoke the stated decay rule to suppress decay widths for the abnormal members. Note this luxury is not possible were $A_{2,I}$ assigned the $(\frac{3}{2},\frac{1}{2})$ or $(\frac{1}{2},\frac{3}{2})$ representation, or a mixture thereof (cf. Ref. 15).

[17]M. Gormley, et al., Phys. Rev. Letters 21, 402 (1968), and 22, 108 (1969).

[18]Like broken SU(3), evidence of SU(2) ⊗ SU(2) is expected to be most prominent in mass spectra with decreasing manifestations in decay and four-body processes.

[19]This reaction is SU(2) ⊗ SU(2) allowed if π is assigned to (0,1) rather than to (1,0), or given mixed assignments (0,1) and (1,0).

[20]J. Rosner, Phys. Rev. Letters 21, 950 (1968), and 22, 689 (1969); V. Barger and D. Cline, Phys. Rev. 182, 1849 (1969).

[21]Note that despite the near ρ^0-ω^0 mass degeneracy, they cannot be assigned to the $(\frac{1}{2},\frac{1}{2})$ representation because these states have opposite G parity.

[22]R. D. Tripp, in Proceedings of the Foureenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 173.

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EXTERIOR THREE-PARTICLE WAVE FUNCTION*

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It is shown that the wave function for three particles interior to the finite volume where all force ranges overlap completely determines the exterior wave function via the solution of a one-variable integral equation with a compact kernel. A unitary parametrization of the interior wave function therefore would supply the equivalent of a phase-shift analysis for three-particle final states, including the exact description of overlapping resonances throughout the Dalitz plot.

For (central, spin-independent) forces of finite range the exterior wave function for two particles can be uniquely specified by a single phase shift in each angular-momentum state, which parameters also specify the scattering cross section. So far, the corresponding description for three-particle final states has not been constructed. At first sight, the Faddeev[1] decomposition into the three channels which asymptotically contain an interacting pair plus a free particle would seem to solve the problem. However, so long as the interacting pair are within the range of the two-particle force, their energy and momentum are not necessarily connected as they will be in the three-free-particle final state, and they can pick up momentum from the outgoing wave in one of the other two channels (cf. Fig. 1). This produces a nonlocal[2] interaction which falls off only with the inverse distance to the third particle, even if the two-particle forces are of finite range. This in turn produces a singularity in the Faddeev equations which must be removed (e.g., by contour rotation or iteration) before they can be solved. In addition, any strongly interacting system can be expected to have three-body forces where all three-particle ranges overlap, and these must be specified, in addition to the

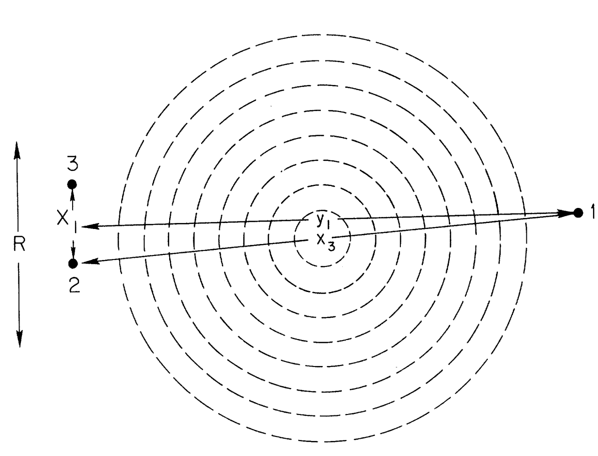

FIG. 1. Scattering in the "three" channel (relative coordinate x_3) produces an outgoing wave which can scatter from the particles in the "one" channel (relative coordinate x_1) so long as they are within the range of forces, R. The effect falls off as $1/y_1$ regardless of the range of forces, and hence is nonlocal.