bo, Istituto di Fisica Marconi, Università di Roma, Nota Interna No. 141, 1967 (unpublished).

[4]We thank C. Cronström for suggesting this solution.

[5]H_I transforms like an octet so that although we use the same symbols c_i also for the symmetry-breaking Hamiltonian that transforms as $(\underline{3}^*\underline{3})$, $(3\underline{3}^*)$, i.e., $c_0u_0 + c_3u_3 + c_8u_8$, the meaning is different.

[6]Symmetry-breaking Hamiltonians that are composed from currents that are neither vector nor axial vector are considered by Y. Ne'eman, Phys. Rev. 172, 1818 (1968).

[7]R. Socolow, Phys. Rev. 137, B1221 (1965). The upper limit follows from $|(\Xi^- - \Xi^0)/(\Xi - \Sigma)|$ and the lower limit from $|(n-p)/(\Lambda - p)|$, where particle symbols denote the corresponding masses.

[8]R. Gatto, G. Sartori, and M. Tonin, Phys. Letters 28B, 128 (1968); and N. Cabibbo and L. Maiani, Phys. Letters 28B, 131 (1968).

EXOTIC EXCHANGE OR KINEMATICAL REFLECTION?*

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Recently observed peripheral peaks in reactions where single-meson exchange is forbidden are explained as kinematical reflections. Experimental and theoretical implications are discussed.

A notable feature of strong interactions is the apparent absence of exotic meson resonances.[1] Recently, however, peripheral forward peaks have been observed in differential cross sections of certain quasi-two-body reactions for which ordinary single-meson exchange is forbidden.[2] Interpreted naively, these peaks suggest a dynamics involving exchange of meson states with exotic quantum numbers, $I \geq \frac{3}{2}$ or $|S| = 2$. In this note, a simple alternative explanation is developed. Peripheral peaks of the type observed are shown to be generated as reflections from competing allowed processes. Calculated magnitudes and shapes are consistent with available data.

Distinctive backward peaks are present in certain stable-particle two-body ($n = 2$) reactions for which the backward exchange (u) channel is exotic. Examples are $\pi^+\pi^- \to \pi^+\pi^-$ and $K^-p \to \bar{K}^0 n$.[3] However, because the peaks have been observed to date only at very low energy, they are often described simply as manifestations of prominent resonances in the direct channel. It is not yet clear whether a dual interpretation is tenable in terms of exchange of exotic single-particle states.[4] The present paper is concerned with possibly exotic exchange phenomena in multiparticle ($n > 2$) data. The point emphasized here is that physically significant reflections are generated by backward peaks associated a priori with stable-particle two-body subchannels. The reflections can imitate exotic exchange. Specifically, for a reaction of type $M^\pm B \to (M^\pm B) + M^\mp$, backward "scattering" in the final M^+M^- (meson-meson) rest frame produces reflections whose characteristics are very similar to reported[2] forbidden peripheral production of decuplet states (M^+B). The reflected effects are large. They also persist to fairly high energy, even though backward scattering in the M^+M^- system is appreciable only at quite small values of M^+M^- invariant mass. These features may explain why forbidden forward peaks are observed in decuplet production processes[2] (e.g., $\pi^-p \to K^+Y_1^*$) but not in similar reactions involving only stable particles (e.g., $\pi^-p \to K^+\Sigma^-$). Because of this added ambiguity, great care is required experimentally; perhaps the search for exotic exchange effects is best confined to reactions (such as $\pi^-p \to \Sigma^-K^+$ or $p\bar{p} \to \Sigma\bar{\Sigma}$) involving particles which are stable with respect to strong interactions. Implications for phenomenology, including $\pi\pi$ scattering, are discussed at the end of the paper.

The model is best illustrated by explicit examples which will be treated in turn:

$$\pi^+n \to \pi^+\pi^-p\ [\to \pi^-\Delta^{++}(1238)], \tag{1}$$

$$\pi^-p \to \pi^-K^+\Lambda\ [\to K^+Y^{*-}(1385)], \tag{2}$$

$$K^-p \to \pi^-K^+\Xi^0\ [\to K^+\Xi^{*-}(1530)]. \tag{3}$$

The pseudoexotic exchange processes are indicated in parentheses.

Prominent features of the data for Reaction (1) include peripheral production of the ρ^0 and f^0 as well as the Δ^{++}. Backward[5] production of Δ^{++} is presumably mediated by baryon exchange; one expects to observe a peak in the center-of-mass differential cross section near $\cos\theta^{\text{c.m.}} = -1$. However, after selecting events for which the invariant mass m_{π^+p} is in the $\Delta^{++}(1238)$ mass band, one observes a pronounced peak near $\cos\theta^{\text{c.m.}}=+1$, also. As will now be demonstrated, an enhancement in the π^+p mass spectrum near the Δ position as well as the peak near $\cos\theta^{\text{c.m.}}=+1$ result from decay of $\pi^+\pi^-$ states formed in the peripheral process $\pi^+n \to (\pi^+\pi^-)p$.

A qualitative argument will be given first. The diagram of Fig. 1(a) illustrates the effect. The $\pi^+\pi^-$ pair is typically produced with a wide spectrum of invariant masses, but with most of the events in the resonance region below ≈ 1.5 GeV. This peripherally produced $\pi\pi$ system, traveling forward with respect to the incident π^+ beam, decays with an angular distribution in its center-of-mass frame characteristic of the resonant and other low partial waves in the $\pi\pi$ spectrum. This distribution changes with $\pi\pi$ mass, but is generally peaked near both $z=-1$ and $z=+1$, or at least has significant intensity in the backward hemisphere.[6] With substantial probability, therefore, the decay π^+ may proceed backwards, i.e., in the direction of the p. The result is an enhancement near threshold in the π^+p system. Moreover, largely for kinematic reasons, the enhancement is associated with very small values of the square of the invariant momentum transfer, $t_{\pi^+\pi^-}$, from incident π^+ to final π^-.

The preceding argument may be made more quantitative. An approximate form for the absolute square of the relevant invariant amplitude [Fig. 1(b)] is

$$\sum |\mathfrak{M}|^2 = 2g^2 \frac{-t_{np}}{(t_{np}-m_\pi^{\ 2})^2} |A_{\pi^+\pi^-}|^2 \times \exp[\lambda(t_{np}-m_\pi^{\ 2})], \qquad (4)$$

$$|A_{\pi^+\pi^-}(s_{\pi^+\pi^-}, z)|^2 = (8\pi)^2 s_{\pi\pi}(d\sigma_{\pi\pi}/d\Omega). \qquad (5)$$

Below $s_{\pi\pi}=2.9$ GeV^2, the physical $\pi^+\pi^-$ elastic-scattering angular distribution $d\sigma_{\pi\pi}/d\Omega$ can be represented by actual, nonextrapolated data[7] restricted to values $|t_{np}| < 0.2$ GeV^2. Above 2.9 GeV^2, a smooth Regge-pole amplitude is employed; it is constructed using parameters for the P, P', and ρ trajectories obtained through factorization[8] and has no backward peak. The value of λ in Eq. (4) was determined to be 6.0 $(\text{GeV}/c)^{-2}$ by a fit to $d\sigma/dt$ for $\pi^-p\to\rho^0 n$. In Eq. (4) the pion-nucleon coupling constant is $g^2/4\pi = 14.5$; there are no free parameters.

Results are shown in Fig. 2; to facilitate comparison with the data,[2] the ρ mass band was excluded from the integrations. As will be noted, the decaying $\pi^+\pi^-$ system generates a sizable reflection near threshold in the π^+p mass spec-

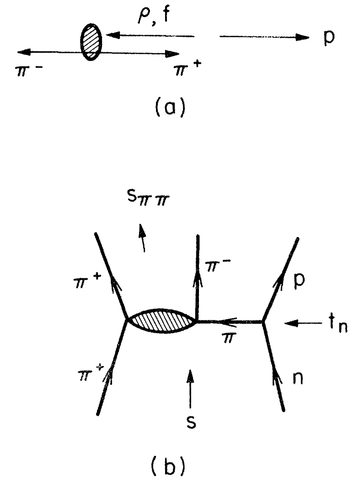

FIG. 1. Diagrams illustrating the process. In (a) only the final-state particles are shown; ρ and f represent, more generally, the peripherally produced $\pi\pi$ system. In (b), the "blob" stands for the complete $\pi\pi$ amplitude.

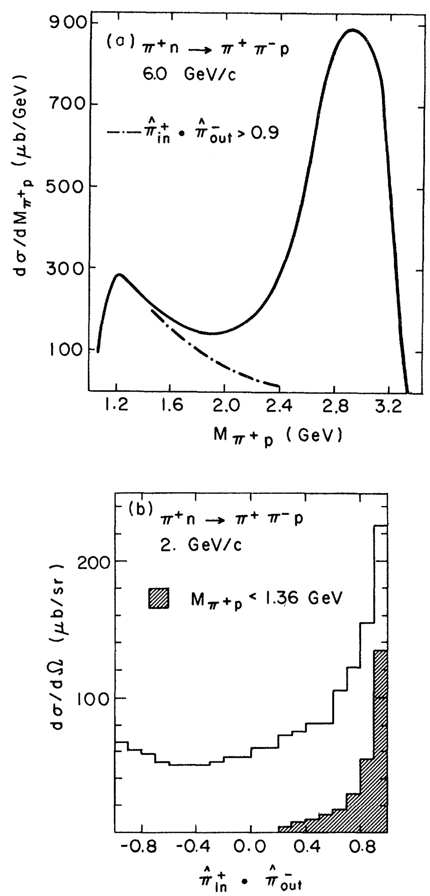

FIG. 2. Distributions obtained from Eq. (4) for (a) invariant mass of π^+p and (b) cosine of the π^+(in) to π^-(out) scattering angle in the overall c.m. frame. The ρ band $(0.64 < m_{\pi^+\pi^-} < 0.88)$ was excluded. Other cuts are indicated; $\hat{\pi}$ denotes the unit vector three momentum of a pion in the overall c.m. frame.

trum. Moreover, the effect is produced very peripherally: For $m_{\pi^+p} < 1.36$ GeV, $d\sigma/dt_{\pi^+\pi^-} \propto \exp(7t_{\pi^+\pi^-})$ at 2.0 GeV/c and $\propto \exp(11t_{\pi^+\pi^-})$ at 6.0 GeV/c. Because of the more restricted phase space, the two peaks shown in Fig. 2(a) are not resolved at 2.0 GeV/c; the near-threshold enhancement appears as a shoulder only. At 6.0 GeV/c, the center-of-mass production angular distribution is qualitatively similar to Fig. 2(b), but considerably more forward peaked, as indicated by the dashed line in Fig. 2(a).

A comparison with sizes of analogous peaks in reactions for which single-meson exchange is allowed demonstrates that the magnitude of the calculated effect is indeed large. For example, the observed forward peak in $d\sigma/d\Omega$ for $\pi^+p \to \pi^0\Delta^{++}$ is only a factor of 2-3 larger.[9] At 2, 4, 6, and 10 GeV/c, with $\Delta^{++} \equiv m_{\pi^+p} < 1.36$ GeV and ρ restored, $\sigma(\pi^+n \to \pi^-\Delta^{++})$ is 0.50, 0.18, 0.07, and 0.02 mb, respectively.

Except, perhaps, for the full width of the mass enhancement, these results are in agreement with the available data.[2,10] The width issue will be discussed after the next example is treated.

A complete treatment of Reaction (2) requires consideration of both pseudoscalar (K) and vector (K^*) exchange components, known to be present in peripheral production of the $K^*(890)$ and $K^*(1420)$.[11] For simplicity, only pseudoscalar is retained here; however, the effect obtained is qualitatively similar with K^* exchange included. For $\pi^-p \to (K^+\pi^-)\Lambda$, via K exchange,

$$\sum|\mathfrak{M}|^2 = g_\Lambda^2 \frac{(m_\Lambda - m_p)^2 - t_{p\Lambda}}{(t_{p\Lambda} - m_K^2)^2} |A_{K\pi}|^2 \times \exp[2(t_{p\Lambda} - m_K^2)\ln s]. \qquad (6)$$

The elastic $K^+\pi^-$ amplitude $A_{K\pi}$ was represented as a sum of Breit-Wigner expressions for the $K^*(890)$ and $K^*(1420)$, with spin $J = 1$ and 2, respectively, and width having energy dependence $(q/q_{\mathrm{res}})^{2J+1}$. In the calculations, the $Kp\Lambda$ coupling constant has been chosen to be $g_\Lambda^2/4\pi = 14$; other choices will scale the results proportionately. An additional t dependence of Regge type ($s^{2\alpha_K}$) has been included[12]; away from the dip at the forward direction, the differential cross section for Λ production integrated over all $K\pi$ masses, is $\propto \exp(4t_{p\Lambda})$.

The resulting peripheral enhancement in the $\pi^-\Lambda$ mass spectrum is shown in Fig. 3. Also illustrated is the direct association of the backward hemisphere in $\pi^-K^+ \to \pi^-K^+$ with low-mass structure in the $\pi^-\Lambda$ system. When the appropriate proportion of K^* exchange is included, the valley between the two peaks of Fig. 3(a) is filled in somewhat, and the low-mass peak moves to 1.5 GeV. For $m_{\pi\Lambda} = 1.4$ GeV, at 2, 6, and 10 GeV/c, $d\sigma/dt(\pi_{\mathrm{in}}^- \to K^+) \propto \exp(bt)$, with $b = 10$, 13, and 16 $(\mathrm{GeV}/c)^{-2}$, respectively. The slope b decreases as $m_{\pi\Lambda}$ is increased, a characteristic feature of doubly peripheral processes.[13]

Forward production of $Y_1^{*-}(1385)$ in kaon-induced reactions may be similarly explained. Consider $K^-p \to \pi^+\pi^-\Lambda$. Here also, an appreciable fraction of the peripherally produced $\pi^+\pi^-$ system will decay in such a manner that the π^- travels backwards and "catches up" with the Λ.

As they stand, the full widths of mass enhancements shown in Figs. 2(a) and 3(a) are too broad for direct identification with the $\Delta^{++}(1238)$ and $Y_1^*(1385)$, respectively. However, the curves do indicate a large probability for the π^+ and p

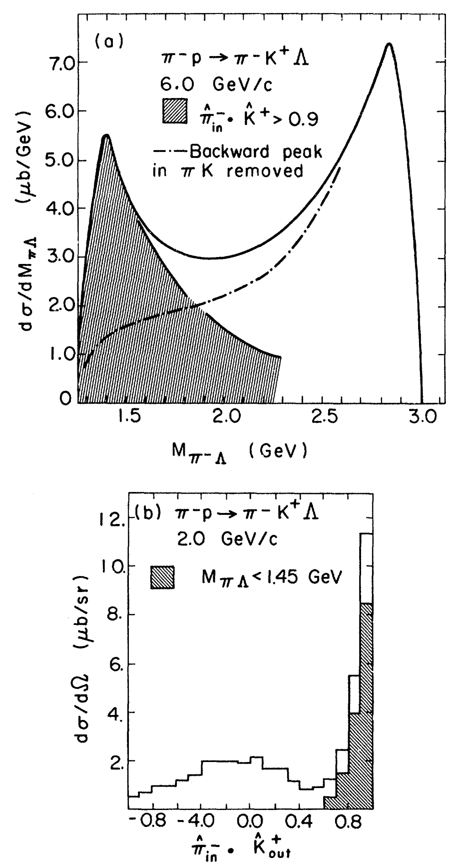

FIG. 3. Distributions calculated from Eq. (6). Cuts are indicated; the dot-dashed curve in (a) was obtained after setting $P_J(z) = P_J(0)$ for $z < 0$ in expressions for the $K^*(890)$ and $K^*(1420)$. Symbols $\hat{\pi}$ and $\hat{K}$ denote unit-vector three-momenta of π and K, respectively, in the overall c.m. frame.

(or π^- and Λ) to have relative momentum characteristic of these resonant states. Moreover, partial-wave analysis of the amplitudes in Eqs. (4) and (6) shows that at low invariant masses the p-wave component in the π^+p and $\pi^-\Lambda$ rest frames is substantial.[14] Inasmuch as strong resonant effects are known to be present in those partial waves, it appears that final-state rescattering effects will be important in reshaping final mass peaks. Indeed, the approximate effect of final-state interaction is to multiply the $J^P = \frac{3}{2}^+$ component of the amplitudes by a Breit-Wigner form.[15] Unfortunately, the multiplicative scale factor associated with final-state rescattering is not determinable unless explicit models are introduced; however, the usual result of an additional attractive interaction is to augment the effect.

The final-state mechanism may be invoked to explain production of Ξ^{*-} in Reaction (3). A two-step process is involved. First, in $K^-p \to K^+K^-\Lambda$, values of the $K^-\Lambda$ invariant mass near threshold are enhanced as a reflection of backward scattering at low energies in the K^-K^+ system. An established $\bar{K}\Lambda$ state above threshold is the $\Xi^*(1815)$, but exothermic rescattering may transform $K^-\Lambda \to \Xi^{*-}(1530) \to \pi^-\Xi^0$.

In addition to the question of exotic states, other consequences of reflections should be noted. (1) The analysis given here generally emphasizes again the need for better understanding of interference effects among competing two-body channels in a three-body final-state reaction. In fact, the discussion can be reversed: The backward peak for $\pi^+\pi^- \to \pi^+\pi^-$, extracted from data on $\pi^-p \to \pi^+\pi^-n$, for example, may be severely biased by the presence of peripherally produced N^{*-} states. This effect has important bearing for current research on $\pi\pi$ and $K\pi$ phase shifts. (2) In phenomenological analyses based on a multiperipheral approach, events are often assigned to specific diagrams based on an ordering of momentum transfer, longitudinal momenta, or related scattering-angle variables. When subenergy values are small, the analysis of this note demonstrates that these ordering schemes are likely to be misleading.[16] (3) Consider $\pi^+p \to \pi^+\pi^0p$. There are no exotic channels, and the peripheral quasi-two-body final state $\pi^0\Delta^{++}$ is presumably mediated by ρ exchange.[9] However, here again, the competing channel $\pi^+p \to (\pi^0\pi^+)p$ complicates the problem by generating a sizable reflection which looks like $\pi^0\Delta^{++}$, is peripheral, and must be treated coherently.

Finally, a comment is in order on the relationship of the approach outlined here to box-diagram or two-meson-exchange schemes.[17] In essence, once final-state interactions are included (e.g., to shape a final-state mass distribution), the topology of the overall scattering diagram is that of a box. In this paper, in the direct channel, for reactions $MB \to MMB$, the intermediate states of the box are B and (MM). An important distinction should be noted. Here, a complete spectrum of (MM) mass values is retained, whereas, usually only narrow resonant components are used. This has dramatic consequences for both overall magnitude and energy dependence of the effective box diagram. In particular, consider $\pi^+n \to \pi^-\Delta^{++}$, discussed earlier. The cross section for the two-pion-exchange box with $\rho\rho$ direct-channel intermediate state has energy dependence p_{lab}^{-4}. However, with the complete $\pi\pi$ spectrum retained (rather than just ρ), numerical values given earlier show that the rapid decrease in fact sets in very slowly, not reaching full strength until $p_{lab} \gtrsim 10$ GeV/c.

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[1]An exotic meson is one whose quantum numbers cannot be generated via the quark model as $(\bar{q}q)$. For a review of the experimental status, see A. H. Rosenfeld, in Meson Spectroscopy, edited by C. Baltay and A. H. Rosenfeld (W. A. Benjamin, Inc., New York, 1968), p. 455.

[2]M. A. Abolins et al., Phys. Rev. Letters 22, 427 (1969); P. M. Dauber et al., Phys. Letters 29B, 609 (1969); A. Abashian et al., in Proceedings of the International Conference on High Energy Physics, Lund, Sweden, 1969 (unpublished).

[3]R. Armenteros et al., Nucl. Phys. B8, 235 (1968).

[4]To substantiate the dual point of view, it will be important to establish that the peaks maintain their integrity for energies above the resonance region and that their decrease with energy is characterized by a power bahavior. Nonexotic interpretation in terms of cuts in the l plane (box diagrams, exchange of two ordinary states is also possible and is subject to similar energy-dependence criteria.

[5]Direction is specified by following the baryon through the reaction; in meson-meson processes, charge can be followed.

[6]In the final-state $\pi^-\pi^+$ rest frame, z is the cosine of the scattering angle from incident to final π^+. It is

not necessary that there be a backward peak, per se; the relative size of the reflection effect is inversely proportional to the ratio $(F-B)/(F+B)$, averaged over the low-energy $\pi\pi$ spectrum. For a compilation, see J. J. Veillet, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, Calif., 1967), p. 130.

[7]P. B. Johnson et al., Phys. Rev. 176, 1651 (1968); J. A. Poirier et al., Phys. Rev. 163, 1462 (1967); D. J. Crennell et al., Phys. Letters 28B, 136 (1968).

[8]E. L. Berger and G. C. Fox, Phys. Rev. (to be published). Factorization gives the parameters of the P and P'; exact exchange degeneracy of the ρ and P' was invoked to obtain the ρ parameters from those of the P'.

[9]R. D. Mathews, Nucl. Phys. B11, 339 (1969).

[10]Recall that the data do have backward Δ^{++} production produced by baryon exchange, in addition.

[11]D. J. Crennell et al., Brookhaven National Laboratory Report No. BNL-10462R, 1966 (unpublished); $\pi^-p\to K^*(890)\Lambda$ is in fact dominated by vector exchange at 6.0 GeV/c so that the angular distribution in the K^* rest frame is fairly flat. However, $\pi^-p\to K^*(1420)\Lambda$ shows strong forward and backward peaking in $\cos\theta$. The $K^*(1420)$ mass region is also more effective in generating the reflection effect discussed here.

[12]The Regge form is not essential. Note that it depends on total s, not on a subenergy variable. None of the effects discussed depend critically on its presence.

[13]E. L. Berger, Phys. Rev. 179, 1567 (1969); see Sec. V-A.

[14]The $l=1$ and higher components in the $\pi^-\Lambda$ system are also produced more peripherally than is the overall enhancement. For example, for $l=1$, $d\sigma/dt(\pi^-\to K^+)\propto \exp(20t)$ at 6-GeV/c lab momentum.

[15]J. D. Jackson, Nuovo Cimento 25, 1038 (1962).

[16]Uncritical application of the longitudinal momentum scheme by W. Ochs [Nucl. Phys. B10, 453 (1969)] indicated the need for exotic exchange.

[17]For a summary of the state of the art, see Proceedings of the 1969 Regge Cut Conference, Madison, Wisconsin, edited by P. Fishbane and L. Simmons (to be published).

ERRATA

VENEZIANO REPRESENTATION FOR CHARGED-PION PHOTOPRODUCTION. M. Ahmad, Fayyazuddin, and Riazuddin [Phys. Rev. Letters 23, 504 (1969)].

We have been informed by Dr. M. B. Halpern that a representation similar to ours has also been considered by R. C. Brower and M. B. Halpern [Phys. Rev. 182, 1779 (1969)].

PARAMETRIC CONVERSION OF X RAYS. Isaac Freund and B. F. Levine [Phys. Rev. Letters 23, 854 (1969)].

In the first paragraph, the third line, "···proton···" should read "···photon···." In Eq. (14), replace θ_B by $2\theta_B$. In Ref. 8 we did not cite the work of Byer and Harris correctly; the correct citation is Phys. Rev. 168, 1064 (1968).