of $\frac{3}{2}^{\pm}$, $\frac{5}{2}^{\pm}$, $\cdots$, resonances decaying into $\pi+N$ will not be contaminated by a Deck background. For $N+N\to N+(\pi\Delta)$ the Deck effect contributes dominantly to $\frac{1}{2}^+$ and $\frac{3}{2}^-$ $(\pi\Delta)$ states, allowing a clean determination of the $\frac{1}{2}^-$, $\frac{3}{2}^+$, $\cdots$, production.

There is also a theoretical complication in estimating the cross sections for high-mass resonances whose production is allowed by our model. For example, in $\pi+N\to\pi^*+N$, the cross sections for the production of the pion recurrences cannot all be equal, since their sum would soon exceed the "constant" total cross section. Presumably this is accomplished by a damping of the production of massive recurrences, though it is hard to estimate what rate of damping one should expect.

In N^* production,[10] we note that the dominant features are peaks at 1400 and 1690 MeV which, in our model, correspond to allowed $\frac{1}{2}^+$ and $\frac{5}{2}^+$ resonances (see Table I). Smaller peaks are also seen at 1520 and 2190 MeV corresponding to the SU(6)-forbidden $\frac{3}{2}^-$, $(\frac{1}{2}^-)$, and $\frac{7}{2}^-$ resonances. The size of the cross sections for production of these resonances is evidence for the degree of SU(6) breaking. No peaks have been seen which correspond to the other allowed states in Table I [N^*(1750, N^*(1860)]. However, these states have been seen only in phase-shift analyses and appear there with large widths.

In meson production, the ambiguities in the data and the previously mentioned difficulties of interpretation prevent a meaningful test of our rules at the present time.

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[4]R. Carlitz (to be published) has made an independent derivation of these rules in a more detailed model involving duality.

[5]Quark-model assignments are given in Table I. A. Donnachie, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968, edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 155.

[6]Had we used a different quark-model classification, including, for example, 56 odd-L states, then our results would not be the same.

[7]This follows from the general rule for production by natural-parity exchange at zero degrees.

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CABIBBO ANGLE AND SELF-CONSISTENCY CONDITION*

K. Tanaka
Research Institute for Theoretical Physics, University of Helsinki, Helsinki, Finland, and Department of Physics, Ohio State University, Columbus, Ohio

and

P. Tarjanne
Research Institute for Theoretical Physics, University of Helsinki, Helsinki, Finland

(Received 25 August 1969)

A solution of the c-number self-consistency condition $a_i - d_{ijk}a_j a_k = h_i$ is obtained, where a_i represents the matrix elements of unitary octet vector charge and h_i is a driving term. With a reasonable choice of a driving term, we obtain $\sin\theta = 0.22-0.14$ for the Cabibbo angle θ. The choice of h_i that leads to $\tan\theta = m_n/m_\Lambda$ (where m_n and m_Λ are the quark masses) is also pointed out.

It is conjectured that the weak and electromagnetic properties of hadrons are determined by self-consistency requirements and, in particular, that the Cabibbo angle is an inherently weak effect. A solution of the c-number equations ($i=1, 2, \cdots, 8$)

$$a_i - d_{ijk}a_j a_k = 0, \tag{1}$$

where a_l represents the matrix elements of the unitary octet vector charge, exists that depends on two parameters.[1] This suggests that the equations underdetermine the amplitudes so that more restrictive equations should be considered.

Equation (1) results from taking the one-particle matrix elements of the relation $Q_i = d_{ijk}Q_jQ_k$ (where Q_i is the vector charge) and saturating the intermediate states by one-particle states.[2] It is consistent with the commutation relation $[Q_i, Q_l] = if_{ilm}Q_m$ as can be seen from

$$[Q_i, Q_l] = d_{ijk}\{Q_j[Q_k, Q_l] + [Q_j, Q_l]Q_k\} = id_{ijk}(f_{klm}Q_jQ_m + f_{jlm}Q_mQ_k) = if_{ilm}Q_m. \tag{2}$$

When Eq. (2) is multiplied by f_{ilp} and summed over i,l, we obtain $Q_l = d_{ljk}Q_jQ_k$ and subsequently Eq. (1).

We consider modifying Eq. (1) by a real c-number driving term[3] h_i,

$$a_i - d_{ijk}a_ja_k \equiv a_i - (a\times a)_i = h_i. \tag{3}$$

The h_i represents a matrix element of some other charge. Physically, it can arise from the symmetry breaking of the strong interaction (in which case θ is no longer an inherently weak effect) or some other effect such as violation of a discrete symmetry. We obtain from (3)

$$(h\times h)_i \equiv d_{ijk}h_jh_k = d_{ijk}[a_j - (a\times a)_j][a_k - (a\times a)_k] = \tfrac{2}{3}[I_3(a) - I_2(a)]a_i + [1 - \tfrac{1}{3}I_2(a)](a\times a)_i, \tag{4}$$

where

$$I_2(a) = a_ia_i, \quad I_3(a) = (a\times a)_ia_i.$$

A solution of (3) follows[4] from eliminating $(a\times a)_i$ from (3) and (4),

$$a_i = \{(h\times h)_i + [1 - \tfrac{1}{3}I_2(a)]h_i\}/I(a), \tag{5}$$

where

$$I(a) = 1 - I_2(a) + \tfrac{2}{3}I_3(a).$$

Let us, for simplicity, choose the driving term such that

$$h_3 = h_6 = h_7 = h_8 = 0, \tag{6}$$

and further envisage a symmetry-breaking Hamiltonian of the current-current form $H_i = d_{ijk}J_jJ_k$ so that[5,6]

$$c_i = d_{ijk}h_jh_k. \tag{7}$$

When (6) and (7) are substituted into (5), one obtains

$$a_i = h_i[1 - \tfrac{1}{3}I_2(a)]/I(a), \quad i = 1, 2, 4, 5 \tag{8}$$

in which a_i is proportional to h_i. The Cabibbo angle θ is then given with the aid of (8) as

$$\tan^2\theta = \frac{a_4 + ia_5}{a_1 + ia_2}\frac{a_4 - ia_5}{a_l - ia_2} = (h^2{}_4 + h^2{}_5)/(h^2{}_1 + h^2{}_2). \tag{9}$$

Solving Eq. (7) in terms of c_i $(i = 3, 8)$, we get

$$h^2{}_1 + h^2{}_2 = c_3 + \sqrt{3}c_8, \quad h^2{}_4 + h^2{}_5 = 2c_3, \tag{10}$$

which after combining with (9) yields[7]

$$\tan^2\theta = 2c_3/(c_3 + \sqrt{3}c_8) = 0.052 - 0.020,$$

or

$$\sin\theta = 0.22 - 0.14. \tag{11}$$

It is amusing to note that if one chooses h_i so that

$$h_ih_i = d_{iik}c_k - (1/\sqrt{6})c_0,$$

and substitutes it into (9), one obtains the well-known result[8]

$$\tan^2\theta = \frac{(\frac{2}{3})^{1/2}c_0 + (1/\sqrt{3})c_8 - c_3}{(\frac{2}{3})^{1/2}c_0 - (2/\sqrt{3})c_8} = \frac{m_n}{m_\Lambda} = 0.053, \tag{12}$$

where m_n and m_Λ are the masses of the n and Λ quarks, respectively.

The relation (10) does not give the electromagnetic condition $a_3 = \sqrt{3}a_8$ but this relation can be satisfied by the introduction of the matrix elements of the axial-vector charges b_i in addition to a_i.[1]

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EXOTIC EXCHANGE OR KINEMATICAL REFLECTION?*

Edmond L. Berger†
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720
(Received 11 August 1969)

Recently observed peripheral peaks in reactions where single-meson exchange is forbidden are explained as kinematical reflections. Experimental and theoretical implications are discussed.

A notable feature of strong interactions is the apparent absence of exotic meson resonances.[1] Recently, however, peripheral forward peaks have been observed in differential cross sections of certain quasi-two-body reactions for which ordinary single-meson exchange is forbidden.[2] Interpreted naively, these peaks suggest a dynamics involving exchange of meson states with exotic quantum numbers, $I \geq \frac{3}{2}$ or $|S|=2$. In this note, a simple alternative explanation is developed. Peripheral peaks of the type observed are shown to be generated as reflections from competing allowed processes. Calculated magnitudes and shapes are consistent with available data.

Distinctive backward peaks are present in certain stable-particle two-body ($n=2$) reactions for which the backward exchange (u) channel is exotic. Examples are $\pi^+\pi^- \to \pi^+\pi^-$ and $K^-p \to \bar{K}^0 n$.[3] However, because the peaks have been observed to date only at very low energy, they are often described simply as manifestations of prominent resonances in the direct channel. It is not yet clear whether a dual interpretation is tenable in terms of exchange of exotic single-particle states.[4] The present paper is concerned with possibly exotic exchange phenomena in multiparticle ($n>2$) data. The point emphasized here is that physically significant reflections are generated by backward peaks associated a priori with stable-particle two-body subchannels. The reflections can imitate exotic exchange. Specifically, for a reaction of type $M^\pm B \to (M^\pm B) + M^\mp$, backward "scattering" in the final M^+M^- (meson-meson) rest frame produces reflections whose characteristics are very similar to reported[2] forbidden peripheral production of decuplet states (M^+B). The reflected effects are large. They also persist to fairly high energy, even though backward scattering in the M^+M^- system is appreciable only at quite small values of M^+M^- invariant mass. These features may explain why forbidden forward peaks are observed in decuplet production processes[2] (e.g., $\pi^-p \to K^+Y_1^*$) but not in similar reactions involving only stable particles (e.g., $\pi^-p \to K^+\Sigma^-$). Because of this added ambiguity, great care is required experimentally; perhaps the search for exotic exchange effects is best confined to reactions (such as $\pi^-p \to \Sigma^-K^+$ or $p\bar{p} \to \Sigma\bar{\Sigma}$) involving particles which are stable with respect to strong interactions. Implications for phenomenology, including $\pi\pi$ scattering, are discussed at the end of the paper.

The model is best illustrated by explicit examples which will be treated in turn:

$$\pi^+n \to \pi^+\pi^-p\ [\to \pi^-\Delta^{++}(1238)], \quad (1)$$

$$\pi^-p \to \pi^-K^+\Lambda\ [\to K^+Y^{*-}(1385)], \quad (2)$$

$$K^-p \to \pi^-K^+\Xi^0\ [\to K^+\Xi^{*-}(1530)]. \quad (3)$$

The pseudoexotic exchange processes are indicated in parentheses.

Prominent features of the data for Reaction (1) include peripheral production of the ρ^0 and f^0 as well as the Δ^{++}. Backward[5] production of Δ^{++} is presumably mediated by baryon exchange; one expects to observe a peak in the center-of-mass differential cross section near $\cos\theta^{\text{c.m.}} = -1$. However, after selecting events for which the invariant mass m_{π^+p} is in the $\Delta^{++}(1238)$ mass