ELECTRIC FIELDS IN ROTATING, MAGNETIC, RELATIVISTIC STARS*

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All the current models for pulsars call for very large magnetic fields in rotating neutron stars. Since general relativistic effects are important in the latter, electromagnetism too must be framed consistently (which has been overlooked so far). It is shown that in a rotating neutron star even a uniform magnetic field, static in the corotating frame, implies in the same frame the static electric field which is crucial to the pulsar emission theories.

Pulsars are now believed to be rotating neutron stars,[1] whose pulses result from a corotating beacon which is regarded in different ways by different authors. A critical review of the models presented to date and an extensive bibliography are given by Chiu.[2] The basic ideas contained therein are accepted here. The emission is believed to take place on the very surface of the neutron star, a point of view which is supported by results based on the estimation of the light curves.[3] Consequently, the magnetic laser device[4-6] appears to be a most likely candidate – indeed the only one available today – for the highly nonthermal emission required (if a reasonable superficial magnetic field of 10^{12} G is allowed).

So far electric fields in pulsars have been derived from a Lorentz transformation, by justifying a relative motion between matter and magnetic field (see also the unipolar induction device of Goldreich and Julian[7]). It must be noted, however, that the built-in general relativistic conditions already imply, in a straightforward way from Maxwell's equations, the existence of an electric field. This Letter presents only a qualitative and preliminary investigation; the reliability of some of the approximations used is even more evident in quantitative treatments of the problem.[8,9]

Maxwell's equations in general relativity have been studied extensively. In the absence of electric charges and currents, they are

$$F^{\alpha\beta}{}_{;\beta}=0 \quad (\alpha, \beta=0, 1, 2, 3), \tag{1}$$

$$F_{\alpha\beta}=A_{\alpha;\beta}-A_{\beta;\alpha}\ (=A_{\alpha,\beta}-A_{\beta,\alpha}), \tag{2}$$

where A_α is interpreted as the ordinary four potential in the locally orthogonal frame:

$$A_\alpha{}^{(0)}=(\psi, -A_x, -A_y, -A_z). \tag{3}$$

Commas stand for ordinary differentiation, semicolons for covariant differentiation. According to a standard procedure[10-12] one derives from (1) and (2) that

$$\Box A^\alpha=R^{\alpha\beta}A_\beta,$$

where $R^{\alpha\beta}$ is the Ricci tensor, and

$$\Box A^\alpha=A^{\alpha;\beta}{}_{;\beta}, \tag{4}$$

and the gauge

$$A^\beta{}_{;\beta}=0 \tag{5}$$

is imposed. The convenience of the contravariant position of the index α in (4) (the direct physical meaning lies in A_α) will be evident in the sequel. Furthermore, for the sake of definiteness, the adopted sign conventions in the Riemann-Christoffel and Ricci tensors are recalled:

$$A^\alpha{}_{;\mu;\nu}-A^\alpha{}_{;\nu;\mu}=-R^\alpha{}_{\beta\mu\nu}A^\beta, \quad R_{\alpha\beta}=R^\mu{}_{\alpha\mu\beta}.$$

Equations (4) may imply that every component of the potential acts as a source for all others when $R^{\alpha\beta}\neq 0$, which is the case for neutron stars and pulsars. (It must be emphasized that in these instances space-time is strongly curved because of the high density of matter, whereas the contribution from the electromagnetic field is entirely negligible.) In particular, for $\alpha=0$ in (4), in the presence of rotation and of a magnetic field, there follows the existence of an electric field in the corotating frame of reference. This is a general relativistic effect: By substituting for $R^{\alpha\beta}$ in terms of Einstein's equations, the right-hand side of (4) turns out to be of the order of magnitude of (R_S/R), where $R_S=2GM/c^2$ is the Schwarzchild radius and R is the radius of the star. Indeed, even a post-Newtonian approximation will suffice to clarify the matter.

Equations (4), being fully covariant, can be studied equally well either in the fixed frame of reference or in the corotating one.[8,9] Let polar coordinates be introduced,

$$x^0=ct,\ x^1=r,\ x^2=\theta,\ x^3=\varphi,$$

and let the corotating frame (angular velocity Ω) be denoted by primed variables. The transformation,

$$t'=t,\ r'=r,\ \theta'=\theta,\ \varphi'=\varphi+\Omega t$$

implies

$$A^0=A'^0,\ A_3=A'_3. \tag{6}$$

In addition let there be, in the corotating frame only, a pure static magnetic field and, for simplicity, let the latter be uniform and parallel to the rotation axis (at least inside the star), so that there is no radiation. Then

$$A'_3=-\tfrac{1}{2}Br^2\sin^2\theta,\ A'_1=A'_2=0, \tag{7}$$

where B is the magnitude of the field. Accordingly, the only nonvanishing components of $F'_{\alpha\beta}$ are

$$F'_{23}=Br^2\sin\theta\cos\theta,\ F'_{31}=-Br\sin^2\theta \tag{8}$$

which imply the expected physical components

$$B'_{(r)}=F'_{23}/(r^2\sin\theta)=B\cos\theta,$$
$$B'_{(\theta)}=F'_{31}/(r\sin\theta)=-B\sin\theta. \tag{9}$$

That there is no ambiguity in the choice (7) is made clear by evaluating the T'^0_0 component of the electromagnetic energy-momentum tensor,

$$T'^{\beta}_{\alpha}=4\pi^{-1}[F'_{\alpha\sigma}F'^{\sigma\beta}+\tfrac{1}{4}\delta^{\beta}_{\alpha}F'_{\sigma\tau}F'^{\sigma\tau}].$$

The latter indeed turns out to be $B^2/8\pi$. [Strictly speaking, the field (9) is not the self-consistent magnetic field, for general relativistic corrections already appear in the empty, nonrotating case.[13] However, as one is interested here primarily in induced electric fields and not in self-consistent magnetic fields, Eqs. (7) may be used. Furthermore, in the present post-Newtonian approximation only the zero-order flat-space magnetic field is needed.]

Now, only in the local orthogonal frame is the definition of the electric potential unambiguous and meaningful (3). There

$$\psi=A_0^{(O)}=A^{0(O)}=A'^0=A^0, \tag{10}$$

where use has been made of the first of (6) in the last equality and, in the preceding, of the fact that the value of any vector is not changed[14] locally by the transformation

$$x^{\alpha(O)}=x^{\alpha}+\left\{ {\alpha \atop \mu\nu} \right\}^{(O)}x^{\mu}x^{\nu}.$$

Thus for $\alpha=0$, because of (10), Eq. (4) yields directly the physical electric potential ψ. One has

$$\Box\psi=R^{00}g_{00}A^0+R^{03}A_3, \tag{11}$$

where the flat-space metric has been used to lower the index in A^0. (In general however, g_{03} is different from zero.[15-17])

To solve Eq. (11), the self-regeneration of A^0 —i.e., the first term on the right-hand side—may be neglected, since it is of the second order in (R_s/R). (A^0 is already of first order and the Ricci tensor introduces another factor R_s/R.) Also, the largest contribution to R^{03} comes from the rest-mass energy density ρc^2, which allows Eq. (11) to be written, remembering the second of (6), as

$$\Box\psi=(8\pi G/c^2)\rho(\Omega/c)A'_3. \tag{12}$$

Then, because of the assumed time independence and the choice of the fixed frame,[8] $\Box\psi=-\nabla_{\psi}{}^2 \cong -\psi/R^2$, and one has an electric field

$$E=-\psi/R\cong(\Omega R_s/c)B. \tag{13}$$

The fact that the latter is actually due to general relativity is made clear by the explicit occurrence of R_s. There is no relative motion between matter and magnetic field.

For a typical pulsar, $\Omega\cong 10\ \mathrm{sec}^{-1}$, $R_s\cong 3$ km,

$$E\cong 10^{-4}B\ (\mathrm{cgs})=3\times10^{10}B/10^{12}\ \mathrm{V/cm}. \tag{14}$$

In the theory developed by Chiu and co-workers,[2,4-6] the electric field component parallel to the magnetic field will cause electrons and ions to be accelerated into opposite directions in coherent beams of high energy. Upon collisions the quantized electrons will be excited from the lowest to higher magnetic states. A population inversion and a laser-type continuum radiation will result. While the general problem is currently under investigation, it is remarked that the general relativistic electric field discussed here is a direct coupling between rotation and the emission from pulsars.

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PRECISION MEASUREMENT OF THE g FACTOR OF THE FREE POSITRON*

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We report a new measurement of the positron g factor. Our result is five times more accurate than the best previous value for this quantity. It shows that the positron and electron g factors are identical to within one part per million.

We have just completed a precision measurement of the positron g factor. With the g factor written in terms of the anomaly a, as $g=2(1+a)$, our result is $a(e^+)_E=(11\,602\pm 11)\times 10^{-7}$. This is five times more accurate than the best previous positron determination.[1] The corresponding value of g agrees, at the 1-ppm level, with both the theoretical (T) value[2] and the previously measured experimental (E) value[3] for the free electron. In terms of a, $a(e^+)_T=a(e^-)_T=0.5(\alpha/\pi)-0.328\,48(\alpha/\pi)^2+1.6(\alpha/\pi)^3=(1\,159\,660\pm 2)\times 10^{-9}$ and $a(e^-)_E=(1\,159\,549\pm 30)\times 10^{-9}$. The error in a_T is due only to error in α which we take as $\alpha^{-1}=137.0361\pm 0.000\,26$. Possible error in the coefficient of $(\alpha/\pi)^3$ is not considered here.

The basic experimental technique is the same as that used in the last Michigan experiment, hereafter $g(e^+)_\text{I}$. A group of positrons from a Co^{58} source is confined in a magnetic mirror trap. After a measured length of time the particles are ejected from the trap into a polarimeter. On emission from the Co^{58} source, the positron "beam" is already polarized parallel to its "average" velocity, $\langle\vec{V}\rangle$, with polarization $\vec{P}=\langle\vec{V}\rangle/c$. While in the field the beam undergoes cyclotron orbital motion at angular frequency ω_C. Simultaneously $\vec{P}$ precesses at an angular frequency ω_S. Since these two frequencies are slightly different, $\vec{P}$ rotates about $\vec{v}$ at the beat or difference frequency $\omega_D=\omega_S-\omega_C$. If we neglect certain small corrections (to be discussed later), we may write

$$a=(m_0c/eB)\omega_D. \tag{1}$$

To determine a we must measure ω_D and B. The constant $(e/m_0c)_{e^+}$ is known to sufficient accuracy.

One detects ω_D by the use of a polarimeter, a polarimeter being any device which has a linear response to the projection of $\vec{P}$ onto a fixed direction, $\hat{h}$, in the laboratory frame. If the beam always enters the polarimeter in exactly the same direction, independent of trapping time, this projection differs by only a constant phase angle from $\vec{P}\cdot\vec{v}/v$. It is proportional to $\cos(\omega_DT+\varphi)$. Here T is the time the particles are trapped in the field and φ is a phase constant. In order to obtain ω_D we measure the output of the polarimeter as a function of T. The data are then fitted by a sinusoid from which ω_D may be in-