has observed that the application of a correction calculated by Chen for nuclear polarization restores the skin thickness to more normal values ($c = 5.75 \pm 0.03$ F, $t = 2.38 \pm 0.08$ F). This application of polarization corrections to muonic x-ray data is consistent with the results of recent work by Anderson et al.[8] on Pb^{206} where "an effect tentatively interpreted as due to the polarization of the nucleus by the muon" was reported.

The close analogy between low-energy electron scattering and muonic x-ray measurements suggests that dispersion corrections should be applied to our data. Previous estimates[9-12] of the dispersion correction as it applied to electron scattering have indicated that the effect should be small at our incident energies. However, inherent in these calculations are drastic assumptions concerning the distribution of electromagnetic strengths; in view of our results, a need for further calculations in indicated.

The authors wish to express their gratitude to C. S. Wu, E. R. Macagno, and Min-Yi Chen for their timely discussions; and to H. A. Bentz for a check run with the Bühring code.

†Work supported by the U. S. Atomic Energy Commission under Contract No. AT(30-1)2726 with Yale University.

[1] L. R. B. Elton, Nuclear Sizes (Oxford University Press, New York, 1961).

[2] R. A. Eisenstein, thesis, Yale University, 1968 (unpublished).

[3] H. A. Bentz, R. Engfer, and W. Bühring, Nucl. Phys. A101, 527 (1967).

[4] D. W. Madsen, Yale Electron Accelerator Internal Report No. EAL 2726-E-71 (unpublished).

[5] G. H. Rawitscher and C. R. Fischer, Phys. Rev. 122, 1330 (1961); C. R. Fischer and G. H. Rawitscher, Phys. Rev. 135, B377 (1964).

[6] T. T. Bardin, R. C. Barrett, R. C. Cohen, S. Devons, D. Hitlin, E. Macagno, C. Nissim-Sabat, J. Rainwater, K. Runge, and C. S. Wu, Phys. Rev. Letters 16, 718 (1966).

[7] E. R. Macagno and M.-Y. Chen, private communication.

[8] H. L. Anderson, C. K. Hargrove, E. P. Hincks, J. D. McAndrew, R. J. McKee, and D. Kessler, Phys. Rev. Letters 22, 221 (1969).

[9] L. I. Schiff, Phys. Rev. 98, 756 (1955).

[10] G. H. Rawitscher, Phys. Rev. 151, 846 (1966).

[11] C. Toepffer, Phys. Letters 26B, 426 (1968).

[12] D. S. Onley, Nucl. Phys. A118, 436 (1968).

OPTICAL-MODEL ANALYSIS OF ALPHA-PARTICLE SCATTERING FROM Mg^{24} †

P. P. Singh, R. E. Malmin, M. High, and D. W. Devins
Indiana University, Bloomington, Indiana 47401
(Received 13 October 1969)

Optical-model analysis of elastic-scattering data taken at 40 MeV produces the familiar potential ambiguities, but at 80 MeV a single set is obtained. The parameters V, W, r_0, and a are found to be highly correlated and hence cannot be uniquely determined. The functions which are determined most uniquely are the rms radius of the real and imaginary potential and the products Vr_{0R}^4 and Wr_{0I}^4.

The success of the optical model[1] in explaining the scattering of nucleons of 10-MeV energy or higher from nuclei of all masses has prompted[2] the application of optical-model techniques in the analysis of alpha-particle scattering. Optical potentials providing a good fit to scattering data would give physical information about the form of the alpha-nucleus interaction and would be useful in distorted-wave and coupled-channel analyses of inelastic alpha-particle scattering. However, such optical-model analyses of the alpha-particle elastic-scattering data generally lead to a number of families of parameters which give fits of similar quality. These ambiguities are found[2] to be either continuous, where a small change in the value of one parameter is compensated by small changes in the values of the others, or discrete, in which families of parameters correspond to different numbers of half-wavelengths of the alpha-particle wave function included within the nuclear potential well. The continuous ambiguities can arise from several causes: The optical model may not be able to account adequately for the nuclear interaction embodied in the data (as one would expect at lower energies), or measurements may not have been made over a sufficiently large angular range; finally, the model may be overparametrized. One can circumvent these difficulties by studying the scattering at progressively higher energies, where there should be no difficulty in justifying the application of the optical model, and by extending the measurements over a large angular range, especially those in the backward hemisphere. However, measurements become more difficult at higher energies because of both reduced cross sections and narrowly spaced diffraction maxima and minima.

Here we report the results of a study of elastic alpha-particle scattering from Mg^{24} at 40 and 80 MeV in which the motivation was to investigate the possibility of obtaining a unique set of optical-model parameters at these energies.

Measurements were made using the Oak Ridge isochronous cyclotron (ORIC). The targets used were self-supporting foils of Mg^{24} from 350 to 2040 $\mu g/cm^2$ in thickness. The scattered alpha particles were detected by an array of two to four lithium-drifted silicon detectors, mounted in the ORIC 30-in. scattering chamber, with an overall resolution of about 200 keV. Measurements were made in 2° steps from 20° to 170° and from 21.5° to 173.5° at 40- and 80-MeV bombarding energies, respectively. The experimental resolution was adequate to resolve the elastic peak in the pulse-height spectrum from the inealstic and contaminant peaks.

The observed cross sections were analyzed in terms of a Woods-Saxon optical potential of the form

$$U(r) = -V(e^x+1)^{-1} - iW(e^{x'}+1)^{-1},$$

where $x=(r-r_{0R}A^{1/3})/a_R$ and $x'=(r-r_{0I}A^{1/3})/a_I$, along with the Coulomb potential of a uniformly charged sphere of radius $r_cA^{1/3}$. The Coulomb radius r_c was set equal to 1.3 fm.

The data were fitted by varying all or some of the six parameters V, W, r_{0R}, r_{0I}, a_R, and a_I of the optical potential in order to minimize χ^2, the mean-square deviation between the experimental and the calculated cross sections. The computer code OPTICAL,[3] as modified and adapted for use on the Indiana Research Computing Center's CDC 3400/3600 computers by G. T. Eckley, was employed for this purpose.

To avoid the ambiguities that could be introduced by overparametrization the initial searches were limited to four parameters,[4] V, W, r_0, and a, by setting the radius and diffuseness for the imaginary potential to be equal to their real counterparts. To trace various possible discrete families of the parameters the searches were initialized by choosing values of V from 5 to 200 MeV in 5-MeV steps.[5] These 40 searches for the 40-MeV data converged to four distinct minima in χ^2 characterized by real depths approximately of 40, 80, 150, and 200 MeV. Figure 1, *A* shows the fit with the 150-MeV potential; other sets gave fits of similar quality. For each case the fit is quite good for forward angles (<90°), but at backward angles only the qualitative features of the data are reproduced. Allowing all six parameters to vary independently also resulted in four sets with similar parameters but without significant improvement (see Fig. 1, *B*) in the overall fit.

A similar optical-model analysis of the 80-MeV data resulted in only one acceptable minimum[6] in the χ^2 surface, with the following parameters:

$$V=92.0 \text{ MeV}, \quad W=47.9 \text{ MeV},$$
$$r_0=1.40 \text{ fm}, \quad a=0.709 \text{ fm},$$
$$\chi_{\min}^2=9. \tag{1}$$

The fit obtained with this set is shown in Fig. 1, *C*. Note the excellent quality of the fit over the

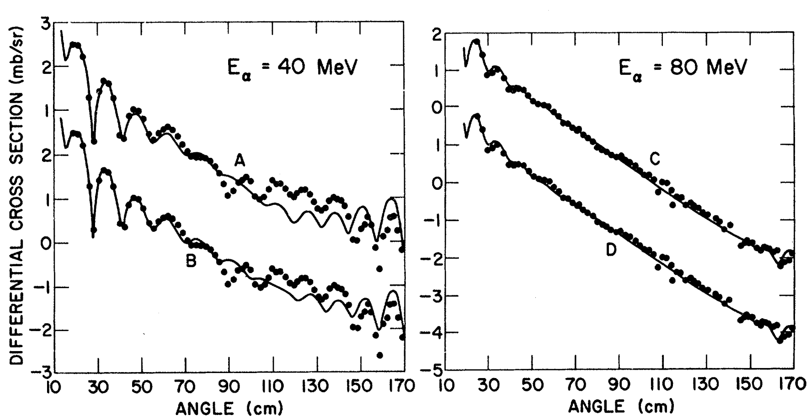

FIG. 1. Differential cross sections versus the center-of-mass scattering angle. The solid circles represent the experimental points and the curves are the best-fit calculations with optical model. *A*: $V=151.6$ MeV, $W=33.9$ MeV, $r_0=1.39$ fm, and $a=0.620$ fm. *B*: $V=125.3$ MeV, $r_{0R}=1.55$ fm, $a_R=0.542$ fm, $W=30.7$ MeV, $r_{0I}=1.59$ fm, and $a_I=0.393$ fm. *C*: $V=92.0$ MeV, $W=47.9$ MeV, $a_I=a_R=0.709$ fm, and $r_{0I}=r_{0R}=1.40$ fm. *D*: $V=120$ MeV, $r_{0R}=1.290$ fm, $a_R=0.754$ fm, $W=47.9$ MeV, $r_{0I}=1.40$ fm, and $a_I=0.709$ fm.

whole angular range.

Six-parameter searches were also tried but they invariably led to a few additional χ^2 minima. However, all of these, except one, had χ^2 which were a factor of 13 to 30 larger than that of set (1). The exceptional case returned values of parameters and χ^2 which were similar to that of set (1). Thus it seems that the four parameters may be more than adequate to account for the data and the additional solutions found with six parameters are due to overparametrization.

Calculations with only forward-angle data (<70°) led to four families of parameters, about 50 MeV apart in real depth. Hence, it appears that the inclusion of data from the backward hemisphere plays a critical role in removing the discrete ambiguities in the real potential strength.

Concurrent with this work, Reed et al.[7] have also studied alpha scattering from Mg^{24} at 81.0 MeV from 8.9° to 113.9° in the lab. Our cross sections agree relative to theirs within 1% and within 20% on absolute scale over the angular range of overlap between the two studies. To test the effect of extreme forward-angle data on the analysis, their 8.9° to 20° data were joined on to our data and the optical-model search was repeated. Again one minimum was found with parameters similar to those of set (1). Apparently, the forward-angle data has a smaller effect in determining the optical-model parameters than large-angle data.

In order to find specific correlations which may exist among the real and imaginary well parameters (these correlations are responsible for the observed continuous ambiguities), searches were performed in which a given parameter was fixed to arbitrary values over a wide range and one or more of the other parameters were selected to vary for best fit. In these studies the parameters for which correlations were not under investigation were fixed to values given in (1). Fits were considered acceptable if the χ^2 was less than or equal to twice the minimum value of χ^2 obtained with the parameters of set (1). Results of these calculations are summarized below.

(1) Three-parameter correlations: For any value of V in the range of 40 to 150 MeV, values of r_{0R} and a_R could be found to give acceptable fits (see Fig. 1, D). In this case $W = 47.9$ MeV; $r_{0I} = 1.40$ fm and $a = 0.709$ fm. However, these values of r_{0R} and a_R were such that the product $Vr_{0R}{}^n$ and the root-mean-square radius remained constant. The exponent n is found to be 4.0 ± 0.2 and the rms radius equal to $4.07^{+0.15}_{-0.1}$ fm. Similarly for any value of W in the range of 25 to 200 MeV, acceptable fits could be obtained with appropriate values of r_{0I} and a_I which conserved the rms radius (4.00 ± 0.3) of the complex potential and the product $Wr_{0I}{}^n$ ($n = 3.8 \pm 0.2$).

(2) Two-parameter correlations: For fixed a_R (e.g., 0.709 fm) appropriate values of r_{0R} could lead to acceptable fits for any V between 75 and 115 MeV. Over this range the product $Vr_{0R}{}^4$ and the rms radius remained constant within 2.5 and 3.8%, respectively. For fixed r_{0R}, however, only for an 87- to 96-MeV range of V were acceptable fits obtained by searching on a_R. Quantities conserved in these calculations are $Va_R{}^m$ ($m = 1.8 \pm 0.1$) and rms radius (4.14 ± 0.06 fm). Only a 2% change in the radius parameter, keeping V fixed, could be compensated by the diffuseness parameter while still maintaining the fit within acceptable limits.

Similar two-parameter correlations were found among the imaginary parameters. For fixed a_I over a range of W from 30 to 100 MeV appropriate values of r_{0I} could give acceptable fits. $Wr_{0I}{}^m$ ($m = 4.6 \pm 0.3$) and the rms radius (4.14 ± 0.38 fm) were again found to be conserved. Keeping r_I fixed, values of a_I could be found for arbitrary values of W in the range of 37 to 70 MeV which gave fits within acceptable limits. The quantity $Wa_I{}^m$, with $m = 1.8 \pm 0.1$, and rms radius $= 4.14 \pm 0.35$ fm, remain constant over the above limits. Just as for the real well potential, only very small changes (~2%) in r_{0I} could be compensated by a_I. It appears that the correlations among the imaginary parameters are not as strong as those found for their real counterparts.

In four-parameter searches, i.e., with $a_I = a_R$ and $r_{0I} = r_{0R}$, it was found that for any arbitrary value of V, in the range 40 to 150 MeV, acceptable fits were obtained for values of W which were a factor of 1.9 ± 0.1 smaller than the value of V.

Thus it seems that rms radius is one property of the nuclear potential which is unambiguously determined. A similar result has been found for nucleon scattering by Greenlees, Pyle, and Tang,[8] and for the He^3 scattering by Luetzelschwab and Hafele.[9] The constancy of the products $Vr_{0R}{}^4$ and $Wr_{0I}{}^4$ seems to be unique to α scattering; however, its significance is not completely understood as yet. It cannot be explained in the same terms as the $Vr_0{}^2$ ambiguity. It may, however, signify that besides the normalized second moment, equivalent to the $\langle r^2 \rangle$, the product of the potential strength and fourth normalized moment is conserved. Note that the dominant term in the fourth moment[10] (normalized) would be proportional to r_0^4. The fact that the parameters of the real or that of the imaginary potentials could be varied independently over a wide range (for a fixed set of values of the other) further supports the above indication that perhaps it is a few characteristic quantities, such as $\langle r^2 \rangle$ or Vr_0^4, which determine the effect of the potential on scattering rather than the details of the form factor.

An analysis of 80-MeV elastic alpha-particle scattering data obtained by Reed et al.[7] from Ne^{20} yields results similar to those for Mg^{24}. The rms radius of Ne^{20} is found to be 3.87 fm. It is interesting to note that the ratio of the rms radii for Ne^{20} and Mg^{24} is equal to the ratio of the cube roots of their atomic numbers within 2%. The quantities $Vr_{0R}{}^n$ and $Wr_{0I}{}^n$ were also found to be constant for the Ne^{20} case with $n = 4.4 \pm 0.4$ over ranges of V and W from 40 to 180 MeV and from 40 to 200 MeV, respectively. A similar analysis of the 119.7-MeV Mg^{24} data of Reed et al.[7] also yielded values of rms radius and the exponent n consistent with those reported above.

In conclusion, it appears from this study that at some high incident energy the optical model with four parameters can account remarkably well for the elastic alpha-scattering data from medium-mass nuclei over the complete angular range. The significant quantities that can be extracted without much ambiguity from such an analysis are the rms radius and the product Vr_0^4 (this may signify the importance of the fourth normalized moment of the potential). The important role that the large-angle data play is worth mentioning again. The implication of these results is that, so far as the alpha-scattering measurements are concerned, (a) it is not possible or meaningful to distinguish a "deep" potential from a "shallow" one, and (b) only the few above-mentioned properties (and not the details) of the nuclear potentials may be determined. However, if the interaction radius or the diffuseness parameter could be determined by some other means, then the interaction strength and the other parameters could be specified with little ambiguity.

We wish to thank Dr. M. L. Halbert of Oak Ridge National Laboratory and Dr. C. R. Bingham of the University of Tennessee for their help in setting up the experiments and for many useful discussions. Discussions with Dr. J. G. Wills of Indiana University are also thankfully acknowledged.

---

†Work supported in part by the National Science Foundation.

[1]F. G. Perey, Phys. Rev. 131, 745 (1963); P. E. Hodgson, Ann. Rev. Phys. 17, 1 (1968); B. A. Watson, P. P. Singh, and R. E. Segel, Phys. Rev. 182, 977 (1969).

[2]For example, see L. McFadden and G. R. Satchler, Nucl. Phys. 84, 177 (1966).

[3]Written by Dr. F. G. Perey of Oak Ridge National Laboratory.

[4]There is some evidence that the shapes for the real and imaginary potentials should be different. See H. W. Brock, J. L. Yntema, B. Buck, and G. R. Satchler, Nucl. Phys. 64, 259 (1965).

[5]Starting values of other parameters were chosen from those given by McFadden and Satchler (Ref. 2); however, particular values had little effect on the results as long as they were of reasonable order of magnitude.

[6]One more minimum was found with the following parameters; $V = 0.001$ MeV, $W = -12.9$ MeV, $r_{0R} = r_{0I} = 2.119$ fm, and $a_R = a_I = 0.312$ fm. χ^2 for this case was 68. Because of large χ^2 and very unreasonable parameters this solution is considered unacceptable.

[7]M. Reed, University of California Radiation Laboratory Report No. UCRL-18414, 1968 (unpublished).

[8]G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. Letters 17, 33 (1966).

[9]J. W. Luetzelschwab and J. C. Hafele, Phys. Rev. 180, 1023 (1969).

[10]By normalized Kth moment of a function $g(r)$ is meant $\int_0^\infty g(r) r^{K+2} dr / \int_0^\infty g(r) r^2 dr$, whereas in the un-normalized Kth moment the denominator is unity.