ANOMALOUS ELECTRON SCATTERING FROM Nd^{142} †

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Yale University's electron accelerator has been used to study elastic electron scattering from Nd^{142}. The energy was varied between 15 and 60 MeV, while angular distributions were obtained at angles between 70° and 150°. The results indicate an anomalously small skin thickness. A comparison with the effect of dispersion corrections on the interpretation of muonic x-ray results suggests that similar corrections are needed in the present case.

The elastic scattering of 40- to 60-MeV electrons from unpolarized deformed nuclei is expected to follow the behavior predicted by a conventional phase-shift calculation for a phenomenological Fermi charge distribution with an abnormally large surface thickness.[1] In order to test further this hypothesis, Yale University's electron accelerator with its associated instrumentation[2] has been used for a systematic study of the charge distributions of $Nd^{142, 146, 150}$. The results from the study of Nd^{142} are interesting enough to be considered alone, and it is the intent here to report our results for this particular isotope of neodymium.

Since the elastic scattering from natural carbon has been well determined at these energies,[3] the scattering from a metallic Nd^{142} (enriched to 97.7%) target was measured relative to carbon. Both the Nd^{142} and the carbon target were thin ($\sim 1.7\times10^{-3}$ radiation length) so that multiple-scattering effects were small and canceled in the ratio; further, both targets were rotated continuously to average out target nonuniformities. Measurements were performed at scattering angles of 70, 90, 110, 130, and 150 deg with energies of 40 and 60 MeV.

The experimental Nd^{142} cross sections were compared with those expected from a theoretical charge distribution of the Fermi type:

$$\rho(r)=\rho_0\{1+\exp[(r-c)/a]\}^{-1}, \tag{1}$$

where c is the half-density radius and $t=4a\ln3$ is the skin thickness. In our analysis c and t were varied until a best fit was obtained using the criterion of minimization of the value for χ^2. Following this procedure we obtained the Fermi distribution parameters listed in Table I. The errors were calculated according to the following recipe: The statistical error in a parameter α may be found by holding other independent variables $\beta, \gamma, \cdots$ fixed at their values $\bar\beta, \bar\gamma, \cdots$ as determined by the minimum χ^2 and then computing $\Delta\bar\alpha$ such that

$$\chi^2(\bar\alpha\pm n\Delta\bar\alpha)=[1+n^2/(N-1)]\chi^2(\bar\alpha), \tag{2}$$

where N is the number of degrees of freedom.[4] The systematic error in the parameter is found by adding the systematic error to the cross-section ratio and refitting the data, obtaining "best-fit" parameters which differ from the old parameters by the systematic error.

The nucleus Nd^{142} is believed to be nearly spherical; it does not display an easily recognizable rotational spectrum. The value of the half-density radius obtained in the present experiment conforms to expectation ($c=1.12A^{1/3}$), but the skin thickness $t=1.79\pm0.14$ F is significantly smaller than anticipated, lying outside the range 2.4 ± 0.2 F expected for spherical nuclei with $A > 16$.[1] We have in these circumstances re-examined the basis for our value for t by asking certain critical questions:

(a) Are the elastic-scattering cross sections correctly deduced from the data? The quantity measured is the ratio of the elastic scattering from the neodymium target to that from the carbon target. The energy resolution of 0.25% is quite sufficient to resolve the inelastic scattering from the lowest excited state in Nd^{142} (1.57 MeV) and, at back angles, to separate the elastic scattering from the oxygen contaminant. Conventional radiative corrections have been applied[2]; even

Table I. Nd^{142} best-fit parameters for a charge distribution of the Fermi type.

	c (F)	t (F)	$\langle r^2\rangle^{1/2}$ (F)	χ^2
Value	5.83	1.79	4.77	1.20
Statistical error	±0.02	±0.06	±0.02	
Systematic error	±0.00	±0.08	±0.02	
Total error	±0.02	±0.14	±0.04	

though these are known only in Born approximation, the use of thin targets and cutoffs 1.5 MeV below the elastic peak are believed to reduce the implied errors to negligible proportions.

(b) Are the errors assigned correctly? The cross-section ratio implies knowledge of the ratio of the target thicknesses, the accuracy of which determines to a large degree the systematic error in t. Measurements such as weight divided by area can give one an accurate average target thickness, but even continuous rotation of the target does not eliminate problems arising from small irregularities. To compensate for this, four measurements were made at a scattering angle of 70° with an incident energy of 15 MeV where the scattering is essentially that from a point nucleus. Thus the target-thickness ratio, as seen by the incident beam, is directly measureable and has been determined to within 1.1%.

The assignment of statistical error described above is strictly applicable only to statistically independent variables; the χ^2 plot in Fig. 1 shows that the statistical errors in c and t are correlated to some degree. Even under the extreme assumption that statistical errors should be assigned from the outer limits of the inner error ellipse, thereby tripling the upper statistical error limit, the upper limit of 2.05 F is uncomfortably small.

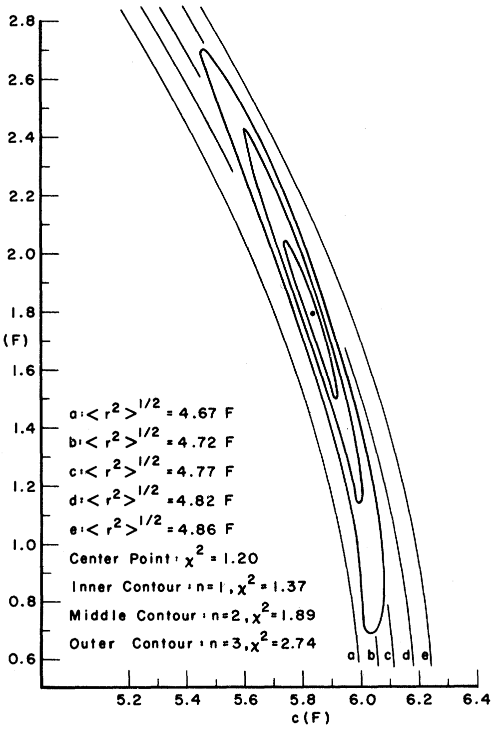

FIG. 1. Constant χ^2 contours and root-mean-square radii as functions of c and t for the elastic scattering from Nd^{142}. "n" is the parameter described in Eq. (2).

(c) Does the anomaly arise from the use of an inappropriate charge distribution? For the electron energies used here, cases have been observed for which equally good fits could be obtained either by varying c and t or by assuming a different charge distribution, as long as the parameters involved were constrained to give the same root-mean-square radius.[2] A variety of charge distributions were tried, with the results listed in Table II. From examination of this table and the graphical results which they summarize, we conclude that conventional analysis of the present experiment does locate the half-density radius and also the slope of the charge distribution in the neighborhood of that point, regardless of the phenomenological prescription used to parametrize the data.

(d) Does the anomaly arise from the use of an inappropriate theoretical analysis? The theoretical cross sections for carbon as well as those for neodymium have been generated using the phase-shift code of Rawitscher and Fischer[5] adopted for the use of any spherically symmetric charge distribution. This code has been checked for numerical accuracy against other codes, including that of Bühring[3] over the range of experimental parameters used; the largest discrepancy was less than 0.1%. However, all the theoretical calculations conventionally used for analysis of electron scattering assume that the charge distribution may be represented in the Dirac equation by a static potential. In view of the anomalously small value of t here obtained, it is interesting to note that Bardin et al.[6] studied the same isotope with muonic x rays and quoted values of $c = 5.90 \pm 0.06$ F and $t = 2.05 \pm 0.15$ F. Macagno and Chen[7] have indicated that the Columbia group

Table II. Comparison of the best-fit root-mean-square radii and χ^2 values for several well-known models. The harmonic well distribution was derived by assuming that the nuclear protons obey the extreme single-particle shell model and using the harmonic-oscillator wave functions.

Charge distribution	Best-fit $\langle r^2\rangle^{1/2}$ (F)	χ^2/N
Fermi	4.77 ±0.04	1.20
Wine bottle	4.77	1.37
Uniform	4.74	2.68
Harmonic well	4.80	2.58
Gaussian	5.00	12.0

has observed that the application of a correction calculated by Chen for nuclear polarization restores the skin thickness to more normal values ($c = 5.75 \pm 0.03$ F, $t = 2.38 \pm 0.08$ F). This application of polarization corrections to muonic x-ray data is consistent with the results of recent work by Anderson et al.[8] on Pb^{206} where "an effect tentatively interpreted as due to the polarization of the nucleus by the muon" was reported.

The close analogy between low-energy electron scattering and muonic x-ray measurements suggests that dispersion corrections should be applied to our data. Previous estimates[9-12] of the dispersion correction as it applied to electron scattering have indicated that the effect should be small at our incident energies. However, inherent in these calculations are drastic assumptions concerning the distribution of electromagnetic strengths; in view of our results, a need for further calculations in indicated.

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[1] L. R. B. Elton, Nuclear Sizes (Oxford University Press, New York, 1961).

[2] R. A. Eisenstein, thesis, Yale University, 1968 (unpublished).

[3] H. A. Bentz, R. Engfer, and W. Bühring, Nucl. Phys. A101, 527 (1967).

[4] D. W. Madsen, Yale Electron Accelerator Internal Report No. EAL 2726-E-71 (unpublished).

[5] G. H. Rawitscher and C. R. Fischer, Phys. Rev. 122, 1330 (1961); C. R. Fischer and G. H. Rawitscher, Phys. Rev. 135, B377 (1964).

[6] T. T. Bardin, R. C. Barrett, R. C. Cohen, S. Devons, D. Hitlin, E. Macagno, C. Nissim-Sabat, J. Rainwater, K. Runge, and C. S. Wu, Phys. Rev. Letters 16, 718 (1966).

[7] E. R. Macagno and M.-Y. Chen, private communication.

[8] H. L. Anderson, C. K. Hargrove, E. P. Hincks, J. D. McAndrew, R. J. McKee, and D. Kessler, Phys. Rev. Letters 22, 221 (1969).

[9] L. I. Schiff, Phys. Rev. 98, 756 (1955).

[10] G. H. Rawitscher, Phys. Rev. 151, 846 (1966).

[11] C. Toepffer, Phys. Letters 26B, 426 (1968).

[12] D. S. Onley, Nucl. Phys. A118, 436 (1968).

OPTICAL-MODEL ANALYSIS OF ALPHA-PARTICLE SCATTERING FROM Mg^{24} †

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Optical-model analysis of elastic-scattering data taken at 40 MeV produces the familiar potential ambiguities, but at 80 MeV a single set is obtained. The parameters V, W, r_0, and a are found to be highly correlated and hence cannot be uniquely determined. The functions which are determined most uniquely are the rms radius of the real and imaginary potential and the products Vr_{0R}^4 and Wr_{0I}^4.

The success of the optical model[1] in explaining the scattering of nucleons of 10-MeV energy or higher from nuclei of all masses has prompted[2] the application of optical-model techniques in the analysis of alpha-particle scattering. Optical potentials providing a good fit to scattering data would give physical information about the form of the alpha-nucleus interaction and would be useful in distorted-wave and coupled-channel analyses of inelastic alpha-particle scattering. However, such optical-model analyses of the alpha-particle elastic-scattering data generally lead to a number of families of parameters which give fits of similar quality. These ambiguities are found[2] to be either continuous, where a small change in the value of one parameter is compensated by small changes in the values of the others, or discrete, in which families of parameters correspond to different numbers of half-wavelengths of the alpha-particle wave function included within the nuclear potential well. The continuous ambiguities can arise from several causes: The optical model may not be able to account adequately for the nuclear interaction embodied in the data (as one would expect at lower energies), or measurements may not have been made over a sufficiently large angular range; finally, the model may be overparametrized. One can circumvent these difficulties by studying the scattering at progressively higher energies, where there should be no difficulty in justifying the application of the optical model, and by extending the measurements over a large angular range, especially those in the backward hemisphere. However, measure-