est.

The most ionic crystal of this type with a tetrahedral structure is expected to be an alloy, $Zn_xCd_{1-x}O$ with x of order 0.1. Again assuming $\Delta s=-5$, we predict $e_{33}=1.86$ C/m^2. This may be a good estimate of the largest piezoelectric constant attainable in a nonferroelectric material.

We are grateful to R. M. Martin for valuable discussions.

Note added in proof. —An alternative explanation of the reversal in sign of the piezoelectric constant in A^NB^{8-N} tetrahedral crystals on going from $N=2$ to $N=3$ is that $e^*(b_0)$ has actually reversed sign. Our present model argues against this possibility, but we would like to point out that the question can be settled experimentally. Millimeter crystals (one could also use powders) of ZnSe:GaAs solid solutions have been prepared [S. M. Ku and L. J. Bodi, J. Phys. Chem Solids 29, 2077 (1968)]. The oscillator strength of the Reststrahlen as a function of composition would be approximately linear if $e^*(b_0)$ did not reverse sign, but would show a pronounced minimum if it did.

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ELECTROEXCITATION OF PARTICLE-HOLE STATES IN O^{16}†

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We present experimental data and theoretical results for electroexcitation of $T=1$ particle-hole states in O^{16}.

The electroexcitation of O^{16} in the energy range 10-30 MeV is studied on the basis of the single-particle-hole model.[1,2] Experimental data at large three-momentum transfer q and large scattering angles θ are presented here and analyzed in the same manner as the data for C^{12} treated previously.[3,4] By working at high q, states of large angular momentum can be strongly excited, giving rise to an excitation spectrum containing only a few large peaks. By working at large θ, transverse multipoles are enhanced and $T=1$ states are preferentially excited in nuclei with $T=0$ ground states such as O^{16}. Here we consider only the $T=1$, 1p-1h (one particle, one hole) states of O^{16}, but include states of all allowed angular momenta.

The experiment was performed on the Stanford Mark III electron accelerator at scattering angles of 135 and 145° and energies between 100 and 400 MeV. Through control of beam-momentum width and target thickness the overall resolution was kept between 0.2 and 0.3 %. The spectrometer and detection apparatus are described elsewhere.[5] The target consisted of plates of ice, 1-4 mm in thickness, which were kept at −180°C. This was accomplished by connecting the copper frame holding the ice by a bellows to a liquid N_2 reservoir. Since at this temperature no significant evaporation in vacuum is expected, the target was used without foils, thus avoiding all background problems due to quasielastic scattering whose threshold would in general be much lower for the foil than for the O^{16} target. Since the proton cross section is accurately known, the hydrogen in the target was used to determine the target thickness. Within 2 %, no change in target thickness could be observed over a period of several days at 1.5 μA, the maximum beam current used.

The cross sections have been obtained after correcting the spectra for radiative effects[6] and, at low momentum transfers, subtracting the radiative tails from unobserved levels at small excitation energies. Figure 1 shows a spectrum at 224-MeV incident energy. The experimental form factors obtained by integrating over energy ranges covering the main features in the excitation spectrum (see Fig. 1), together with data from Vanpraet and Barber,[7] Fuller and Hayward,[8] Drake, Tomusiak, and Caplan,[9] Vanpraet,[10] and Goldemberg and Barber,[11] are given in Figs. 2 and 3. For the present data given in Fig. 3(a) the calculated[12] quasielastic contributions have been subtracted. This makes little difference for the large peak centered at 18.7 MeV, but is almost wholly responsible for the estimated errors in the feature seen at 20.4 MeV.

In analyzing these data we take the ground state of O^{16} to contain closed 1*s* and 1*p* shells and describe the states excited in electron scattering as linear combinations of single-particle-hole states. The particular linear combinations (configuration-mixed states) used are determined by the residual interaction which is taken here to be a Serber-Yukawa force.[13] All 1p-1h states of all allowed angular momenta which can be obtained by promoting a particle from the 1*p* shell to the 2*s*-1*d* shell are considered. Configuration-mixed states which lie nearby in energy are grouped together into complexes whose total electron-scattering form factors are to be compared with experiment. Following the idea of considering the single-particle-hole states to be the doorway states[14,15] in electroexcitation, this comparison is made only with data averaged over energy intervals on the order of 1 MeV. Of course in addition to the strong well-defined peaks observed,

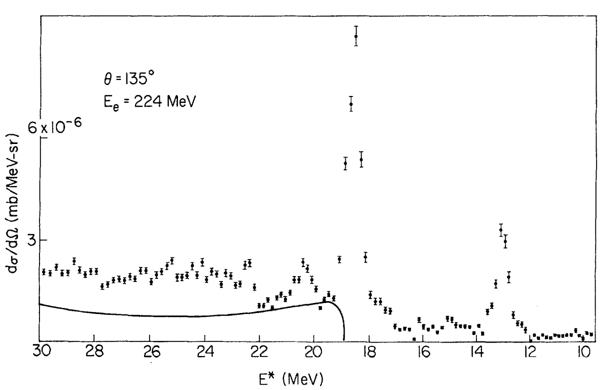

FIG. 1. The spectrum shown is unfolded for radiative processes. The cross sections have been obtained by integrating over the energy intervals 12.6-13.6, 17.0-18.0, 18.0-20.8, 20.0-20.8, and 20.8-26.0 MeV and subtracting the various backgrounds. The computed quasielastic spectrum is shown as a solid curve.

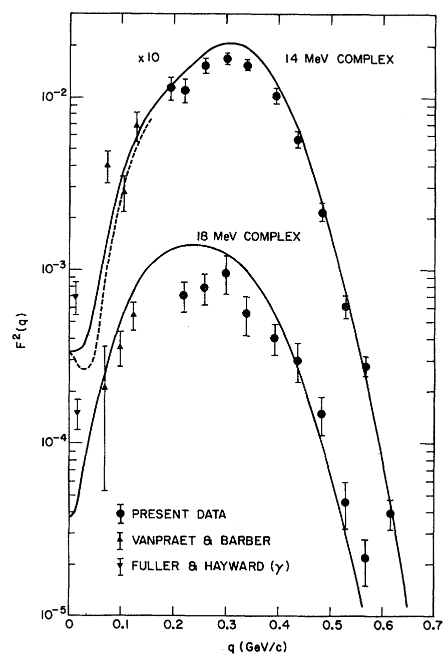

FIG. 2. Form factors

$$F^2(q) = \frac{d\sigma/d\Omega}{[4\pi\sigma_{\mathrm{Mott}}(\frac{1}{2} + \tan^2\frac{1}{2}\theta)]}$$

for the 14- and 18-MeV complexes. The solid curves are for $\theta = 135°$, the scattering angle in the present experiment except at 191 and 219 MeV/*c* where $\theta = 145°$. The dashed curve is for $\theta = 180°$ to compare with the data below 120 MeV/*c*. Longitudinal contributions are relatively small for both complexes. The calculated curves for the 14-MeV complex have been reduced in amplitude by 1.7, the 18-MeV complex by 1.4.

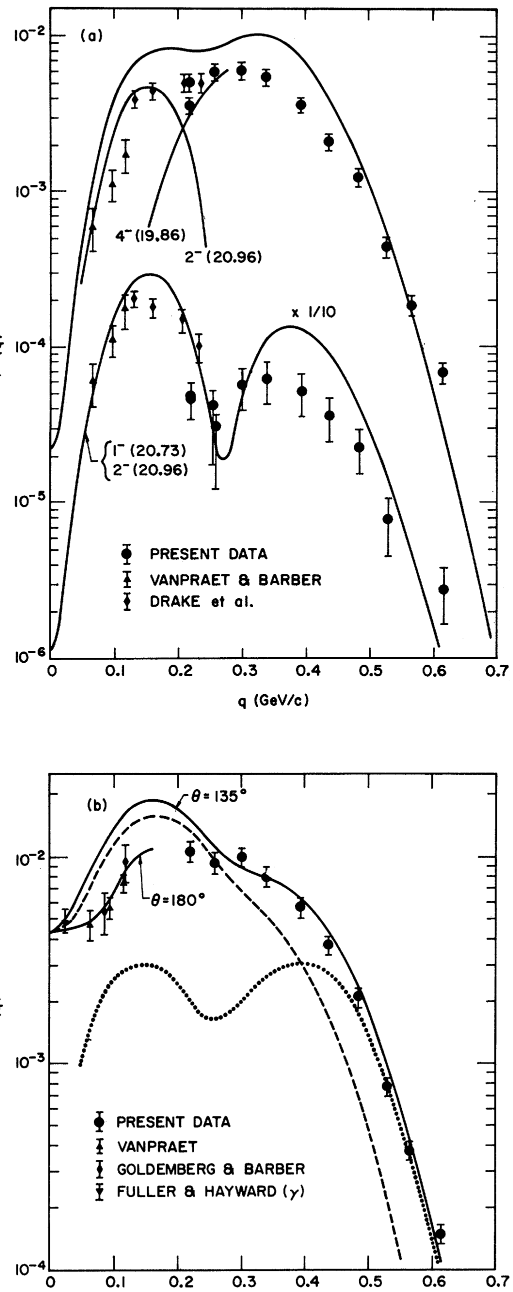

FIG. 3. Form factors as for Fig. 2. (a) 20-MeV region for $\theta = 135°$ (the longitudinal contributions are, in fact, small). Only the lower curve for the 1^- and 2^- states has been reduced in amplitude (by 1.4). The comparison there is for data at about 20.4 MeV. (b) Giant resonance region. The dotted curve is the quasielastic contribution and the dashed curve is the form factor for the discrete levels, both curves for $\theta = 135°$. The solid curves are the total. Data below 120 MeV/c are at $\theta = 180°$; the rest are at 135° except for the point at 219 MeV/c which, having $\theta = 145°$, lies relatively lower.

the experimental excitation spectrum contains fine structure, but this should be considered only within the framework of a model having more complicated admixtures of many-particle, many-hole states.

The oscillator parameter $b = (\hbar/M\omega)^{1/2}$ is taken throughout to be 1.77 F, the value obtained[16] in considering the elastic form factor of O^{16}, although somewhat larger than the value 1.68 F obtained from Coulomb-energy differences.[17] Previously deForest[18] considered the 0^-, 1^-, and 2^- states in O^{16}, using the same model but with $b = 1.67$ F. His results are thus quantitatively slightly different although qualitatively the same at low q where high-spin states are unimportant. The pure 1p-1h states and their unperturbed energies are the following: $(1p_{1/2})^{-1}(1d_{5/2})_{2^-,3^-}$ at 11.52 MeV, $(1p_{1/2})^{-1}(2s_{1/2})_{0^-,1^-}$ at 12.39 MeV, $(1p_{1/2})^{-1}(1d_{3/2})_{1^-,2^-}$ at 16.60 MeV, $(1p_{3/2})^{-1} \times (1d_{5/2})_{1^-,2^-,3^-,4^-}$ at 17.68 MeV, $(1p_{3/2})^{-1} \times (2s_{1/2})_{1^-,2^-}$ at 18.55 MeV, and $(1p_{3/2})^{-1} \times (1d_{3/2})_{0^-,1^-,2^-,3^-}$ at 22.76 MeV. The 0^- states are not excited in electron scattering. These are all the 1p-1h states that can be formed in O^{16} by promoting a particle one oscillator spacing. Using the Serber-Yukawa residual interaction these pure 1p-1h states are mixed and shifted in energy and we find at least four clearly defined groupings of levels. The energies of the configuration-mixed states and consequently the complexes considered are the following: 14-MeV complex, 3^- (13.57 MeV), 2^- (13.59 MeV), and 1^- (14.38 MeV); 18-MeV complex, 2^- (18.45 MeV) and 1^- (18.46 MeV); 20-MeV complex, 3^- (19.17 MeV), 2^- (19.77 MeV), 4^- (19.86 MeV), 1^- (20.73 MeV), and 2^- (20.96 MeV); and giant resonance region, 1^- (23.26 MeV), 2^- (24.28 MeV), 3^- (25.30 MeV), and 1^- (26.13 MeV). In addition, the sum of the form factors for the 1^- and 2^- states at 20.73 and 20.96 MeV, respectively, is considered separately. The existence of the 14-MeV complex is well known: The experimental energies are 2^- (12.96 MeV), 1^- (13.09 MeV), and 3^- (13.25 MeV). The major peaks in photoexcitation are seen[19] at 17.3, 19.6, 22.3, and 24.3 MeV. Here we identify our 1^- at 18.46 MeV with the 1^-, $T=1$ state seen experimentally[20] at 17.3 MeV and take this 1^-, and the 2^- which is nearly degenerate with it, together as the 18-MeV complex. Furthermore, from previous electron-scattering studies,[7] the existence of a 2^- state at 20.2 MeV (20.4 MeV in the present experiment) can be inferred. This state is strongly excited at low momentum transfer and so we are led to identify it with our 2^- state at 20.96 MeV, the giant magnetic quadrupole. The agreement here between our calculated energies and their experimental values where known is good to about 1 MeV. In fact the calculated eigenvalues are consistently higher than experiment by about 1 MeV.

The form factors for the complexes considered are shown in Figs. 2 and 3. In some cases the form factors have been reduced in amplitude to permit direct comparison with experiment of the momentum-transfer dependence. Reduction factors of about the size used here are known to be required when the simple 1p-1h model is used.[21] This point is discussed in more detail at the end.

In Fig. 2 we show the results for the 14-MeV complex where an empirical reduction in amplitude by 1.7 has been used. At very low q only the 1^- form factor is significant, but already at $q \sim 100$ MeV/c the 2^- form factor is dominant and remains so up to $q \sim 200$ MeV/c. The 3^- form factor is small up to about 200 MeV/c and then at higher momentum transfer the 3^- and 1^- form factors are both large and of comparable strength.

The 18-MeV–complex form factor is also shown in Fig. 2, reduced in amplitude by a factor 1.4. Here the roles of the 1^- and 2^- states change with momentum transfer: At low q the 1^- is dominant and the 2^- is only weakly excited; at high q the reverse situation holds. The 1^- and 2^- form factors are equal at about 200 MeV/c.

In Fig. 3(a) the form factor for the 20-MeV complex is shown with no reduction in amplitude, although a reduction factor of about 1.4 appears to be necessary. At low q the 2^- at 20.96 MeV contains the major strength. However, at $q \sim 200$ MeV/c and higher, the 4^- state is extremely important. In fact, the 2^- (20.96-MeV) form factor has a diffraction zero at 270 MeV/c (see lower curve), while for $q \sim 300$ MeV/c the 4^- state is by far the most strongly excited, containing about 65% of the strength in that region. At high q the 20.96-MeV 2^- and the 4^- contribute about equally to the cross section. The remaining states in the 20-MeV complex are never dominant, but nevertheless together contribute up to 40% to the cross section. The region up to about 240 MeV/c has been analyzed in two different ways in the past. At lower q (up to 120 MeV/c) Vanpraet and Barber[7] observed a feature at an excitation energy of 20.2 MeV, which from its q dependence was attributed to a 2^- state. On the other hand at higher q (between 160 and 240 MeV/c) Drake, Tomusiak, and Caplan[9] found peaks at 19.1 and 20.5 MeV. The former they took to be the 2^- giant quadrupole and the latter to be a 1^+ state (as it was all transverse in character). However from the form factors shown in Fig. 3(a) an alternative interpretation is possible; that the feature seen at about 20.4 MeV is the 2^- giant quadrupole state (20.96 MeV in our calculation) and the feature Drake, Tomusiak, and Caplan saw at 19.1 MeV is the 4^- state which we calculate to be about 1 MeV lower than our 2^-. The peak seen at 18.7 MeV in the present experiment may then be attributed mainly to our proposed 4^- state with the peak at 20.4 MeV resulting from the giant quadrupole. At low q, where the resolution is highest in the present experiment, fine structure is seen in the region between 18 and 20 MeV.

In Fig. 3(a) the sum of the 1^- (20.73-MeV) and 2^- (20.96-MeV) form factors is also shown (reduced by 1.4 in amplitude). The 2^- form factor dominates over the 1^- except at small q and where the 2^- has its diffraction zero. Note that in the present data alone the existence of a diffraction feature is indicated.

Finally in Fig. 3(b) the form factor in the giant resonance region (20.8-26.0 MeV) is shown. The form factor for the 1p-1h states in this region goes from being all giant dipole resonance (23.26 MeV) at low q to being more spin-flip dipole resonance (26.13 MeV) at intermediate q, and finally to being primarily 3^- at high q. The 2^- at 24.28 MeV contributes about 13% at most to the cross section. Following the discussion in Lewis and Walecka[2] the Coulomb dipole form factors have been reduced by a factor of 2 (in the cross section). In addition to the form factors for the 1^-, 2^-, and 3^- discrete levels in this energy range, the quasielastic form factor has been included. This is calculated in the framework of a square-well shell model where all significant multipole transitions from bound states into the continuum are summed. The quasielastic cross section is compared with the experimental excitation function shown in Fig. 1. The total form factor presented is then the quasielastic form factor integrated in excitation energy between 20.8 and 26.0 MeV added to the form factor for the discrete levels. A detailed report on this quasielastic calculation for O^{16} and C^{12} will be presented in the near future.[12]

If 2p-2h states are allowed as components in the ground state, and 3p-3h states are admixed with the 1p-1h excited states, then more detailed structure in the excitation spectrum is obtained.[21] The main effect of introducing such higher configurations on the present calculation, where only complexes of $T=1$ levels are considered, is to lower the amplitude in most cases while leaving the momentum-transfer dependence relatively unaffected.

This rather simple result is obtained partly because the nonvanishing $|2p\text{-}2h\rangle \to |1p\text{-}1h\rangle$ transition matrix elements are often proportional to the appropriate $|\text{closed-shell}\rangle \to |1p\text{-}1h\rangle$ matrix elements. The constant of proportionality is independent of q and in many cases (e.g., the important isovector magnetic-moment contribution in electron scattering to odd-parity excited states of O^{16}) has opposite phase to the usual single-particle transition matrix element, so that destructive interference occurs. Of course the usual transition matrix elements are reduced because the amount of closed-shell configuration in the ground state (g.s.) is lowered. Assuming that the 2p-2h component adopted in Ref. 17 comprises 25% of the g.s., and ignoring the effect of 3p-3h excited states, we find a reduction of 1.25 to 1.8 in the cross section (amplitude squared) to individual states. The larger reduction factors are obtained for the lowest 1^-, 2^-, and 3^- states where the destructive interference mechanism is most important. A larger reduction, 1.4 to 3 in cross sections to individual final states, may be obtained by assuming that the g.s. of O^{16} is 36% 2p-2h or by mixing 3p-3h states with the usual 1p-1h excited states. The details of these considerations as well as a discussion of the further implications of the results for $T=0$ and even-parity states will be reported elsewhere.[22] Reduction factors of this nature have also been reported in random-phase approximation calculations.[23,24]

In conclusion we see that this simple single-particle-hole model for O^{16} is very successful in describing the gross structure observed in electroexcitation in the energy range 10-30 MeV. The existence of clearly defined complexes of levels, containing in some cases states of high angular momentum, is predicted on the basis of this model and results in form factors which are in excellent agreement with experimental data over a wide range of momentum transfer.

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