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LOW-TEMPERATURE SATURATION OF THE SUPERCONDUCTING PROPERTIES INDUCED IN SILVER BY THE PROXIMITY EFFECT*

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Critical-field measurements of silver layers sandwiched between two superconducting layers indicate that supercooling appears below a temperature T^* and that the supercooling field stays constant at low temperatures. We propose a simple theoretical model which correctly predicts the temperature of saturation as well as the observed amount of supercooling, and leads to an estimate of 0.1 for NV for silver. This model further suggests that a $1/x$ spatial dependence of the order parameter in the silver should be observed in the He^3 temperature range and below.

The proximity of a superconductor (S) may induce superconductivity in an otherwise normal metal (N). This effect has previously been studied by measurements of transition temperature, tunneling, critical current, and surface impedance. Attempts have been made to deduce from these experiments the value of the electron-electron interaction potential V, in particular to determine whether it is positive in N, which would imply that the normal metal would become an intrinsic superconductor at some lower temperature.[1,2]

The thermal conductivity also leads to information about the energy gap and the interaction potential. In contrast to tunneling and critical-current measurements, it is sensitive to electrons traveling parallel to the SN boundary so that one is not limited by problems associated with the mismatch across the boundary. We have made such measurements on PbBi/Ag/PbBi triple layers.[3] In the course of this research it turned out that a value of V can also be deduced from a study of the critical fields, and we report here on this aspect of our work.

The specimens were evaporated onto room-temperature substrates of No. 00 microscope cover glass. The thermal conductivity was measured as a function of magnetic field applied parallel to the SNS interface down to a temperature of 0.3°K. At a field below the lower critical field H_{c1} of the PbBi layers, the thermal conductivity shows a transition of the silver layer from the induced superconducting state to the normal state.

The thermal conductance of the substrate was measured in a separate run so that the conductance of the specimen could be obtained by subtraction. The measurements show that below 1.4°K and below H_{c1} the thermal conductance of the PbBi layers is negligible so that the specimen conductance is then entirely that of the silver in either its normal or its superconducting state. The normal-state Ag conductivity agrees with measurements on films of the same thickness without the PbBi layers. This agreement indicates that interdiffusion does not have any deleterious effect on our specimens.

In increasing fields the thermal conductivity of the silver reaches its normal value at some field H_s; in decreasing fields superconductivity nucleates at a field H_n. At high temperatures the two fields are equal, but below a temperature T^* the two fields separate with H_n less than H_s, indicating that H_n is a supercooling field and that the field transition is of first order. Below T^*, H_n is proportional to temperature down to a temperature T_s below which it stays constant.

Figure 1 shows the behavior of a film of 2000-Å Ag between two 2000-Å layers of PbBi. A second specimen of similar composition was measured at 0.6°K. The fields H_n and H_s were about 12% larger, but the ratio H_n/H_s (which is used in the subsequent analysis) was within 3% of the

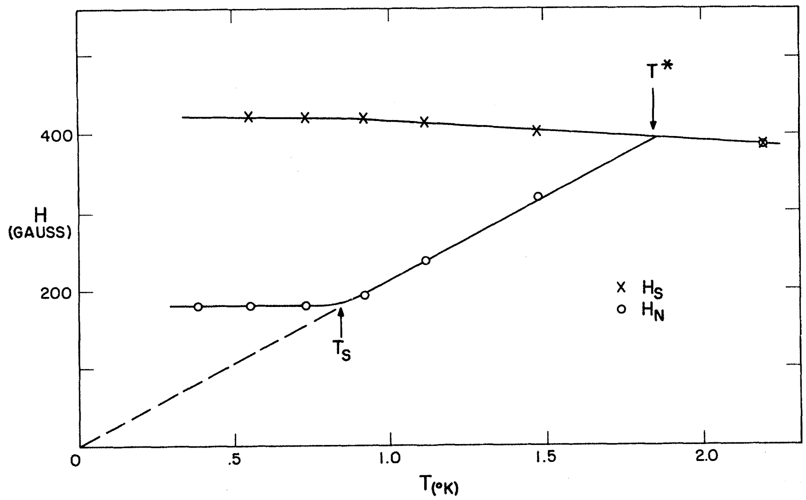

FIG. 1. Critical fields of a 2000-Å silver layer sandwiched between two layers of 2000-Å PbBi (5 at. %), as a function of the temperature. The normal state is restored at H_s in increasing fields, and the superconducting state nucleates at H_n in decreasing fields.

value of H_n/H_s for the specimen of Fig. 1 below T_s. We also measured a specimen of 5000-Å Ag between two 2000-Å layers of PbBi, for which T^* was about 1.3°K, and for which T_s was not reached at the lowest temperature (0.4°K).

The behavior which we observe above T_s can be understood on the basis of the theory of the proximity effect for the case where the pair potential Δ is small compared with kT.[1] In that case Δ decreases exponentially in N with a characteristic length K^{-1} given in the dirty case ($l \ll K^{-1}$) for the limit where the critical temperature T_{cn} of the normal metal goes to zero by

$$K^{-1} = (\hbar v_F l/6\pi k T)^{1/2}, \qquad (1)$$

where l is the mean free path, and v_F the Fermi velocity in N.

The screening currents are described by a position-dependent penetration depth[4]

$$\lambda(x, T) = \frac{\beta}{5^{1/2}}\left[(\hbar c^2 k T\rho)^{1/2}\frac{1}{\Delta_s}\right]\frac{\Delta_n(x=0)}{\Delta_n(x)}, \qquad (2)$$

where ρ is the normal-state resistivity in N, Δ_s the value of the pair potential in S at the interface ($x=0$), and $\beta = [(N_n/N_s)+0.5(NV)_s]^{-1}$. N_n and N_s are the densities of states, and V_n and V_s the electron-electron interactions in N and S. Equation (2) makes use of the de Gennes boundary condition that Δ/NV is continuous across the interface.[1] Equations (1) and (2) predict that $\kappa(x, T) = \lambda(x, T)/K^{-1}$ is proportional to temperature.

We assume that there is little or no superheating (since this is normally difficult to achieve) so that H_s is close to the thermodynamic critical field. Below T^* we can then write $H_n/H_s = \kappa(0, T)/\kappa(0, T^*)$.[5] Since H_s is nearly independent of T below T^*, this implies that H_n is proportional to $\kappa(0, T)$ and, hence, to T. This is in agreement with the linear variation of $H_n(T)$ which we observe between T_s and T^*.

The temperature T^* can be calculated using Eqs. (1) and (2) and the relation[5]

$$d_N/2\lambda(0, T^*) = \tfrac{3}{2}\cosh[d_N/2K^{-1}(T^*)], \qquad (3)$$

where d_N is the thickness of N. The calculated value of T^* is 1.98°K, in very good agreement with the experiment. Table I shows the values of the parameters which have been used. The ratio $(NV)_n/(NV)_s$ was taken from thermal-conductivity measurements of the gap induced in silver in specimens similar to the one described here.[3]

On the other hand the behavior observed below T_s contradicts Eqs. (1) and (2). It is known that right at T_{cn} the value of κ is the intrinsic value for the normal metal given by the Gor'kov-Goodman relation.[5] This is the result of the fact that the linear term of the Ginzburg-Landau equation is zero for $T = T_{cn}$, and the behavior is then governed by the nonlinear term which describes saturation effects. The Ginzburg-Landau equation for induced Δ is $C\Delta'' = A\Delta + B\Delta^3$ where C and B are constants and A goes to zero at T_{cn}.[6] When the linear term is dominant, Δ has the usual exponential variation and κ is given by Eqs. (1) and (2). But when the nonlinear term is dominant, namely at $T = T_{cn}$, it varies as $1/x$ far from the interface[7] and κ has the Gor'kov-Goodman value κ_{GG}.

It appears likely that the saturation effects

Table I. Parameters for the specimen of Fig. 1.

v_F	1.4×10^8 cm/sec [a]	$\kappa(0,T)$	$0.173T$
γ	590 erg/cm^3 deg^2 [a]	κ_{GG}	0.142
ρl	0.733×10^{-11} Ω cm^2 [b]	T_s(theor)	0.82°K
N_n/N_s	0.405 [a]	T_s(exp)	0.83°K
$(NV)_n/(NV)_s$	0.25 [c]	T^*(exp)	1.86°K
ρ	0.785×10^{-6} Ω cm	T^*(theor)	1.98°K
$\lambda(0,T)$	$397T^{1/2}$ Å	$(H_n/H_s)_{\rm sat}$(theor)	0.41
$K^{-1}(T)$	$2290/T^{1/2}$ Å	$(H_n/H_s)_{\rm sat}$(expt)	0.43

[a]C. Kittel, Introduction to Solid State Physics (John Wiley & Sons, Inc., New York, 1966), 3rd ed.
[b]M. Claude Boulesteix, Compt. Rend. 260, 6845 (1965).
[c]See Ref. 4.

which we observe in the silver far above any possible critical temperature are of the same nature. We therefore postulate that the observed value of $\kappa(x,T)$, instead of going to zero as expected from Eqs. (1) and (2), saturates at κ_{GG}, and that the saturation temperature T_s is defined by the relation

$$\kappa(x,T_s)=\kappa_{GG}. \tag{4}$$

There are two experimental results which confirm this postulate. The first is that, in the saturation regime ($T<T_s$), H_n/H_s is very close to the value $\kappa_{GG}/\kappa(0,T^*)$ (see Table I). The second is the magnitude of T_s. From Eqs. (1), (2), and (4) we can write

$$T_s(x)=\left[T_{cs}\frac{1.5}{\beta}\frac{1}{12\pi}(v_F\rho l\gamma)^{1/2}\right]\frac{\Delta_n(x)}{\Delta_n(0)}, \tag{5}$$

where T_{cs} is the transition temperature of the superconductor and γ is the electronic specific-heat coefficient of Ag. For our specimen $\Delta_n(x)/\Delta_n(0)$ is close to 1 near T_s since K^{-1} is larger than $\frac{1}{2}d_N$. Equation (5) then gives $T_s(0)=0.82$°K, again in very good agreement with experiment.

Equation (5) shows that T_s depends on $(NV)_n/(NV)_s$ through β but is independent of ρ (since ρl is constant) and, when d_N is sufficiently small, independent also of specimen thickness. Hence a measurement of T_s leads to a value of NV in "normal" metals. If we ignore all other calculations in this paper and use the experimental value of T_s and the values of ρl, γ, and v_F in Table I, Eq. (5) gives $(NV)=0.1$. [If $(NV)_n$ were equal to zero, Eq. (5) would give $T_s=0.62$°K.]

T_s is position dependent through $\Delta(x)$ and decreases for increasing x. Since the measured value is that for $x=\frac{1}{2}d_N$, this is in agreement with our observation that T_s decreases when d_N increases. Conversely, at a given temperature $T<T_s(0)$, Eq. (5) can be solved for x, giving a value $x_s(T)$: For $x\ll x_s(T)$, there is saturation and Δ will vary as $1/x$; for $x\gg x_s(T)$, there will be the usual exponential dependence. When $T\to0$, $x_s(T)$ goes to infinity.

In a recent Letter Freake and Adkins[8] report tunneling experiments done at 0.06°K which show a $1/d_N$ dependence of the gap induced by the proximity effect in copper up to a thickness of the order of 7000 Å. These results have been interpreted in terms of the theory of McMillan[9] which assumes a uniform order parameter in N. It is unlikely that this condition holds for their thickest films, and we suggest that the experimental result might well be explained by a $1/x$ dependence of the order parameter resulting from saturation effects in N, as described in this Letter.

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