NUCLEAR PRODUCTION AS A TECHNIQUE FOR DISTINGUISHING BETWEEN KINEMATIC ENHANCEMENTS AND RESONANCES*

J. S. Trefil
Physics Department, University of Illinois, Urbana, Illinois 61801
(Received 28 July 1969)

The problem of distinguishing between kinematic enhancements and true resonance production is discussed. It is shown that a systematic study of production on nuclei should provide a way of making this distinction.

A long controversy surrounds the appearance of enhancements above phase space in the mass spectrum for the reaction

$$\pi + p \to (\rho\pi) + p \tag{1}$$

in the region of the A_1 meson. On the one hand, we are accustomed to interpreting such enhancements as resonances, but on the other, it has been pointed out that enhancements of this type can occur through the Deck-type mechanism[1] with no mention of resonances at all. In this note, we investigate the possibility of resolving this dilemma experimentally by a systematic study of the production of the "A_1" on nuclei. We shall see that such a resolution is indeed possible, and is within the reach of present experimental techniques.

The basic property of nuclear production which allows us to do this is the fact that after the A_1 has been produced on a single nucleon, it then scatters off other nucleons in the nucleus before leaving the nucleus and subsequently decaying into the observed $\rho\pi$ system. We shall consider only the two extreme interactions possible in the $\rho\pi$ system. If the ρ and the π are strongly bound together, so that they interact with subsequent nucleons as a single particle, rather than as some sort of composite system, we shall call the $\rho\pi$ system a resonance. At the other extreme, if the $\rho\pi$ system is very weakly bound, so that it interacts with subsequent nucleons as a pair of free particles, we shall term the system an uncorrelated pair. Although the true case probably falls between these extremes, it often proves useful to analyze idealized cases in trying to examine a complicated phenomenon.

It is well known that the ordinary Glauber theory[2] can be extended to cover the production of resonances on nuclei.[3,4] More recently, it has been pointed out that processes in which two or more particles are produced can also be handled by the multiple-scattering theory.[5] Thus, for the two cases which we are considering, namely, the case where the A_1 is a true resonance in the sense defined above and the case where the A_1 is an uncorrelated $\rho\pi$ pair which has enjoyed a kinematic enhancement, it is possible to write down a theoretical expression for the production of the A_1 from nuclei.

If we write the nuclear wave function as

$$\psi = \left(\frac{1}{\pi^{1/2}R}\right)^3 \prod_{j=1}^{N} \exp\left(-\frac{r_j^2}{R^2}\right) \tag{2}$$

(a discussion of the range of validity of this approximation is given in Ref. 4),

$$g_j(\delta) = \frac{G(i+\gamma)}{4\pi} p e^{-g\delta^2/2} \exp\left[i\left(\frac{m_R^2 - m_\pi^2}{2p}\right) z_j\right] \tag{3}$$

as the amplitude to produce a resonance of mass m_R on the jth nucleon,

$$h_j(\delta) = \frac{H(i+\eta)}{4\pi} p e^{-h\delta^2/2} \exp\left[i\left(\frac{m_R^2 - m_\pi^2}{2p}\right) z_j\right] \tag{4}$$

as the amplitude to produce an uncorrelated pair of the same mass off the same nucleon, and

$$f_{qN,n}(\delta) = [\sigma_{qN}(i+\alpha_q)/4\pi] p e^{-aq\delta^2/2}$$

as the amplitude for a particle of type q to scatter off the nth nucleon, then the expression for production of a resonant A_1 from a nucleus f_A, is given by[3]

$$f_A = \frac{ip}{2}\exp\left(\frac{R^2\Delta^2}{4A}\right)\exp\left[-\frac{1}{2}\left(\frac{m_R{}^2-m_\pi{}^2}{2p}\right)^2 R^2\right]\frac{G(1-i\gamma)}{2\pi(R^2+2g)}$$
$$\times\sum_{K=1}^{A}\sum_{n=1}^{K}\binom{A}{K}(-1)^{K+1}\left[\frac{\sigma_{\pi N}(1-i\alpha_\pi)}{2\pi(R^2+2a_\pi)}\right]^{K-n}\left[\frac{\sigma_{AN}(1-i\alpha_A)}{2\pi(R^2+2a_A)}\right]^{n-1}\frac{1}{E}e^{-\Delta^2/4E}, \quad (5)$$

where Δ is the transverse momentum transfer to the nucleus, and E is defined by

$$E = \frac{K-n}{R^2+2a_\pi} + \frac{n-1}{R^2+2a_A} + \frac{1}{R^2+2g}. \quad (6)$$

The equation which describes the production of uncorrelated $\rho\pi$ pairs of mass m_R on a nucleus is[5]

$$f_{\rho\pi} = \frac{ip}{2}\frac{H(1-i\eta)}{2\pi(R^2+2h)}\exp\left[-\frac{1}{2}\left(\frac{m_R{}^2-m_\pi{}^2}{2p}\right)^2 R^2\right]\exp\left(\frac{\Delta^2R^2}{4A}\right)\sum_{l=0}^{A-1}\sum_{m=0}^{A-l-1}\sum_{n=0}^{A-l-1}\sum_{j=j_{\rm min}}^{j_{\rm max}}\binom{A}{l+m+n+1-j}$$
$$\times(F_\pi)^{l+m-j}(F_\rho)^{n-j}\left[\frac{\sigma_{\rho N}\sigma_{\pi N}(1-i\alpha_\rho)(1-i\alpha_\pi)}{8\pi^2(a_\rho+a_\pi)[R^2+2a_\rho a_\pi/(a_\rho+a_\pi)]}\right]^j r^2 e^{-\Delta^2 r^2/4}, \quad (7)$$

where we have defined

$$F_{\pi(\rho)} = -\frac{\sigma_{\pi(\rho)}[i+\alpha_{\pi(\rho)}]}{2\pi[R^2+2a_{\pi(\rho)}]} \quad (8)$$

and

$$\frac{1}{r^2} = \frac{l}{R^2+2a_\pi} + \frac{1}{R^2+2h} + \frac{m+n-j}{R^2+2a_\rho} + \frac{j}{R^2+2a_\rho a_\pi/(a_\rho+a_\pi)}. \quad (9)$$

To understand the physical basis of the difference between these two formulas, consider a term in the amplitudes involving double scattering. For resonance production, there is only one such term, corresponding to the production of the A_1 on the first nucleon, and the elastic scattering of the A_1 on the second. For uncorrelated pairs, however, there are two terms, corresponding to either the ρ or the π scattering on the second nucleon. Clearly, the differences will become greater for higher order terms in the mulliple-scattering series.

There are, in principle, three ways in which one might expect this type of difference to show up experimentally. First, it should affect the relative magnitudes of production cross sections from different nuclei; second, it should have some small effect on the shape of the differential cross section for production from a single nucleus; and, third, it might affect the shape of the observed mass spectrum in the A_1 region. In fact, except for the usual suppression of higher masses by the nuclear form factors, the third effect above is found to be very small indeed, so we shall confine our attention to an investigation of the dependence of the production on A, the atomic number, and on Δ^2.

The most general amplitude for the production of a $\rho\pi$ system is

$$f = (1-\beta)\{xf_A + f_{\rho\pi}\}, \quad (10)$$

where $x = \beta/(1-\beta)$ is the ratio of resonant to nonresonant production on hydrogen. We shall assume that x is real, since both resonance production and Deck mechanisms are primarily diffractive in nature. Of course, in the event that an appreciable nondiffractive production occurs in hydrogen, Eq. (10) must be understood to refer only to the diffractive part, and in this case the nondiffractive contribution would have to be subtracted from the hydrogen data before it could be used in the analysis of the nuclear data.

In this case, there are two unknown quantities in Eq. (10). They are σ_{AN}, the A_1-nucleon cross section, and x, the ratio of resonant to nonresonant production on hydrogen. In general, both must be varied when attempting to fit experimental data.

In Fig. 1, we show the effect of varying these parameters on the A dependence of nuclear A_1 production and the effect on the Δ^2 dependence in

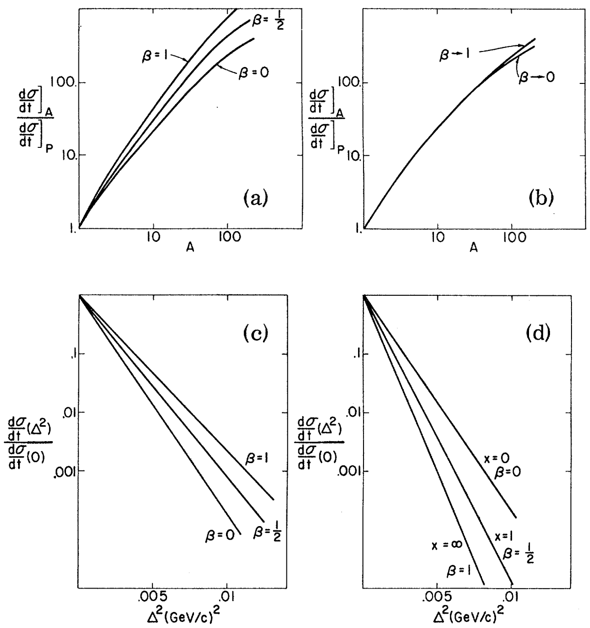

FIG. 1. (a), (b) The results of calculations from Eq. (10) for the A dependence of the production cross section for $\pi+A\rightarrow(\rho\pi)+A$ at $\Delta^2=0$, for various values of β, and for $\sigma_{AN}=10$ and 50 mb, respectively. (c), (d) The Δ^2 dependence of the reaction $\pi+\mathrm{Pb}\rightarrow(\rho\pi)+\mathrm{Pb}$ for the same range of parameters and the same respective values of σ_{AN}. In these the curves are normalized to unity at $\Delta^2=0$ [the difference in values at this point can be read from (a) and (b)].

the case of the lead target. Some discussion of these curves is in order. Consider the A dependence first. Clearly, in the case that $\beta=0$, i.e., when we have only the production of uncorrelated $\rho\pi$ pairs, the results must be independent of σ_{AN}. Furthermore, we would expect the $\rho\pi$ system to be rather strongly absorbed in the nucleus, with a mean free path corresponding to $\sigma\sim30$-60 mb. Thus as we let β differ from zero and start adding in resonance production, we expect a strong dependence on β if σ_{AN} is appreciably different from the above range, and not much dependence on β as σ_{AN} gets into the above range. In fact, from Figs. 1(a) and 1(b) we see that there is a striking β dependence in the plot of $(d\sigma/dt)_{\Delta^2=0}$ vs A for $\sigma_{AN}=10$ mb, but virtually no β dependence at all for 50 mb. There is a more or less smooth transition between these extremes as σ_{AN} is varied.

It is well known that the shape of the differential cross section depends on the cross section of the produced particles in resonance production, and a description of this phenomenon in terms of the multiple-scattering series has been given.[3] Small σ_{AN}, corresponding to small absorption, leads to rather flat distributions compared with larger σ_{AN}. As in the A dependence, we know that for the case $\beta=0$, the result for $d\sigma/dt$ on a given nucleus must be independent of σ_{AN}, and as β is allowed to approach 1, the slope would be expected to change from that appropriate to uncorrelated $\rho\pi$ production to that appropriate to resonance production at whatever value of σ_{AN} we are considering. In Figs. 1(c) and 1(d) we show how this works in the case of ^{208}Pb targets. If σ_{AN} is small, the resonance-production slope is flatter than that for $\rho\pi$ production, so as β increases, the slope decreases. For 50 mb, however, we have just the opposite situation—the resonance-production slope is steeper than that for uncorrelated $\rho\pi$ production, so as β increases, the slope also increases. The crossover point, where the slopes of resonant and nonresonant production are nearly equal so that the total slope is independent of β, occurs at about 30 mb.

There is a rather nice way of presenting these results which, although it involves some approximation, provides a concise way of presenting the dependence of the results on σ_{AN} and β. We note that for $A\lesssim100$, we can write

$$(d\sigma/dt)_{\Delta^2=0}\sim A^n, \tag{11}$$

where n is a parameter which characterizes the slope of the lines in Figs. 1(a) and 1(b). Similarly, for small Δ^2, we can write

$$d\sigma(\pi+\mathrm{Pb}\rightarrow\rho\pi+\mathrm{Pb})/dt\sim e^{-\alpha\Delta^2}, \tag{12}$$

where α is the slope of the differential cross section on a given nucleus (in our case, on lead). If we then plot n vs α for each σ_{AN} value, we get the type of curves shown in Fig. 2. When $\beta=0$, all values of σ_{AN} give the same A dependence and slope, as discussed above. For $\sigma_{AN}=10$ mb, the observed steepening of the slope in the A dependence and flattening of the slope in the Δ^2 dependence as β is increased is represented by a line moving upward and to the left from the $\beta=0$ point. Similarly, for $\sigma_{AN}=50$ mb, the observed insensitivity of the A plot to changes in β, and the increase in slope on the Δ^2 plot as β is increased, is represented by a curve moving almost vertically to the right from the $\beta=0$ point.

A recent comparison of A_1 production on hydrogen and in heavy-liquid chambers[6] first introduced the basic idea of separating resonant from nonresonant production through nuclear techniques which we have investigated here. However, because these workers had only two targets and relatively poor statistics, a systematic anal-

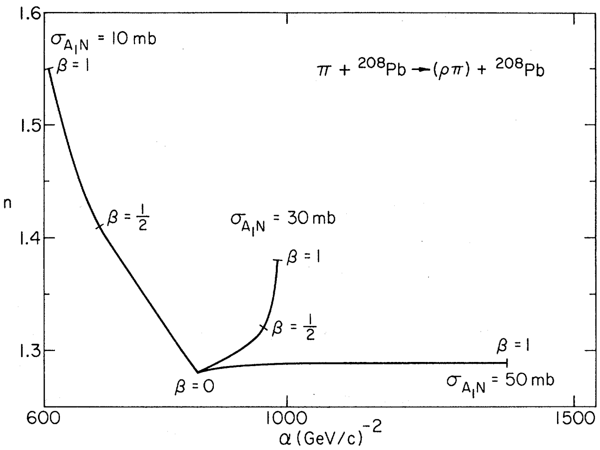

FIG. 2. The plot of n vs α discussed in the text. The values of α are considerably larger than those usually reported because here they represent the slope of the coherent cross section only at $\Delta^2 = 0$, whereas the usual values of α which are quoted represent the slope for both coherent and incoherent processes averaged over a large range of Δ^2.

ysis of the type suggested here could not be carried out. Data which could be analyzed as we have suggested have been taken,[7] and we await the results of that work with interest.

Finally, it should be noted that the theoretical analysis which has been presented here could be checked by looking at processes in which it is known that resonances are produced (e.g., the reaction $\gamma + A \to \rho^0 + A$), and where there is only one free parameter (the parameter x), and verifying that in mass bands near the resonance, we have $x \to 1$, and in mass bands far from the resonance, $x \to 0$. This investigation, as well as calculations of the effect of using wave functions other than (2), are under way and will be reported when completed.

It is a pleasure for the author to thank Professor U. Kruse and Professor T. O'Halloran for many helpful discussions, and N. Sanders for assistance in the preparation of the manuscript.

*Work supported by the National Science Foundation under Grant No. NSF GP 9273.

[1]S. D. Drell and K. Hiida, Phys. Rev. Letters 7, 199 (1961); R. T. Deck, Phys. Rev. Letters 13, 169 (1964); U. Maor and T. O'Halloran, Phys. Letters 15, 281 (1965); L. Resnick, Phys. Rev. 150, 1292 (1966); U. Maor, Ann. Phys. 41, 456 (1967); D. R. O. Morrison, Phys. Letters 22, 226 (1966); L. Stodolsky, Phys. Rev. Letters 18, 973 (1967).

[2]R. J. Glauber, in Lectures Delivered at the Summer Institute for Theoretical Physics, University of Colorado, 1958-59, edited by W. E. Brittin and G. Dunham (Interscience Publishers, Inc., New York, 1959), Vol. I, p. 315, and High Energy Physics and Nuclear Structure, edited by G. Alexander (North Holland Publishing Company, Amsterdam, The Netherlands, 1967).

[3]J. S. Trefil and J. Formanek, Nucl. Phys. B3, 155 (1967), and B4, 165 (1968); B. Margolis, Phys. Letters 26B, 524 (1968), and Nucl. Phys. B4, 433 (1968).

[4]J. S. Trefil, Phys. Rev. 180, 1366, 1379 (1969). This reference also contains a discussion of the validity of the Glauber theory and references to earlier work.

[5]J. S. Trefil, to be published.

[6]A. S. Goldhaber, C. J. Joachain, H. J. Lubatti, and J. J. Veillet, Phys. Rev. Letters 22, 802 (1969).

[7]C. Bemporad, in Proceedings of the Conference on High Energy Reactions of Elementary Particles with Nuclei, Stony Brook, New York, 2-3 May 1969 (unpublished), and private communication.

REAL PARTS OF VECTOR-MESON SCATTERING AMPLITUDES*

J. Swartz and R. Talman
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 20 August 1969)

The determination of the ρ-photon coupling constant by photoproduction from complex nuclei can be brought into agreement with other determinations by invoking a real part of the ρ-nucleon scattering amplitude, thus determining the magnitude of the real part. A similar analysis with φ mesons is possible with available data.

In analyses up to this time[1-3] of photoproduction of vector mesons from complex nuclei it has been assumed that the real part of the ρ-nucleon scattering amplitude is negligible. We argue here that the presence of a substantial real part can clear up the present discrepancy for ρ's between the value of the photon-ρ coupling constant $\gamma_\rho^2/4\pi$ as determined by photoproduction and as determined by other methods. At the same time the value of $\sigma_{\rho N}$, the ρ-nucleon total cross section, is brought into agreement with the quark-model prediction. A similar analysis of φ photo-