VENEZIANO AMPLITUDE: INTERFERENCE MODELS AND ARGAND LOOPS*

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In the Veneziano model, at large s and fixed t, the s-channel resonances sum to give $\Gamma(1-\alpha(t))(-s)^{\alpha(t)-1}$ and the u-channel resonances sum to $\Gamma(1-\alpha(t))s^{\alpha(t)-1}$. Using this feature, we propose a new interference model for intermediate energies which gives a significantly improved fit to the backward π^-p data, and we make comments on the Schmid Argand loops.

The Veneziano representation[1] contains the very desirable features of direct-channel poles (narrow resonances), asymptotic Regge-pole behavior, and crossing symmetry. As noted by Veneziano, in his model (for $\pi\pi\to\pi\omega$), the full Regge behavior for large s and fixed t

$$\Gamma(1-\alpha(t))(1-e^{-i\pi\alpha(t)})s^{\alpha(t)-1} \tag{1a}$$

originates from two parts. The first term

$$\Gamma(1-\alpha(t))s^{\alpha(t)-1} \tag{1b}$$

comes from the u-channel resonances, and the second term

$$\Gamma(1-\alpha(t))(-s)^{\alpha(t)-1} \tag{1c}$$

from the s- or direct-channel resonances. If we generalize this feature of his model, we can make a number of observations which shed considerable light on the questions of interference models,[2] Schmid Argand loops,[3] and exchange degeneracy.[4]

(i) Interference models: At large s and fixed t we have the Regge behavior (1a) for the amplitude. At moderate energies, interference models[2] have been used extensively to parametrize the amplitude. These interference models have used direct-channel resonances interfering with Regge-pole terms of the form (1a). Many papers have been concerned with the problem of double counting in this model.[5] The opposite approach has been used by Dikmen[6] who uses just s-channel resonances (and no Regge terms) for the amplitude. Both of these extreme approaches appear to yield good fits to the data. We observe that neither of these two models is in agreement with the Veneziano representation which gives an interference model (intermediate to these two) in which s-channel resonances are added to the Regge form (1b) without the term (1c).[7] Our model applied to the backward π^-p data gives a significantly improved fit (see Fig. 1) as compared with the calculations of Refs. 2 and 6.

(ii) Schmid Argand loops: Schmid[4] has performed partial-wave projections of the Regge amplitude (1a) and shown that loops in Argand diagrams occur. These loops are then identified as direct-channel resonances. We show, in Fig. 2, that these loops come entirely from the direct-channel term (1c) and that the "exchange" term (1b) just adds an almost constant background to the "resonance" loop.

(iii) Exchange degeneracy: Following Schmid's argument above, whenever one could have a Regge-exchange term, one expected to see direct channel resonances. When none occurred, as in K^+p scattering, one invoked a cancellation due to two trajectories which were exchange degenerate.[3,4] In the light of our remarks in (ii), we claim that the use of the full amplitude (1a) already implies the existence of the direct-channel resonances. If none exists then the Regge term is of the form (1b)[8] which does not yield the Schmid loops. There is no imaginary part of the amplitude associated with the crossed channel term (1b). Any imaginary part (for physical s) must come from the s-channel cut (or s-channel poles in the Veneziano model). The features of direct-channel resonances and exchange-degenerate trajectories appear as a result of the dynamics or symmetries in nature and not mysteriously from Regge behavior.

To illustrate our points we choose the simplest form of the Veneziano amplitude,

$$T(x,y,z)=A(x,y)+A(x,z)+A(y,z), \tag{2}$$

where

$$A(x,y)=\frac{\Gamma(1-x)\Gamma(1-y)}{\Gamma(2-x-y)} \tag{3}$$

and $x=\alpha(s)$, $y=\alpha(t)$, $z=\alpha(u)$. We assume that the same trajectory is exchanged in each channel and that the trajectory is linear $[\alpha(s)=\alpha_0+\alpha's]$ so that

$$x+y+z=3\alpha_0+\alpha'(s+t+u)=\text{const}=c.$$

We restrict ourselves to the physical s channel where $1-y>0$ and $1-z>0$. In this region we can expand $A(x,y)$ and $A(x,z)$ exactly as a sum of

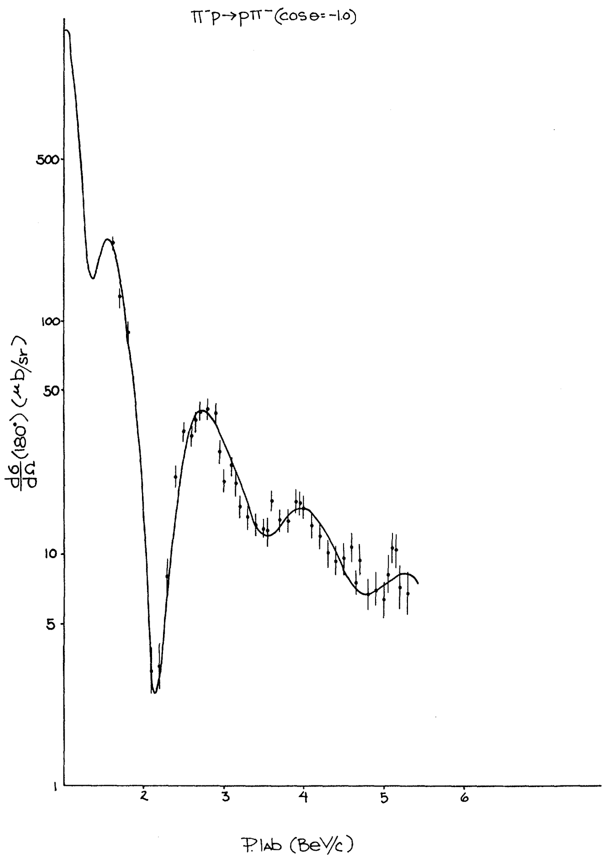

FIG. 1. Theoretical curve for the 180° π^-p elastic-scattering differential cross section as a function of laboratory momentum. The following resonances [with their respective spin-and-parity assignments, widths (in BeV), and elasticities] are used:

$\Delta_\delta(1236)$, $\frac{3}{2}^+$, 0.12, 1.0; $\Delta_\delta(1940)$, $\frac{7}{2}^+$, 0.21, 0.35; $\Delta_\delta(2420)$, $\frac{11}{2}^+$, 0.31, 0.167; $\Delta_\delta(2850)$, $\frac{15}{2}^+$, 0.40, 0.07; $\Delta_\delta(3230)$, $\frac{19}{2}^+$, 0.44, 0.03; $N_\gamma(1515)$, $\frac{3}{2}^-$, 0.115, 0.60; $N_\gamma(2190)$, $\frac{7}{2}^-$, 0.30, 0.18; $N_\gamma(2650)$, $\frac{11}{2}^-$, 0.36, 0.035; $N_\gamma(3030)$, $\frac{15}{2}^-$, 0.40, 0.014; $N_\gamma(3350)$, $\frac{19}{2}^-$, 0.10, 0.007; $N_\alpha(1690)$, $\frac{5}{2}^+$, 0.125, 0.60; $N_\gamma(2210)$, $\frac{9}{2}^+$, 0.22, 0.092; $N_\alpha(2610)$, $\frac{13}{2}^+$, 0.31, 0.025.

Our Regge contribution from the exchange of the Δ_δ trajectory is of the same form as that of Ref. 2 without the signature factor. The residue, γ, is here taken to be 0.06. The experimental points are those of S. W. Kormanyos et al., Phys. Rev. 164, 1661 (1967).

poles in x,[9]

$$A(x,y)=\Gamma(1-y)\sum_{n=1}^{\infty}\frac{(-1)^n}{(n-1)!\,\Gamma(2-n-y)}\frac{1}{x-n}, \quad (4)$$

$$A(x,z)=\Gamma(1-z)\sum_{n=1}^{\infty}\frac{(-1)^n}{(n-1)!\,\Gamma(2-n-z)}\frac{1}{x-n}. \quad (5)$$

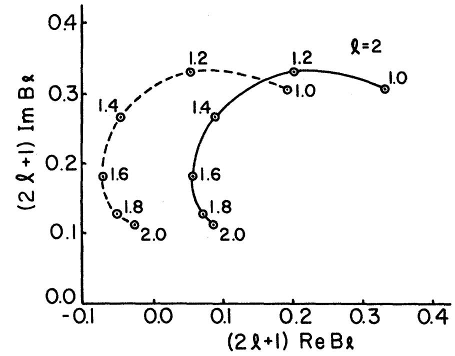

FIG. 2. Argand plot of the amplitude. The solid and dashed lines refer to the $l=2$ partial-wave projections of Eqs. (12) and (13), respectively. The numbers refer to the incident laboratory energy of the pion in BeV. We normalize by setting $\beta(t)=1$, as in Fig. 1 of Ref. 3.

If we keep y fixed and let $x\to\infty$ (letting Im$x\to\infty$ also so that Stirling's formula can be used), then

$$A(x,y)\sim\Gamma(1-y)(-x)^{y-1} \quad (6)$$

so that $A(x,y)$ has Regge-like behavior $s^{\alpha(t)-1}$. The t-channel singularities in $A(x,y)$ are contained in the overall factor $\Gamma(1-y)$. Thus the Regge behavior for large s comes from the sum over s-channel poles. A similar statement can be made for $A(x,z)$ for fixed u [see Eq. (10)].

The term

$$A(y,z)=\frac{\Gamma(1-y)\Gamma(1-z)}{\Gamma(2-y-z)}$$

has no s-channel poles since $1-y$ and $1-z$ are both positive in the physical s channel. Since $1-y>0$ we can expand $A(y,z)$ as a sum of poles in z (or y). Then if we hold y fixed and let $x\to\infty$, we find that the Regge behavior of $A(y,z)$ at large x comes from the sum over the u-channel poles.

We now write down the asymptotic behavior of each term individually for the case that y is held fixed and $x\to\infty$:

$$A(x,y)\sim\Gamma(1-y)(-x)^{y-1}, \quad (7)$$

$$A(x,z)\sim\frac{\sin\pi(y-c)}{\sin\pi x}\Gamma(c-y-1)x^{y+1-c}, \quad (8)$$

$$A(y,z)\sim\Gamma(1-y)x^{y-1}. \quad (9)$$

Since we let $x\to\infty$ in such a way that its complex part becomes large, the term $A(x,z)$ will be relatively unimportant at large s. Thus the signature factor ordinarily found in Regge theory is a result of adding $A(x,y)$, a term containing s-channel resonances, and $A(y,z)$, a term containing no s-channel poles. Clearly, there is interference between the direct-channel resonances and a Regge term asymptotically. Instead of predicting that there is no interference, the Veneziano model gives an explicit method for adding an interference term to the usual s-channel resonances. By using the Veneziano amplitude we have a model to tell us how much double counting was done in the interference model of Barger and Cline[2] and how much is left out in the resonance model used by Dikmen.[6] For example, in our simple model we would add up the s-channel resonances plus a Regge term (1b) without a signature factor [i.e., we omit the term $(-1)^{\alpha(t)}$]. The full Regge amplitude (1a) would be built up asymptotically as a result of the interference between the resonances (1c) and the crossed channel Regge term (1b).

In the Barger-Cline model[2] we hold u fixed. In this case the important amplitudes are[10] (for large x)

$$A(x,z)\sim\Gamma(1-\alpha(u))(-s)^{\alpha(u)-1}, \tag{10}$$

and

$$A(y,z)\sim\Gamma(1-\alpha(u))(-t)^{\alpha(u)-1} \approx\Gamma(1-\alpha(u))s^{\alpha(u)-1}. \tag{11}$$

Then the proper signature factor comes from adding these two terms. The factor $\Gamma(1-\alpha(u))$ contains the u-channel singularities and the asymptotic behavior $(-t)^{\alpha(u)-1}$ comes from the sum over the t-channel poles. Our interpretation of the Veneziano model implies that Barger and Cline should not have included the entire signature factor in their interference term. The part involving $(-1)^{\alpha(u)}$ should be omitted since it comes asymptotically from the direct-channel resonances. Following Barger and Cline[2] we fit the backward π^-p data by having the u-channel Δ_δ trajectory interfere with the direct s-channel resonances. Using the same s-channel resonances and our form for the Δ_δ exchange, we obtain the curve presented in Fig. 1. Our fit has a χ^2 smaller by a factor of 3 than does the fit presented by Barger and Cline[2] or that of Dikmen[6] in the direct-channel resonance model. Our model also predicts backward π^-p polarization which is significantly different from the prediction of Barger and Cline. These details will be presented in a future publication.

Schmid did a partial-wave analysis of the Regge amplitude

$$B(s,t)=\beta(t)\left(\frac{E}{E_0}\right)^{\alpha-1}\frac{1-e^{-i\pi\alpha}}{\Gamma(\alpha)\sin\pi\alpha}, \tag{12}$$

where $\alpha=0.57+1.08t$, $E_0=0.7$ BeV gives a fit to πN charge-exchange scattering. Schmid found that a plot of $\mathrm{Re}B_l$ vs $\mathrm{Im}B_l$ revealed counterclockwise loops in the Argand diagram which one might be tempted to call resonances. According to the idea presented here the term $e^{-i\pi\alpha}$ actually comes asymptotically from the sum of direct-channel resonances while the rest comes from the crossed channel. We show in Fig. 2 that if we make a partial-wave analysis of

$$B'(s,t)=-\beta(t)\left(\frac{E}{E_0}\right)^{\alpha-1}\frac{e^{-i\pi\alpha}}{\Gamma(\alpha)\sin\pi\alpha} \tag{13}$$

then all of the "resonant" behavior in fact comes from this term and the remainder of B merely contributes a slowly varying background. Even though the imaginary parts of B and B' are identical it is not immediately obvious that the term B' should contain all of the resonant behavior. For example, it is entirely possible that B' could describe clockwise loops in the Argand diagram because of the difference in the real parts of B and B'. We have the interesting situation in which the asymptotic form of a sum of resonances apparently retains some "memory" of the resonances and yields resonantlike behavior in the Argand diagram.

We have seen that the direct-channel resonances do not build up asymptotically to give all of the amplitude, but rather the direct-channel resonances interfere with the cross-channel Regge term (1b) so that the correct full Regge amplitude (1a) is produced asymptotically.[11] We use this result to propose a new interference model which gives a significantly improved fit to the backward π^-p data. Further applications of this model will be published elsewhere. Our interpretation of the Veneziano amplitude also leads to some understanding of the Schmid Argand loops.

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[1]G. Veneziano, Nuovo Cimento 57A, 190 (1968).
[2]V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966); and Phys. Rev. 155, 1792 (1967).
[3]C. Schmid, Phys. Rev. Letters 20, 689 (1968).
[4]See for example R. C. Arnold, Phys. Rev. 153, 1506 (1967).
[5]L. Durand, Phys. Rev. 166, 1680 (1968).
[6]F. N. Dikmen, Phys. Rev. Letters 18, 798 (1967).
[7]See Ref. 10.
[8]It is shown by T. Gaisser and C. E. Jones (to be pub-

lished) that an amplitude without signature can be continued in l.

[9]E. T. Whittaker and G. N. Watson, Modern Analysis (Cambridge University Press, New York, 1958), p. 260.

[10]In πN scattering a typical (t,u) term is $\Gamma(1-\alpha_\rho(t)) \times\Gamma(\frac{1}{2}-\alpha_\Delta(u))/\Gamma(\frac{3}{2}-\alpha_\rho(t)-\alpha_\Delta(u))$ which for large s and fixed u becomes $\Gamma(\frac{1}{2}-\alpha_\Delta(u))(-\alpha_\rho(t))^{\alpha_\Delta(u)-1/2}\propto s^{\alpha_\Delta(u)-1/2}$ (we use Δ for the Δ_δ trajectory). The corresponding (s,u) term containing the s-channel resonances adds in to give the full Regge signature for fixed u. For the full πN amplitude we refer the reader to K. Igi, Phys. Letters 28, 330 (1968).

[11]None of what we have said applies to the Pomeranchuk trajectory. There is no simple way to include it in the Veneziano model. See F. Gilman, H. Harari, and Y. Zarmi, Phys. Rev. Letters 21, 323 (1968).

ERRATUM

HIGHLY INELASTIC pp SCATTERING AND ITS INTERPRETATION. D. Garelick [Phys. Rev. Letters 22, 674 (1969)].

The right-hand side of Eq. (4) should read

$$=\frac{d^2\sigma}{d\Omega_3 dp_3}\frac{1}{p_3^{\,2}}\frac{P_\perp}{19.2}\text{ (lab system)},$$

not

$$\frac{d^2\sigma}{d\Omega_3 dp_3}\frac{P_\perp}{P_3^{\,2}}\frac{P_\perp}{19.2}\text{ (lab system)},$$

and the end of Ref. 5 should read "$\cdots$a factor of 3, where $X_\perp \simeq 2.0\ \mathrm{GeV}^2$."

SELECTION RULE FOR NONLEPTONIC HYPERON DECAYS AS A CONSEQUENCE OF DUALITY. Mahiko Suzuki [Phys. Rev. Letters 22, 1217 (1969)].

The discussions on elimination of meson-pole transitions and on lack of resonances in the $(S\Sigma^-)$ system are incomplete and misleading. The conclusion of this Letter is invalid without additional assumptions.