be obtained by doing prior-form exact finite-range calculations with various radial cutoffs in the projectile interior wave function: Eliminating successively greater amounts of the projectile interior from the calculation in 1-fm increments changes at every step mainly the magnitude of the cross section and leaves its shape relatively unchanged, implying that any interior wave function would yield approximately the same shape as any other.

The calculations were performed on the University of Texas CDC 6600 computer, and I wish to thank the computation center there for making time available.

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INVESTIGATION OF BAND-MIXING ANOMALIES IN Sm^{152} †

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Multiple Coulomb-excitation measurements of absolute transition-matrix elements from β-band states in Sm^{152} indicate that the previously reported discrepancies with band mixing arise from a source different from that previously suggested; the matrix elements for the $2_\beta \to 2$, $0_\beta \to 2$, and $2_\beta \to 0$ transitions are consistent with a band-mixing parameter $Z_0 \simeq 0.08$, although an inconsistency remains for the $2_\beta \to 4$ transition. Angular correlation studies show that $\delta(2_\beta \to 2) = -67^{+54}_{-\infty}$ and $\delta(4_\beta \to 4) = -2.94^{+0.85}_{-1.86}$.

Recent data[1-6] on the relative magnitudes of $E2$ matrix elements connecting β-vibrational and ground-state-rotational bands in deformed nuclei indicate an apparent inconsistency with the angular-momentum dependence of the $E2$ intensity rules predicted by the axially symmetric rotational model modified to include coupling of intrinsic and rotational motions. This is in contrast to the more satisfactory situation involving the γ-vibrational bands.[5-7] In a recent review paper[8] Mottelson drew attention to these differences and noted that, since the theory used for these comparisons is founded on a very general treatment of rotational band structure, the apparent anomalies observed for the β-band de-excitations pose a fundamental challenge to the rotational description. We have obtained new evidence, based on measurements reported herein, which indicates that these anomalies arise from a different source than hitherto conjectured; significant discrepancies remain, however, and their elucidation may provide new insight into nuclear collectivity in the regions studied.

Within the general framework of the rotational description, the simple rotational-model intensity rules appear as the leading order term in a series expansion of the transition matrix elements as powers of the total angular momentum, with higher order terms representing the effects of coupling between the intrinsic and rotational motion. If terms up to first order in the spin-dependent corrections are included, the $E2$ reduced transition probabilities for the de-excitation of β- and γ-band states to ground-band states are given, respectively, by[8-10]

$$B(E2; K=0, I_\beta \to K=0, I) = \langle I_\beta 020 | I0\rangle^2 |M_1{}^\beta + M_2{}^\beta [I(I+1) - I_\beta(I_\beta+1)]|^2 \tag{1}$$

$$= \langle I_\beta 020 | I0\rangle^2 |M_1{}^\beta|^2 \{1 + Z_0[I(I+1) - I_\beta(I_\beta+1)]\}^2, \tag{1a}$$

$$B(E2; K=2, I_\gamma \to K=0, I) = 2\langle I_\gamma 22-2 | I0\rangle^2 |M_1{}^\gamma + M_2{}^\gamma [I(I+1) - I_\gamma(I_\gamma+1)]|^2 \tag{2}$$

$$= 2\langle I_\gamma 22-2 | I0\rangle^2 |M_1{}^\gamma|^2 [1 + Z_2 f_2(I_\gamma I)]^2, \tag{2a}$$

where $M_1{}^\beta$ and $M_1{}^\gamma$ are the leading-order matrix elements that connect the β and γ bands, respectively, with the ground-state band. In addition to the more general results given by Eqs. (1) and (2), we have included Eqs. (1a) and (2a) which are parametrized for the familiar special case where the deviations from the simple rotational model $E2$ intensity rules are attributed to a direct coupling between the ground-state band and the β and γ bands. The band-mixing parameters[10] Z_0 and Z_2 are related to the matrix elements in Eqs. (1) and (2) by $Z_0 = M_2{}^\beta/M_1{}^\beta$, and $Z_2 = 2M_2{}^\gamma/(M_1{}^\gamma - 4M_2{}^\gamma)$.[8,9]

Most of the available data which demonstrate the inconsistencies noted above derive from measurements of branching ratios for the decay of β-band states in the nuclei Sm^{152}, Gd^{154}, Gd^{156}, and Hf^{178}.[1-6] In none of these cases can a consistent Z_0 be obtained to fit all the relative transition strengths from the 2_β and 4_β states. The most accurate and complete branching-ratio information has been obtained on the decay of the 2_β state. It indicates that the strengths of the $2_\beta \to 2$ transitions are, for all cases, a factor of 2 or 3 larger than is required for consistency with Eqs. (1) and (1a). A suggestion[1] that the discrepancy with Eqs. (1) and (1a) can be removed by the presence of a large $M1$ admixture in the $2_\beta \to 2$ transitions has been tested in Gd^{154} [11] and Hf^{178} [12] with mixed results. The former measurement shows essentially pure $E2$, while the latter indicates $(17 \pm 10)\%$ $E2$. We may note that recent measurements[5] of branching ratios from the 2_γ states show that the matrix elements $\langle 2_\gamma \| E2 \| 2_\beta \rangle$ in Sm^{152} and Gd^{154} are large, which implies a large mixing between the β and γ bands in both these nuclei. However, consideration of the next group of terms in the expansion in Eqs. (1) and (1a), reflecting higher order mixing or differences in the intrinsic quadrupole moments between the ground-state band and the β and γ bands, does not appreciably improve the agreement between the theory and the branching-ratio data on the de-excitation of the β-band states.[5] This discrepancy may imply inapplicability of the perturbation expansion in these particular cases.

Since the branching ratios presently available only provide a consistency check for the one-parameter band-mixing model, it is desirable that the experimental information be expanded to include measurements of absolute transition rates as well as $M1$ admixtures. This paper reports such studies on Sm^{152}.

Measurements of multiple Coulomb excitation induced by both S^{32} and O^{16} ions have provided absolute transition matrix elements, branching ratios, and detailed angular-distribution data. The availability of high-energy S^{32} ions was of particular importance in these studies because of the enhancement factor of approximately 20 for S^{32}-induced, compared with O^{16}-induced, excitation probabilities for the 2_β and 4_β states. We emphasize that the tests for the presence of $M1$ admixture in the $I \to I$ transitions were performed with coincidence measurements where the correlations are large and the presence of a large A_4 term is usually decisive in resolving the ambiguity in the magnitude of the decuded $E2/M1$ mixing ratio δ.

The O^{16} projectile energy was varied from 48 to 60 MeV. Excitation probabilities were obtained for the 2^+, 4^+, 6^+, 8^+, and 10^+ states in the ground-state band; 0^+, 2^+, and 4^+ states in the β band; and 2^+ and 4^+ states in the γ band. The discussion of the ground-state–band transitions is presented separately in an accompanying Letter, although it should be pointed out that the strong coupling between states dictates that all the states listed be included simultaneously in the analysis procedure, as well as other states that are involved in the multiple Coulomb-excitation process.

The γ-ray relative intensities and reduced transition probabilities for the de-excitation of the 0_β, 2_β, 4_β, 2_γ, and 4_γ states are listed in Table I. These data again illustrate the band-mixing parameter inconsistencies[1,2,5,6] noted above. The measured angular-distribution coefficients obtained from the coincidence measurements for the $I \to I$ transitions, as well as the corresponding $E2/M1$ mixing ratios, are given in Table I and Fig. 1. Except for possibly the $4_\gamma \to 4$ transition, for which two alternative solutions exist, in no case is there any sizable $M1$ admixture. For the latter transition the experimental data greatly favor the solution near pure $E2$, but 40% $M1$ is not excluded: There is some indication that the $M1$ admixtures increase with increasing spin. We note that the alternate method of determining the $E2/M1$ mixing ratio by measuring de-excitation branching ratios and excitation $B(E2; I \to I')$ values proved not to be accurate because of the insensitivity of the 2_β-state population to the values of the matrix element $\langle 2 \| E2 \times \| 2_\beta \rangle$.

The analysis for absolute reduced transition probabilities was performed with the aid of the Winther–deBoer multiple Coulomb-excitation

Table I. Relative γ-ray intensities, absolute $B(E2)$ values, band-mixing parameters, angular-correlation coefficients, and $E2/M1$ mixing ratios for Sm^{152}.

Transition $I_i \to I_f$	Relative gamma-ray intensity	$B(E2, I_i \to I_f)$[a] $e^2 \cdot 10^{-50} cm^4$	Z_0, Z_2 × 100	A_2[b]	A_4[b]	δ[c]
$0_\beta \to 2$		18 ± 2				
$2_\beta \to 0$	1	0.456 ± 0.034[e]				
$2_\beta \to 2$	2.50 ± 0.18	2.58 ± 0.26	8.3 ± 0.3 2.8 ± 0.4	-0.131 ± 0.09	-0.50 ± 0.15	$-67^{+54}_{-\infty}$
$2_\beta \to 4$	0.98 ± 0.10	9.1 ± 1.1				
$4_\beta \to 2$	1	0.53[f]	4.6 ± 0.3			
$4_\beta \to 4$	1.56 ± 0.32	3.7[f]	or 9.7 ± 0.3	0.014 ± 0.13	-0.59 ± 0.23	$-2.94^{+0.85}_{-1.86}$
$2_\gamma \to 0$	1	1.63 ± 0.11[e]				
$2_\gamma \to 2$	1.41 ± 0.11	4.17 ± 0.42	10 ± 2 6.7 ± 1.1	-0.207 ± 0.036	-0.55 ± 0.07	27^{+55}_{-11}
$2_\gamma \to 4$	0.0324 ± 0.0013[d]	0.416 ± 0.032				
$4_\gamma \to 2$	1	0.35 ± 0.013				
$4_\gamma \to 4$	3.56 ± 0.30	3.7 ± 0.13	7.6 ± 0.9	-0.36 ± 0.06	-0.51 ± 0.10	$12.9^{+\infty}_{-7.6}$[g]

[a]Obtained from relative intensity after correction for $E2/M1$ mixing.
[b]A_2 and A_4 describe the correlation of γ rays emitted by the $m=0$ magnetic substate. Corrections for finite solid angles of the particle and γ-ray detectors have been applied.
[c]δ is determined from A_2 only.
[d]From Ref. 5.
[e]From F. K. McGowan, R. O. Sayer, P. H. Stelson, R. L Robinson, and W. T. Milner, Bull. Am. Phys. Soc. 13, 895 (1968).
[f]Subject to large uncertainty; see text.
[g]$\delta = 1.22^{+0.32}_{-0.18}$ not excluded.

computer code.[13] A set of input matrix elements was sought for which the calculated excitation probabilities reproduced the experimental excitation probabilities for all states as functions of projectile energy and species. The matrix elements $\langle 0 \| E2 \| 2\rangle = 1.844e$ b,[14] $\langle 0 \| E2 \| 2_\beta\rangle = 0.151e$ b,[15] and $\langle 0 \| E2 \| 2_\gamma\rangle = 0.285e$ b[15] were used as inputs to the calculations, and the relative values of other matrix elements were fixed by the measured branching ratios.[5,6,16] Extensive studies were carried out with the computer code, using $E1$, $E2$, $E3$, and $E4$ multipole excitations, to explore the sensitivity of the excitation probability of each state to changes in the magnitude and phase of matrix elements, and to determine the effect of including both positive- and negative-parity, high-spin states not directly observed in the experiments. Details of these studies will be discussed in another publication. Here we point out that with all phases fixed to conform with the collective model, the value of the matrix element $\langle 2 \| E2 \| 0_\beta\rangle$ has been found to be insensitive to reasonable variations in the values for the other matrix elements for O^{16} bombardment. However, the same conclusions do not apply for the matrix elements associated with the 4_β state, which depend sensitively on the magnitude and sign of matrix elements connecting the β and γ bands. Since the information on the β- to γ-band transitions is incomplete, the absolute values of matrix elements from the 4_β state must be qualified as giving the best fit to the data based on existing experimental information only, and hence subject to large uncertainty. We have omitted the 4_β state in the comparisons presented below.

Figures 2(a) and 2(b) compare the angular-momentum dependence of the measured $E2$ matrix elements for 0_β, 2_β, 2_γ, and 4_γ states with the predictions of Eqs. (1a) and (2a). The fit to the γ-band de-excitations with a unique Z_2 is reasonable. In Fig. 2(a) the addition of the newly measured $B(E2; 0_\beta \to 2) = 0.18 \pm 0.02(e\ \mathrm{b})^2$ indicates that a typical value of $Z_0 = 0.055$, based on earlier measurements of the ratio $B(E2; 2_\beta \to 4)/B(E2;$

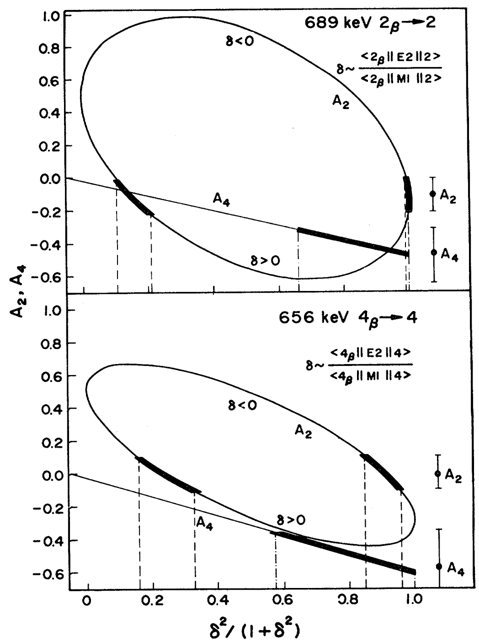

FIG. 1. Experimental values for the angular-correlation coefficients A_2 and A_4. $\delta^2/(1+\delta^2)$ is the fraction of the total transition rate proceeding by $E2$ multipolarity.

$2_\beta \to 0$), is inconsistent with at least two of the measurements. A value of $Z_0 = 0.08$, however, is consistent with three of the four data points. If we still choose to interpret the data in the context of Eq. (1a), it now appears that a value of

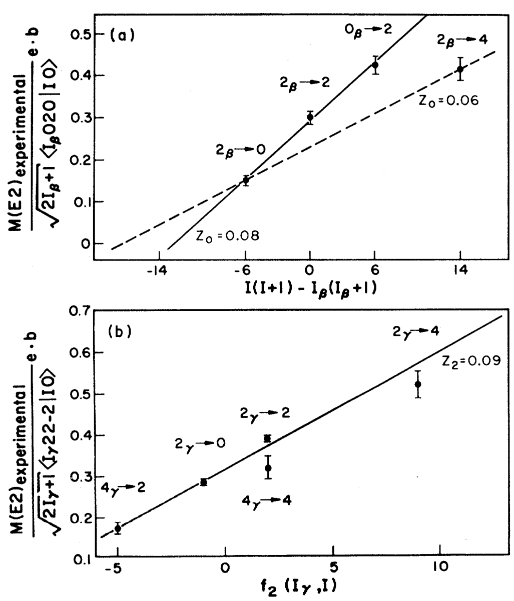

FIG. 2. Absolute $E2$ transition matrix elements measured for (a) β-band and (b) γ-band de-excitations plotted versus the angular-momentum–dependent factors in Eqs. (1a) and (2a).

$Z_0 = 0.08$ is more representative of the band mixing, and that the intensity of the $2_\beta \to 2$ transition is not anomalous, contrary to previous conjectures. The incompatibility of the $2_\beta \to 4$ transition matrix element with the latter value for Z_0, however, again questions the validity of applying the perturbation expansion to these transition nuclei.

We note again that attempts to reproduce all the known branching ratios in Sm^{152} by including higher terms in the expansion (1) have not been successful.[5] It is clear that a more exact theoretical treatment of the mixing of at least the three bands appears essential if we are to retain the inherent simplicity of the collective models in providing a representation of these nuclei.

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E2 MATRIX ELEMENTS BETWEEN HIGH-SPIN STATES IN THE GROUND-STATE BANDS OF Sm^{152} AND Sm^{154} †

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Measurements of transition-matrix elements between states with spin up to 10^+ in the ground-state bands of Sm^{152} and Sm^{154} indicate large deviations from the adiabatic rotational-model predictions in Sm^{152} but not in Sm^{154}. The nonrotational effects are compared with band mixing deduced from β-band de-excitation matrix elements and with measurements of changes in the mean-square radius with increasing spin.

One of the most direct tests for rotational character of nuclear collective motion is the constancy of the intrinsic quadrupole moment for states within a rotational band. Although a number of measurements[1-4] have been reported on low-lying members, experimental considerations have precluded systematic studies of matrix elements at higher excitation energies where nonrotational effects might be expected to be most prominent. The recently available S^{32} and O^{16} beams from Yale's Model MP tandem accelerator have permitted such studies,[5] and we report herein on the situation in Sm^{152} and Sm^{154}. In the former nucleus, but not in the latter, we find large deviations from the adiabatic rotational-model predictions.

The deviations may be attributed in part to the presence of significant couplings between rotational and intrinsic motion which are neglected in the adiabatic approximation.[6] It has been shown that, within a general description where the rotational character dominates, the angular-momentum dependence of the admixture amplitude does not depend on the specific form of the interaction couplings; the mixing effects appear as correction terms in an expansion of the transition-matrix elements in the powers of the total angular momentum.[7] To order I^2 in the intrinsic moment, the angular-momentum dependence for transitions within a $K=0$ band may be written as[7]

$$\frac{B(E2;K=0,I_i\rightarrow K=0,I_f)}{B(E2;K=0,I=0\rightarrow K=0,I=2)} = \frac{\langle I_i020|I_f0\rangle^2}{\langle 0020|20\rangle^2}\left[1+(\alpha_0+\alpha_2)(I_i^{\,2}+3I_i)\right]^2, \quad (1)$$

for $I_f=I_i+2$.

If the specific mixing mechanisms are considered, it is expected that mixing the low-lying β and γ bands may account for a large part of the correction term to the leading-order $E2$ intensity rules. For this particular case the parameters α_0 and α_2 are related to the mixing parameters Z_0 and Z_2 for the β and γ bands, respectively, through the following relationships[3]:

$$\alpha_0=\left[\frac{2Z_0}{(1-6Z_0)^2}\right]\left[\frac{B(E2;0\rightarrow 2_\beta)}{B(E2;0\rightarrow 2)}\right], \quad (2)$$

$$\alpha_2=\left[\frac{Z_2}{6(1-Z_2)^2}\right]\left[\frac{B(E2;0\rightarrow 2_\gamma)}{B(E2;0\rightarrow 2)}\right]. \quad (3)$$

Of course there are other effects,[8] in addition to the above mechanism of centrifugal stretching, which may contribute to changes in nuclear properties under the influence of rotations. The measurements of $B(E2)$ values in conjunction with isomer-shift and level-energy measurements of ground-state–band members provide one of the more direct methods for evaluating the influence of centrifugal stretching and antipairing induced by the Coriolis force.[8-11]

We report here on measurements of $E2$ matrix elements in the ground-state bands of Sm^{152} and Sm^{154} using multiple Coulomb excitation with O^{16} and S^{32} projectiles. These neighboring nuclei were selected for study because of energy-level spacings in the ground-state band and the differences in location of the β- and γ-band heads, implying much larger rotation-induced effects on the $B(E2)$ values in Sm^{152} than in Sm^{154}. We note