[2] G. D. Sprouse and G. M. Kalvius, in Mössbauer Effect Methodology, edited by I. Gruverman (Plenum Press, Inc., New York, 1968), Vol. 4, p. 37.

[3] F. Asaro and K. Siegbahn, in Perturbed Angular Correlations, edited by E. Karlsson, E. Matthias, and K. Siegbahn (North-Holland Publishing Co., Amsterdam, The Netherlands, 1964), p. 233; F. Falk, S. Tornkuist, J. E. Thun, H. Snellman, K. Siegbahn, and F. Asaro, Z. Physik 198, 106 (1967).

[4] G. Goldring, in Hyperfine Structure and Nuclear Radiations, edited by E. Matthias and D. A. Shirley (North-Holland Publishing Co., Amsterdam, The Netherlands, 1968), p. 640; I. Ben-Zvi, P. Gilad, M. Goldberg, G. Goldring, A. Schwarzschild, A. Spinzak, and Z. Vager, Nucl. Phys. A121, 592 (1968).

[5] In this earlier experiment the alignment was detected by observing the polarization of unsplit radiation from nuclei implanted in a field-free medium. See Ref. 1.

[6] W. J. Childs and L. S. Goodman, Phys. Rev. 148, 74 (1966).

[7] G. Goldring and K. Nakai, private communication.

SUBSTANTIAL NUCLEAR ORDERING IN SOLID He^3†

R. T. Johnson,* O. G. Symko, and J. C. Wheatley
Department of Physics, University of California at San Diego, La Jolla, California 92037

(Received 22 July 1969)

Evidence for a substantial nuclear ordering in solid bcc He^3 at melting pressure has been inferred from solid entropies s_S deduced from measurements of the slope of the melting curve for a liquid-solid mixture cooled by adiabatic compression. At 3.4 mK we find experimentally that $s_S/R = 0.83\ln 2$. It is likely that much lower temperatures and solid entropies are obtained during parts of the experiments. The possibility for bulk nuclear polarization is also discussed.

It is possible to convert an ordered Fermi liquid to a solid with ordered nuclear spins.[1] More specifically, highly ordered liquid He^3 may be produced readily at temperatures of 10 to 20 mK, a temperature region easily attained by dilution refrigeration, since liquid He^3 is a Fermi liquid with an effective Fermi temperature much greater than 20 mK. Isentropic conversion of liquid to solid should then produce a highly nuclear-spin-ordered solid. Quantitatively, the ordering of the solid may be measured by its entropy. In this work we determined the solid entropy by measuring the slope dP/dT of the melting curve and using the Clausius-Clapeyron equation

$$\frac{dP}{dT} = -\left(\frac{s_S - s_L}{v_L - v_S}\right), \tag{1}$$

where s_S, v_S and s_L, v_L are the molar entropy and volume on the melting curve for the solid and liquid phases, respectively. Alternatively, nuclear ordering could be observed by forming the solid in a strong magnetic field and measuring the bulk nuclear magnetic moment. This has not been done in the present experiments, but work along these lines is in progress.

At temperatures below a few tenths of a kelvin the entropy of solid He^3 results from disorder in the nuclear-spin system. In zero external field, the spins may be described by an effective spin Hamiltonian[2]

$$\mathcal{H} = -J\sum_{ij}\vec{I}_i \cdot \vec{I}_j, \tag{2}$$

where J is a parameter, and the sum is usually assumed to be over nearest neighbors. Using data of Anderson, Reese, and Wheatley[3] we showed[4,5] that $J < 0$, in agreement with recent measurements by Pipes and Fairbank[6]; Kirk, Osgood, and Garber[7]; and Sites, Osheroff, Richardson, and Lee.[8] Hence in zero external field, antiferromagnetic nuclear ordering is to be expected at low enough temperatures.

In recent work we demonstrated[4] that it is possible to cool a two-phase mixture of liquid and solid He^3 mechanically to temperatures below 3 mK. However, we did not determine the resultant nuclear ordering. In some preliminary work we measured the nuclear-spin diffusion coefficient D in a solid formed by adiabatic compression. We found that, at the lowest temperatures, D had decreased to about $\frac{2}{3}$ of its temperature-independent value at higher temperatures.[9] This result indicated that something interesting was happening in the nuclear-spin system. However, we interrupted the diffusion measurements in favor of the melting-curve measurements since these have the double advantage of yielding a direct measurement of solid entropy and of being much less sensitive to thermal disequilibrium in the solid nuclear-spin system.

From 0.3 K down to about 30 mK the solid molar entropy is essentially $R\ln 2$ (corresponding to random orientations of the spin-$\frac{1}{2}$ nuclei) as deduced from the liquid entropy at the minimum of the melting curve[10] and from solid heat-capacity measurements[11] at a few hundredths of a kelvin. The liquid entropy at the melting curve at low temperatures may be calculated self-consistently using the liquid heat capacity at 27.0 atm given by Abel, Anderson, Black, and Wheatley[12] extended to the melting curve, using the expansion coefficient and compressibility given by Anderson, Reese, and Wheatley.[13] Since the difference $v_L - v_S$ between the liquid and solid molar volumes is known at low temperatures from the work of Scribner, Panczyk, and Adams,[14] measurement of dP/dT along the melting curve allows the solid entropy to be determined from Eq. (1). Furthermore, the temperature calibration can be, and in this work is, determined by taking the solid molar entropy at higher temperatures to be $R\ln 2$.

The low-temperature portion of our apparatus is shown in Fig. 1. Details concerning its operation will be given in a paper currently in preparation. The He^3 is compressed via the flexible-walled section by pressurizing liquid He^4.[15] The epoxy appendix to the flexible section contains the thermometer and strain gauge for the measurements to be presented here. The cell is precooled by mounting it with the upper portion immersed in the mixing chamber of a continuously operating dilution refrigerator.[16] At low temperatures thermal isolation is provided by the thermal boundary resistance between the helium and the container walls.

The pressure of the He^3 is measured with a beryllium-copper strain gauge[14] having a sensitivity of 10^{-4} atm, which is calibrated at 1 K with only liquid in the cell. A magnetic temperature T^* is determined by measuring the 17-Hz susceptibility of powdered cerium magnesium nitrate (CMN) uniformly packed in a right-circular cylinder with diameter equal to height. For CMN we assume[17, 18] $T = T^* + \Delta$ down to 3 mK with $|\Delta|$ less than about 0.2 mK. We have taken $\Delta = 0$ in all computations. The slope and intercept of the straight line relating mutual inductance and T^{*-1} were adjusted as indicated above using Eq. (1) in the range 20 to 38 mK to give a best fit of the data to the calculated curve of dP/dT vs T. The resultant melting curve is in general agreement with that of Ref. 14.

Averaged results obtained during the slow warm-up following compression are shown for five runs in Fig. 2. Owing to uncertainties in the temperature scale and/or problems with thermal equilibrium, lower-temperature data were dis-

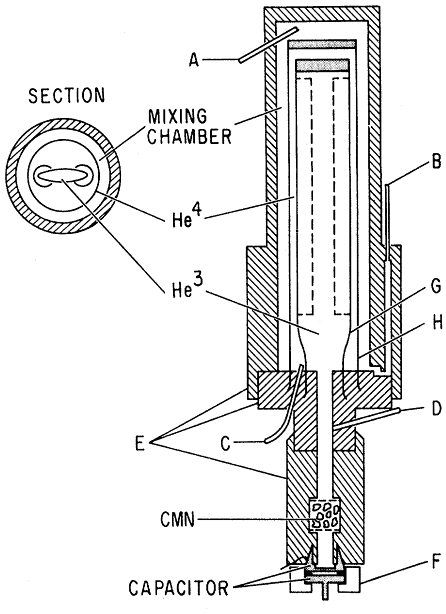

FIG. 1. Schematic of compression cell mounted in the mixing chamber of the continuous dilution refrigerator. A, inlet to mixing chamber. B, outlet of mixing chamber. C, He^4 inlet. D, He^3 inlet. E, heat-curing epoxy resin. F, nylon. G, flexible part of He^3 cell consisting of a flattened 9.52-mm-o.d.× 0.41-mm-wall 70-30 cupronickel (CuNi) tube with slit 4.76-mm-o.d. ×0.25-mm-wall CuNi tubes for springs. H, outer wall of He^4 space consisting of a 15.9-mm-o.d.× 0.41-mm-wall CuNi tube. CMN, powdered cerium magnesium nitrate. CAPACITOR, beryllium copper plates separated by 0.025-mm-thick Mylar ring.

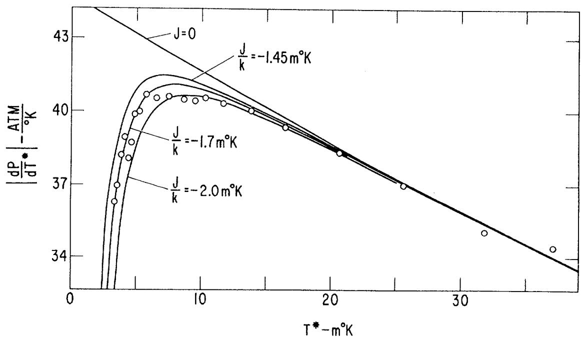

FIG. 2. The slope of the melting curve versus temperature as determined experimentally and as theoretically determined for several values of the parameter J in the Hamiltonian $\mathcal{H} = -J\sum \vec{I}_i \cdot \vec{I}_j$. The theoretical curves assume $T = T^*$. The experimental points are averages of five warming runs.

carded. Also shown on the figure are several curves calculated, for several values of the interaction parameter J in Eq. (2), by means of the ten-term high-temperature expansion for a bcc Heisenberg antiferromagnet and reported by Baker, Gilbert, Eve, and Rushbrooke.[19] According to the $(\partial P/\partial T)_V$ measurements of Panczyk et al.,[20] a value of $|J|/k = 1.44$ mK is to be expected for the solid at its melting pressure near $T=0$. We do not believe that our measurements are accurate enough to allow J to be determined well. However our results, favoring a somewhat higher value of $|J|$ than that given by Panczyk et al.,[20] are sufficiently close to the theoretical curve, calculated using their parameter, to give us confidence in the general correctness of our data.

There is already substantial nuclear ordering at the lowest temperatures for which we have dP/dT measurements. At 3.4 mK we calculate from our dP/dT data and Eq. (1) that $s_S = 0.58R = (0.83) \times R\ln 2$. This is independent of any theoretical model. To give some perspective to this value for the entropy, consider a different physical system which consists of independent spin-$\frac{1}{2}$ particles in a magnetic field. A molar entropy of $0.58R$ corresponds to a polarization of 46% in such a system.

Although our magnetic thermometry is no longer adequate below 3 mK, the melting pressure should be a meaningful thermometer. At 3 mK, the observed melting pressure is 33.91 atm while our highest observed pressure is 33.95 atm. Thompson and Meyer[21] have calculated the melting curve to $T=0$ for $J/k = -1.2$ mK and for $J=0$. We have performed a similar calculation of the melting curve for several values of J using our calculation for s_L, $v_L - v_S$ from Ref. 14, s_S calculated using the expansion in Ref. 19 above $T_N (=1.374|J|/k)$, and the spin-wave T^3 expression in Ref. 14 at low temperatures with a smooth graphical connection through the intermediate region. Starting from 5.0 mK and 33.831 atm, a point on our observed melting curve, we calculate at $T=0$ a melting pressure of 33.967 atm for $J/k = -1.2$ mK, 33.952 atm for -1.44 mK, and 33.935 atm for -1.7 mK. For $J/k = -1.44$ mK, as determined in Ref. 20, we calculate a melting pressure of 33.910 atm at 3 mK in agreement with observation. Our maximum pressure corresponds, for this J, to about 1.5 mK and to $s_S = (0.1_7)R\ln 2$. These figures refer to the liquid-solid interface.

When the solid He^3 is formed in a magnetic field,[22] the effective nuclear-orienting field $H_{\rm eff}$ is less than the applied field $H_{\rm appl}$ as a result of the effective antiferromagnetic spin interaction. Using $J/k = -1.5$ mK we estimate

$$H_{\rm eff} \simeq H_{\rm appl} - 4\times 10^{-4} P_n \text{ G}, \tag{3}$$

where P_n is the nuclear polarization, defined to be positive. If liquid He^3 is converted isentropically to solid in an applied field of 70 000 G, a 76% nuclear polarization (corresponding to $|g_N \beta_N| H_{\rm eff}/2kT = 1$) will be attained at a temperature of 3 mK. Starting with liquid at 20 mK, only 22% of the He^3 needs to be converted to solid to achieve this result. Furthermore, by suitable heating[8] the solid can be formed where required. Whether or not such a high polarization can be attained in practice depends at least on mechanical irreversibilities in the device. In parts of our present work to be presented elsewhere, we find such irreversibilities small. A more serious problem appears to be the attainment of thermal equilibrium within the solid nuclear-spin system itself.

A more detailed account of our work on the melting curve is in preparation. We wish to acknowledge here the important contributions made to measurements of the melting curve, preliminary to those presented here, by Dr. Ralph Rosenbaum and Professor Olli Lounasmaa.

[†] Research supported by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-34, P. A. 143.

[*] Work in partial fulfillment by R. T. Johnson of the requirements for the degree of Doctor of Philosophy at the University of Illinois.

[1] I. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. 20, 919 (1950). For the first experimental demonstration which, however, did not produce significant nuclear ordering, see Yu. D. Anufriyev, Zh. Eksperim. i Teor. Fiz.—Pis'ma Redakt. 1, No. 6, 1 (1965) [translation: JETP Letters 1, 155 (1965)].

[2] D. J. Thouless, Proc. Phys. Soc. (London) 86, 893 (1965).

[3] A. C. Anderson, W. Reese, and J. C. Wheatley, Phys. Rev. Letters 7, 366 (1961).

[4] R. T. Johnson, R. Rosenbaum, O. G. Symko, and J. C. Wheatley, Phys. Rev. Letters 22, 449 (1969).

[5] R. T. Johnson and J. C. Wheatley, to be published.

[6] P. B. Pipes and W. M. Fairbank, Phys. Rev. Letters 23, 520 (1969).

[7] W. P. Kirk, E. B. Osgood, and M. Garber, Phys. Rev. Letters 23, 833 (1969).

[8] J. R. Sites, D. D. Osheroff, R. C. Richardson, and D. M. Lee, Phys. Rev. Letters 23, 835 (1969).

[9] H. Meyer, J. Appl. Phys. 39, 390 (1968).

[10] A. C. Anderson, W. Reese, and J. C. Wheatley,

Phys. Rev. 130, 495 (1963).

[11]A. C. Anderson, G. L. Salinger, W. A. Steyert, and J. C. Wheatley, Phys. Rev. Letters 7, 295 (1961).

[12]W. R. Abel, A. C. Anderson, W. C. Black, and J. C. Wheatley, Phys. Rev. 147, 111 (1966).

[13]A. C. Anderson, W. Reese, and J. C. Wheatley, Phys. Rev. 130, 1644 (1963).

[14]R. A. Scribner, M. F. Panczyk, and E. D. Adams, J. Low Temp. Phys. 1, 313 (1969).

[15]S. G. Sydoriak, R. L. Mills, and E. R. Grilly, Phys. Rev. Letters 4, 495 (1960). R. L. Mills, E. R. Grilly, and S. G. Sydoriak, Ann. Phys. (N.Y.) 12, 41 (1961).

[16]J. C. Wheatley, O. E. Vilches, and W. R. Abel, Physics 4, 1 (1968).

[17]W. R. Abel and J. C. Wheatley, Phys. Rev. Letters 21, 597 (1968).

[18]W. C. Black, R. T. Johnson, and J. C. Wheatley, to be published.

[19]G. A. Baker, Jr., H. E. Gilbert, J. Eve, and G. S. Rushbrooke, Phys. Rev. 164, 800 (1967).

[20]M. F. Panczyk, R. A. Scribner, G. C. Straty, and E. D. Adams, Phys. Rev. Letters 19, 1102 (1967); see also M. F. Panczyk and E. D. Adams, to be published.

[21]J. R. Thompson and H. Meyer, Cryogen. 7, 296 (1967).

[22]L. Goldstein, Phys. Rev. 171, 194 (1968).

EFFECT OF ELECTRON TRAPPING ON THE ION-WAVE INSTABILITY*

K. Nishikawa† and Ching-Sheng Wu‡
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91103
(Received 6 October 1969)

It is found that in many cases electron trapping can have an important effect on ion waves. Trapping can render the linear stability theory invalid, and the conditions under which this may occur are discussed. Assuming the trapping process to be the mechanism whereby an unstable plasma reaches a quasistationary turbulent state, we also derive an expression for the saturation energy spectrum.

In this Letter we discuss the effect of electron trapping on the ion-wave instability in a plasma, the instability arising because of the presence of an electron current. Two fundamental questions are raised: One is whether the stability criterion derived from the linear theory could be modified by trapping; the other is whether, even when linear stability theory does remain valid, the trapping process can act as a mechanism which brings an unstable plasma to a quasistationary turbulent state.

All linear theories[1-7] of ion-wave instability implicitly assume that electron trapping is unimportant. This assumption is reasonable only if we insist that the waves have "very small" amplitude. However, in reality there is a natural lower bound to the amplitude. Fluctuations associated with the thermally generated ion oscillations provide a lower bound for the energy of an ion wave and so for its amplitude. Restricting ourselves to thermal ion-wave fluctuations, we can estimate the bounce frequency of the trapped electrons in a typical potential trough, the trough having a certain characteristic width of the order of the electron Debye length, say. The electron may be said to be trapped only when the bounce period is much shorter than the lifetime of the potential trough (or more precisely, the correlation time of the fluctuating fields of a certain wave packet). This time is at least of the order of some typical ion-wave period, say ω_s^{-1}. Evidently when the bounce frequency ω_b greatly exceeds the wave frequency ω_s the linear stability theory can be in error, unless some other mechanisms, such as collisions, suppress the trapping process. In fact it is interesting, as we show later, that the Coulomb collisions can efficiently prevent trapping from taking place.

We now examine the ratio ω_b/ω_s in order to find under what conditions it exceeds unity. Since the electron Debye length, λ_e, represents a typical length scale for the ion oscillations, we are interested in the most probable potential depth, φ, with width of the order of this characteristic length. Because, in general, the fluctuation fields consist of a large number of propagating modes, the potential φ may be estimated in some average sense. Thus, using the autocorrelation function, we write

$$\varphi^2 = |\langle \varphi(\vec r)\varphi(\vec r + \vec\lambda_e)\rangle|, \tag{1}$$

where $\langle\,\rangle$ denotes the ensemble average. Assuming isotropic fluctuation and using the random phase approximation, i.e.,

$$\langle \varphi_{\vec k}\varphi_{-\vec k'}\rangle = (2\pi)^3\delta[\vec k - \vec k']I_k, \tag{2}$$

where φ_k is the Fourier transform of $\varphi(r)$, we can rewrite (1) as

$$\phi^2 = \frac{1}{2\pi^2}\left|\int_0^{k_{\max}} dk\, k^2 \frac{\sin k\lambda_e}{k\lambda_e} I_k\right|. \tag{3}$$