BROKEN CHIRAL SYMMETRY AND THE VENEZIANO MODEL*

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We investigate chiral-symmetry breaking within the $K\pi$ system when a Veneziano model is assumed for various off-mass-shell extrapolations. We derive expressions for the matrix elements of σ terms and the divergence of the strangeness-changing current and discuss the implications to chiral-symmetry-breaking interactions. Application of our results to the K_{l3} form factors gives $f_K/f_\pi f_+(0) = 1.35$ and the relation $(f_K/f_\pi)^2 = \cos[\alpha_{K^*}'(0)(\mu_K^2-\mu_\pi^2)]$.

In this note we would like to report on some results we have obtained on chiral-symmetry breaking assuming the pseudoscalar-pseudoscalar scattering amplitude to be known from the Veneziano model.[1] To relate symmetry breaking to the scattering amplitude we must extrapolate the latter to off-mass-shell values; however, the Veneziano model can be made to satisfy Adler's self-consistency condition for both the pion and kaon partial conservation of axial-vector currents (PCAC) and this procedure has some justification.[2] Representations for the matrix elements of the commutators of charges and current divergences (σ terms) are related to the Veneziano amplitude and we find that the usual models[3] for symmetry breaking whose components belong to a single $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ representation of SU(3) $\otimes$ SU(3) cannot describe our results. These representations for the σ terms are a generalization of Adler's condition[4] and provide a connection between the possible existence of exotic resonances and σ terms carrying these exotic quantum numbers. Finally the matrix elements of the divergence of the strangeness-changing vector current are determined and the K_{l3} parameters are discussed. In particular we find $f_K/f_\pi f_+(0) = 1.35$.

We begin with the following expression for the off-mass-shell pseudoscalar-pseudoscalar scattering amplitude:

$$i(2\pi)^4\delta^4(\textstyle\sum q_l)M_{jkln}(-q_l^2, s, t, u)$$
$$\equiv \prod_{l=1}^{4}\int dx_l^4 \frac{q_l^2+\mu_l^2}{f_l\mu_l^2}e^{iq_l\cdot x_l}\, T\langle 0|\partial_\mu A_\mu^{\,j}(x_1)\partial_\nu A_\nu^{\,k}(x_2)\partial_\rho A_\rho^{\,l}(x_3)\partial_\sigma A_\sigma^{\,n}(x_4)|0\rangle, \tag{1}$$

where $(2q_0)^{1/2}\langle 0|\partial_\mu A_\mu^{\,j}(0)|P_k(q)\rangle \equiv \delta_{jk}f_k\mu_k^2$ $(j,k=1,2,\cdots,8)$ and $s=-(q_1+q_2)^2$, $t=-(q_1+q_3)^2$, $u=-(q_1+q_4)^2$. The σ term, which is a measure of chiral-symmetry breaking, is defined in general by

$$\int d^3y[A_0^{\,i}(\vec{y},x_0), \partial_\mu A_\mu^{\,j}(x)] \equiv i\sigma_{ij}(x). \tag{2}$$

With these definitions we prove some simple theorems relating the matrix elements of σ terms to the off-mass-shell scattering amplitude. Principal among these for our purposes is the case for two particles on the mass shell. The prototype of relations that can be obtained here are found by integrating (1) by parts, say, with respect to x_4 and letting $q_4\to 0$ with q_1 and q_2 on mass shell:

$$M(\mu_1^2, \mu_2^2, s, 0; s, \mu_2^2, \mu_1^2) = \frac{\mu_3^2-s}{f_3f_4\mu_3^2}(4E_1E_2)^{1/2}\langle 0|\sigma_{nl}(0)|P_j(q_1)P_k(q_2)\rangle. \tag{3}$$

The following statements can be made about this equation [similar statements can be made in t and u channels by keeping a different pair of particles on mass shell].

(a) It is a generalization of the Adler condition[4]; the factor μ_3^2-s guarantees that M vanish at the point $s=\mu_3^2$, $t=\mu_2^2$, $u=\mu_1^2$ but it may further require the off-mass-shell scattering amplitude in some cases to vanish for all s at $t=\mu_2^2$ and $u=\mu_1^2$. This happens if $\sigma_{nl}\equiv 0$, as for example when $n=K^+$, $l=\pi^+$ or vice versa and the σ term belongs to one or more of the $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ and $(\underline{8},\underline{1})\oplus(\underline{1},\underline{8})$ representations of SU(3)$\otimes$SU(3). Thus the question of the existence of exotic resonances, i.e., bosons carrying other than octet or singlet SU(3) quantum numbers, in the off-mass-shell scattering amplitude is equivalent to the existence of the σ terms containing these exotic quantum numbers. That this is so for the on-mass-shell amplitude has not been shown, however. In this regard the above statement is on the same footing as the Adler condition itself which refers to the off-mass-shell amplitude but is known to give interesting results if one makes certain smoothness assumptions.

(b) Under very general conditions the matrix elements of the divergence of the vector current can be related to the off-mass-shell scattering amplitude. Using the equal-time commutation relations for the integrated charge densities of SU(3)⊗SU(3) together with the Heisenberg equation of motion $\partial_\mu F(x) = -i[P_\mu, F(x)]$, one obtains the following relation in the center of mass of 1 and 2 $[\vec{q}_1+\vec{q}_2=0]$:

$$\langle 0|\sigma_{nl}-\sigma_{ln}|P_j(q_1)P_k(q_2)\rangle = f_{nlm}\langle 0|\partial_\mu V_\mu{}^m|P_j(q_1)P_k(q_2)\rangle, \tag{4}$$

and in particular

$$\langle 0|\sigma_{\pi^-K^+}-\sigma_{K^+\pi^-}|k^+\pi^-\rangle = \tfrac{1}{2}i\langle 0|\partial_\mu V_\mu{}^{6-i7}|K^+\pi^-\rangle = 2^{-1/2}i\langle 0|\partial_\mu V_\mu{}^{4-i5}|K^+\pi^0\rangle, \tag{5a}$$

$$\langle 0|\sigma_{\pi^+K^0}-\sigma_{K^0\pi^+}|k^0\pi^+\rangle = \tfrac{1}{2}i\langle 0|\partial_\mu V_\mu{}^{4-i5}|K^0\pi^+\rangle = -2^{-1/2}i\langle 0|\partial_\mu V_\mu{}^{6-i7}|K^0\pi^0\rangle. \tag{5b}$$

The restriction to the center of mass has the advantage that it minimizes assumptions about the nature of Schwinger terms in local equal-time commutation relations. Using Eq. (4), the divergence of the vector current can be related to the scattering amplitude through Eq. (3) and covariance is restored.

We now apply these results to $K\pi$ scattering within the framework of the Veneziano model. The essential assumption we make is that the off-mass-shell amplitude defined in Eq. (1) can be approximated by a sum of functions of the form $\Gamma(1-\alpha_x(t))\Gamma(1-\alpha_y(u))/\Gamma(1-\alpha_x(t)-\alpha_y(u))$ together with similar expressions where s, t, u are interchanged. It will be sufficient to discuss $K^+\pi^+$ elastic scattering as the other charged modes may be obtained from crossing symmetry and isospin invariance. We shall also assume that $\sigma_{K^+\pi^+}=\sigma_{\pi^+K^+}\equiv 0$ and hence no exotic resonances. Then $M_{K^+\pi^+}$ can be written as[5]

$$M_{K^+\pi^+}(s,t,u) = M_{K^+\pi^-}(u,t,s) = \beta\frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_{K^*}(u))}{\Gamma(1-\alpha_\rho(t)-\alpha_{K^*}(u))}. \tag{6}$$

Equation (3) then requires that $M_{K^+\pi^+}$ vanish at $t=\mu_\pi^2$, $u=\mu_K^2$, and $t=\mu_K^2$, $u=\mu_\pi^2$ which is the Adler condition for pion and kaon PCAC, respectively, and is satisfied if $\alpha_\rho(\mu_\pi^2)+\alpha_{K^*}(\mu_K^2)=\alpha_\rho(\mu_K^2)+\alpha_{K^*}(\mu_\pi^2)=1$. This implies[5] $\alpha_{K^*}(\mu_K^2)=\frac{1}{2}$ when $\alpha_\rho(\mu_\pi^2)=\frac{1}{2}$ is used and a constant universal slope for all trajectories gives the mass formula $m_{K^*}{}^2(J)-\mu_K^2 = m_\rho{}^2(J)-\mu_\pi{}^2 = (2J-1)[m_{K^*}{}^2(1)-\mu_K{}^2]$ which is well satisfied by the known resonances, (K^*,ρ) and (K^T,f). Evidently, α_ρ and α_{K^*} are exchange degenerate with α_f and α_{K^T}, respectively, from (6). Encouraged by these results based on both pion and kaon PCAC we proceed further to obtain the following relations from (3) and (6);

$$(4q_0k_0)^{1/2}\langle 0|\sigma_{\pi^-K^+}(0)|K^+(k)\pi^-(q)\rangle = \frac{f_\pi f_K b\beta\mu_K{}^2\Gamma(\frac{1}{2})\Gamma(1-\alpha_{K^*}(s))}{\Gamma(\frac{3}{2}-\alpha_{K^*}(s))}, \tag{7}$$

$$(4q_0k_0)^{1/2}\langle 0|\sigma_{K^+\pi^-}(0)|K^+(k)\pi^-(q)\rangle = \frac{f_\pi f_K b\mu_\pi{}^2\beta\Gamma(1-\alpha_\rho(\mu_K{}^2))\Gamma(1-\alpha_{K^*}(s))}{\Gamma(\frac{3}{2}-\alpha_\rho(s))}, \tag{8}$$

where b is the slope of the trajectory functions and we have made use of the relations $\alpha_\rho(\mu_\pi{}^2)=\alpha_{K^*}(\mu_K{}^2)=\frac{1}{2}$ in (8). At this point we make the following observation:

(c) Any model in which $\sigma_{K^+\pi^-}$ is proportional to $\alpha_{\pi^-K^+}$ is not compatible with Eqs. (7) and (8) because they have different dependence on s $(\alpha_{K^*}\neq\alpha_\rho)$. In particular when the symmetry breaking is represented[3] by a Hamiltonian $H'\equiv u_0+cu_8$ with u_0 and u_8 belonging to the same $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ representation, one finds

$$\sigma_{K^+\pi^-} = [(\sqrt{2}+c)/(\sqrt{2}-\tfrac{1}{2}c)]\sigma_{\pi^-K^+}.$$

Thus such a model could not give the results in (7) and (8). This does not mean that symmetry breaking necessarily contains representations other than the $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ but only that u_0 and u_8 cannot be in the same $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ representation. Thus σ terms with exotic quantum numbers may still vanish. In fact, one can easily convince himself that the pion in (7) and the kaon in (8) may be reduced without introducing unknown quantities, if (1) no σ terms carrying exotic quantum numbers of $|\Delta S|=1$ and $|\Delta I|\geq\frac{3}{2}$, (2) the algebra of charges, and (3) the Jacobi identity are assumed. Reducing the kaon in (7) or the pion in (8) would require knowledge of the commutators $[Q_5{}^{K^-},[Q_5{}^{\pi^-},\partial_\nu A_\nu{}^{K^+}]]$ and $[Q_5{}^{\pi^+},[Q_5{}^{K^+},\partial_\nu A_\nu{}^{\pi^-}]]$ which need a more detailed model for the σ terms. As remarked above the simple models based on a single representation of the type $(\underline{3},\underline{3}^*)$ [or $(\underline{8},\underline{1})$] for the symmetry breaking cannot be used. The interesting question of representations for symmetry breaking will not be pursued further here; instead we would like to present other consequences of this model.

From (5), (7), and (8), we obtain

$$f_+(s)(\mu_K^2-\mu_\pi^2)+f_-(s)s=-2f_\pi f_K b\beta\left[\frac{\mu_K^2\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{2}-\alpha_{K^*}(s))}-\frac{\mu_\pi^2\Gamma(\alpha_{K^*}(\mu_\pi^2))}{\Gamma(\frac{3}{2}-\alpha_\rho(s))}\right]\Gamma(1-\alpha_{K^*}(s)), \tag{9}$$

where K_{l3} form factors $f_\pm(s)$ are defined by the matrix element

$$(4q_0k_0)^{1/2}\langle 0|V_\mu^{K^+}(0)|\pi^+(q)K^0(k)\rangle=2^{-1/2}[f_+(s)(k-q)_\mu+f_-(s)(k+q)_\mu]. \tag{10}$$

Notice from (9) that a scalar kaon with the K^* mass appears naturally in the Veneziano amplitude and is associated with the divergence of the strange vector current. We can calculate from (9) the usual K_{l3} parameters $\lambda_\pm$ and ξ defined by $f_\pm(s)=f_\pm(0)(1-\lambda_\pm s/\mu_\pi^2)^{-1}$ and $\xi=f_-(0)/f_+(0)$. We find

$$[1/f_+(0)](f_K/f_\pi)=1.348, \tag{11}$$

$$\lambda_-=0.0953, \tag{12}$$

and

$$\xi=-0.034. \tag{13}$$

In deriving (11) we have used $\beta=-(2f_\pi^2b\pi)^{-1}$ which follows from (7) by reducing the pion further, and to obtain (12) and (13) we have assumed

$$f_+(s)=\mu_{K^*}^2f_+(0)/(\mu_{K^*}^2-s) \tag{14}$$

which seems to be favored by the experiments.[6] The result (11) is in good agreement with the Cabibbo theory relating $f_+(0)$ to the measured amplitudes for K_{e3}, π_{e3}, $K_{\mu2}$, and $\pi_{\mu2}$ decay,

$$\frac{1}{f_+(0)}\frac{f_K}{f_\pi}=\left(\frac{\tan\theta_A}{\tan\theta_V}\right)_{\rm eff}=1.28,$$

and the value of ξ given by (13) is consistent with the branching-ratio determinations. To calculate $f_+(0)$, we need independent information about f_K/f_π. A further reduction of the kaon in (8) gives

$$\beta=-\{2f_K^2b\Gamma(1-\alpha_{K^*}(\mu_\pi^2))\Gamma(\alpha_{K^*}(\mu_\pi^2))\}^{-1}$$

which yields

$$(f_K/f_\pi)^2=\pi\{\Gamma(\alpha_{K^*}(\mu_\pi^2))\Gamma(1-\alpha_{K^*}(\mu_\pi^2))\}^{-1}=\cos[b(\mu_K^2-\mu_\pi^2)]=0.802 \tag{15}$$

so that

$$f_K/f_\pi=0.895, \tag{16}$$

$$f_+(0)=0.655. \tag{17}$$

We note that (16) implies $|f_K|<|f_\pi|$ and the symmetry-breaking effect in $f_+(0)$ is about 35%, although it should be only of second order in the SU(3) breaking from the Ademollo-Gatto theorem.[7] Moreover, these numbers would require the Cabbibo angles commonly associated with the vector and axial vector to satisfy $\sin\theta_V=0.32$ and $\sin\theta_A=0.29$ to be compared with the observed values of 0.21 and 0.26, respectively. However, it should be emphasized that (16) and (17) are the result of a larger extrapolation than used in obtaining (11) and are therefore less reliable. A more detailed study of the implications of this model on the question of symmetry breaking and other applications will be presented elsewhere.[8]

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[1]G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[2]C. Lovelace, Phys. Letters 28B, 265 (1968); M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Letters 22, 83 (1969); K. Kawarabayashi, S. Kitakado, and H. Yabuki, Phys. Letters 28B, 432 (1969), and Phys. Rev. (to be published).

[3]See for example, M. Gell-Mann, Physics 1, 63 (1964); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[4]S. L. Adler, Phys. Rev. 137, B1022 (1964).

[5]Kawarabayashi, Kitakado, and Yabuki, Ref. 2.

[6]Recent experimental results can be found in the Proceedings of the Topical Conference on Weak Interaction, CERN, Geneva, Switzerland, 14-17 January 1969 (CERN Scientific Information Service, Geneva, Switzerland, 1969).

[7]M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

[8]Another interesting point to be discussed is to modify Eqs. (7) and (8) by allowing some external mass dependence in the Veneziano amplitude so as to make the two equations consistent with the usual symmetry-breaking model of $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ representation. After submission of the present work for publication, we received a Queen Mary College preprint entitled "Veneziano Amplitude and Current Algebra" by H. Osborn in which some aspect of this point is touched upon. We would like to thank Dr. Osborn for bringing his work to our attention.

ERRATA

LIGHT-SCATTERING MEASUREMENT OF CONCENTRATION FLUCTUATIONS IN PHENOL-WATER NEAR ITS CRITICAL POINT. P. N. Pusey and W. I. Goldburg [Phys. Rev. Letters 23, 67 (1969)].

Line 9, column 1 on p. 69 reads "$\cdots \ln\Gamma_c$ was proportional to $|T-T_c| \cdots$." This should be changed to read "$\cdots \ln\Gamma_c$ was proportional to $\ln|T-T_c| \cdots$."

Reference 11 should read: Benedek, Ref. 4.

Reference 13 should read: See Benedek, Ref. 4, $\cdots$.

DUALITY, ABSENCE OF EXOTIC RESONANCES, AND THE $\Delta I = \frac{1}{2}$ RULE IN NONLEPTONIC DECAYS. Ken Kawarabayshi and Shinsaku Kitakado [Phys. Rev. Letters 23, 440 (1969)].

The statement that there are six independent conditions on six s-u dual amplitudes for octet baryon decays is incorrect. Actually, only five of these conditions are independent.

As a result, for each pv and pc amplitude of octet baryon decays, contributions from the 27-plet spurion need not to vanish in general, but are described by a single s-u dual amplitude. However, for pv decays, CP invariance requires that this amplitude should vanish. On the other hand, no such restriction is found for pc decays. Explicit calculation shows, however, that the relevant amplitude gives no contribution to Ξ decays.

In summary then, in the SU(3) limit, the $\Delta I = \frac{1}{2}$ rule is proved for all observable pv decay amplitudes, including $\Omega^- \to \Xi + \pi$ and $K \to 2\pi$. For pc decay amplitudes, the same is valid for $\Omega^- \to \Xi + \pi$, $K \to 3\pi$, and $\Xi \to \Lambda + \pi$. The deviations from the $\Delta I = \frac{1}{2}$ rule for Λ and Σ decays are related through the following sum rule:

$$(\tfrac{2}{3})^{1/2}[P(\Lambda_-{}^0) + \sqrt{2}P(\Lambda_0{}^0)] = -\sqrt{2}P(\Sigma_0{}^+) + P(\Sigma_+{}^+) - P(\Sigma_-{}^-).$$

In the case of broken SU(3), the $\Delta I = \frac{1}{2}$ rule remains valid for $\Omega^- \to \Xi + \pi$ and $K \to 2\pi(3\pi)$. The question of whether or not the same rule is derivable, within our approach, for octet baryon decays deserves further study.

One of us (K.K.) thanks Dr. A. Schwimmer for pointing out the error contained in our paper.

VENEZIANO-TYPE FORM FACTORS FOR THE PION. H. Suura [Phys. Rev. Letters 23, 551 (1969)].

β_0 in Eq. (1) should have the value $\beta_0 = 2g_{\rho\pi\pi}{}^2$ instead of $g_{\rho\pi\pi}{}^2$.

Accordingly, the value of f_ρ given at the end of the paper should be corrected to $f_\rho{}^2/4\pi = \frac{1}{32}(m_\rho/f_\pi)^2 \sim 2$.