ther better nor worse than that shown in Table I. At the same time, we are aware of the suggestive nature of (1). One of us (P.J.K.) is studying a model that would lead to that formula.

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π^--NUCLEUS SCATTERING LENGTHS*

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S-wave π^--nucleus scattering lengths for nuclei of mass number 3 to 24 are extracted from all available $2P$-$1S$ transition-energy data in π^--mesonic atoms.

It is well known that the studies of π^--mesonic atoms are useful to examine the very low-energy π^--nucleus interaction. Detailed optical potentials for the π^--nucleus interaction have been constructed from them[1] and the *S*-wave π^--nucleus scattering lengths were obtained[2] from the first measurements of the $2P$-$1S$ transition energies using solid-state detectors.[3]

In this Letter we report the *S*-wave π^--nucleus scattering lengths for nuclei of mass numbers $3 \leq A \leq 24$ obtained from analysis of all available $2P$-$1S$ transition-energy measurements using solid-state detectors.[3-5] The analysis is similar to the one in SC: First, the best-fit optical potentials are obtained to give the observed atomic transition energies. Then the scattering lengths are calculated from the potentials in the absence of electromagnetic interactions. The scattering lengths thus calculated are free from Coulomb interference and are appropriate to the strong interaction alone.

If the π^--nucleus bound states were due to the strong interaction, we would expect the zero-range approximation to determine the scattering length very well from the sole knowledge of the location of the bound state close to threshold, particularly, because of the large pion Compton wavelength. In reality, Coulomb interaction is responsible for the formation of the atomic bound states and the strong interaction itself does not have any bound state. We expect, however, that knowledge of the strong-interaction potential at the atomic binding energy (<0.4 MeV) alone would determine the scattering length uniquely, and, furthermore, that the scattering length is insensitive to the shape of the potential: The shape could not be determined uniquely from the binding-energy data alone, just as in the case of low-energy nucleon-nucleon interaction phenomena. In the latter part of this Letter, using an example, we will demonstrate that this is the case. At any rate, we choose the simplest form of the optical potential, a complex square well, to describe the π^--nucleus strong interaction.

The finite-charge-distribution (FCD) effect is examined by assuming a uniform charge distribution. The radius of the distribution and of the strong interaction, R, is expressed in terms of the nuclear charge radius R_n and an assumed pion charge radius r. We write

$$R = (R_n^2 + r^2)^{1/2},$$

where R_n is a weighted average of the radii obtained from μ^--atom experiments[6] and from electron scattering experiments.[7] We take r to be 0.7 ± 0.7 F, which is a reasonable guess considering the large uncertainty in the determination of this radius up to now.[8] By solving the Schrödinger equation as done in SC we find that the uncertainty in R produces an uncertainty of about 10-30% in the FCD effect. This is less than the current experimental errors of the transition-energy measurements for $A < 11$ nuclei, but is equal to or a little larger than the errors for the heavier nuclei. Because of this relatively large uncertainty, we regard it as an estimate of the total uncertainty of the electromagnetic effects. This includes uncertainty in the nuclear polarization effect and the effects of the FCD and the strong interaction upon the vacuum polarization (VP) effect.[9]

We calculate the shifts and spreads due to the

Table I. The energy level shifts (ΔE_{SH}) and spread (Γ) due to the strong interactions, and the scattering length calculated. Errors in ΔE_{SH} are a combination of those due to uncertainties in effective charge radii (see text) and of experimental errors in the transition energy measurements. Errors in Γ are experimental ones alone.

	Ref.	ΔE_{SH}	$-i(\Gamma/2)$ (KeV)	Scattering Length (F.)	
He^3	5	-0.03 ± 0.06		-0.0578 ± 0.113	
He^4	5	0.07 ± 0.05	$-i(0.005\pm0.030)$	0.144 ± 0.095	$-i(0.010\pm0.06)$
	5	0.08 ± 0.06	$-i(0.0\ \pm0.043)$	0.163 ± 0.115	$-i(0.0\ \pm0.08)$
Li^6	3	0.62 ± 0.20	$-i(0.195\pm0.180)$	0.367 ± 0.119	$-i(0.118\pm0.11)$
	5	0.56 ± 0.06	$-i(0.075\pm0.025)$	0.202 ± 0.035	$-i(0.044\pm0.015)$
Li^7	3	0.82 ± 0.20	$-i(0.285\pm0.150)$	0.483 ± 0.120	$-i(0.174\pm0.10)$
	5	0.56 ± 0.06	$-i(0.095\pm0.025)$	0.328 ± 0.036	$-i(0.057\pm0.015)$
Be^9	3	1.86 ± 0.20	$-i(0.425\pm0.140)$	0.472 ± 0.053	$-i(0.114\pm0.039)$
	4	1.58 ± 0.20	$-i(0.535\pm0.150)$	0.397 ± 0.052	$-i(0.141\pm0.042)$
	5	1.64 ± 0.06	$-i(0.290\pm0.025)$	0.414 ± 0.016	$-i(0.077\pm0.007)$
B^{10}	3	3.80 ± 0.20	$-i(0.70\ \pm0.25)$	0.542 ± 0.028	$-i(0.102\pm0.037)$
	4	2.76 ± 0.20	$-i(0.635\pm0.125)$	0.393 ± 0.028	$-i(0.089\pm0.018)$
	5	2.91 ± 0.14	$-i(0.840\pm0.060)$	0.414 ± 0.020	$-i(0.119\pm0.009)$
B^{11}	3	4.39 ± 0.20	$-i(1.15\pm0.25)$	0.614 ± 0.028	$-i(0.171\pm0.039)$
	4	3.91 ± 0.20	$-i(0.935\pm0.125)$	0.544 ± 0.029	$-i(0.136\pm0.019)$
	5	3.89 ± 0.14	$-i(0.86\pm0.075)$	0.542 ± 0.020	$-i(0.126\pm0.012)$
C^{12}	3	5.78 ± 0.24	$-i(1.30\pm0.25)$	0.474 ± 0.017	$-i(0.113\pm0.023)$
	4	6.14 ± 0.21	$-i(1.480\pm0.125)$	0.505 ± 0.018	$-i(0.130\pm0.011)$
	5	5.89 ± 0.18	$-i(1.625\pm0.075)$	0.483 ± 0.016	$-i(0.142\pm0.007)$
N^{14}	3	10.95 ± 0.56	$-i(2.05\ \pm0.20)$	0.611 ± 0.035	$-i(0.121\pm0.013)$
	4	10.11 ± 0.29	$-i(2.24\ \pm0.15)$	0.561 ± 0.017	$-i(0.131\pm0.010)$
O^{16}	3	15.05 ± 0.80	$-i(4.50\ \pm1.0)$	0.607 ± 0.031	$-i(0.189\pm0.044)$
	4	15.70 ± 0.47	$-i(3.78\pm0.25)$	0.635 ± 0.019	$-i(0.160\pm0.012)$
O^{18}	4	20.59 ± 0.46	$-i(4.335\pm0.350)$	0.845 ± 0.020	$-i(0.195\pm0.017)$
F^{19}	3	24.93 ± 0.80	$-i(2.3\ \pm1.0)$	0.760 ± 0.026	$-i(0.075\pm0.034)$
	4	25.53 ± 0.80	$-i(4.70\ \pm0.75)$	0.777 ± 0.026	$-i(0.154\pm0.026)$
Na^{23}	3	51.26 ± 1.56	$-i(2.3\ \pm1.5)$	0.945 ± 0.033	$-i(0.048\pm0.032)$
	4	52.26 ± 1.56	$-i(5.15\pm2.00)$	0.965 ± 0.032	$-i(0.108\pm0.043)$
	5	50.76 ± 1.30	$-i(3.10\pm0.60)$	0.934 ± 0.027	$-i(0.065\pm0.013)$
Mg^{24}	3	58.58 ± 1.87		0.881 ± 0.032	
	5	57.08 ± 2.27	$-i(3.9\ \pm2.5)$	0.856 ± 0.037	$-i(0.064\pm0.042)$

strong interaction and the FCD by subtracting the Klein-Gordon $2P$-$1S$ energies and the VP effects for charges from the measured transition energies.[10] The uncertainty in the shifts caused by the uncertainty in R is folded into the uncertainty of the transition-energy measurements. These shifts and spreads are used to find the best-fit optical potentials of the radii R using the Schrödinger equation with the FCD potentials. The Schrödinger equation is integrated numerically using a predictor-corrector method to ensure the desired accuracy. In Table I, we show the shifts and spreads due to the strong interaction alone and the scattering lengths obtained from the best-fit potentials.

To examine a variation of the scattering lengths a with the various nuclei, we assume that a depends linearly on the nuclear mass number A and $N-Z=2T_3$. The best linear curve is found to be, for the real parts,

$$\mathrm{Re}a=(0.0671\pm0.0211)+(0.0349\pm0.0015)A+(0.0689\pm0.0115)2T_3 \quad (\chi^2=56.9 \text{ for } 31 \text{ points}),$$

and, for imaginary parts,

$$\mathrm{Im}a=(-0.0868\pm0.0209)+(-0.0028\pm0.0016)A+(0.0173\pm0.0116)2T_3 \quad (\chi^2=138 \text{ for } 29 \text{ points}).$$

The A dependence of the imaginary parts is clearly more than a linear one. In particular, for $A \geq 20$ nuclei all values are smaller than the best-fit linear curve by 1 to 4 standard deviations. This disagreement is a reflection of the experimental puzzle that the widths of the $2P$-$1S$ transition energy do not increase as Z^4 for these nuclei.[3-5] On the other hand, the linear curve gives a relatively good fit to the real parts of the scattering lengths: The best linear-fit a is close to zero when $A = 0$ and clearly shows the isospin shift.

It was shown[11] that, if the Adler-Weisberger sum rule and Goldberger-Treiman relation are valid, a relation

$$\mathrm{Re}\tfrac{1}{2}\{(1+\mu/m)[a(n)-a(p)]-(1+\mu/M)[a(N+1,Z-1)-a(N-1,Z+1)]\}=0$$

has to be satisfied. Unfortunately, the isospin shift in the best linear fit is too small by a factor of about 2 to satisfy the relation. This discrepancy should not, however, be regarded as evidence that the sum rule and relation are invalid, since the value of the isospin shift depends strongly on the assumed linear A dependence and on the individual value of a. For some nuclei, particularly for the nuclei important in determination of the isospin shift, Li and B, Table I shows appreciable fluctuations among the values of a depending on experiments from which they are extracted. The question whether the relation is valid should be examined more closely in analyses of future mesonic atom data.

As argued in the beginning of the Letter, a choice of the strong-interaction radius does not influence appreciably the value of the scattering length. To demonstrate this point more clearly, we calculate the potential, its volume integral, and the scattering length for a weighted average of the π^--C^{12} data with the interaction radius varying from 1.5 to 4.5 F. The results are shown in Fig. 1. For this variation of the radius, the value of the best-fit potential decreases by a factor of about 0.03 while the scattering length varies by, at most, ±10% from the central value. The variation of the volume integral of the potential is somewhat similar to this variation. In view of the size of the nucleus and the short-range nature of the strong interaction, it is hard to believe that the actual strong-interaction range can deviate from the value of R we have used by even as much as we have considered. Therefore, we conclude that our choice of the square-well potential and of its range would have little consequence in the determination of the scattering lengths.[13]

It is well known that the π^--nucleus interaction has a rather appreciable momentum dependence. This dependence, however, has little effect on determination of the S-wave scattering lengths.[13] We estimate this effect by approximating the π^--mesonic atom wave function inside of the strong interaction to have a quadratic radial dependence.[14] We obtain, for a potential of $V(r) = (\hbar^2/2\mu)\alpha\nabla^2$,

$$\delta a \sim -(2/15)(R/B)^2 R\alpha,$$

where B is the Bohr radius. Applying $\alpha \sim 0.63 + 0.10i$, which is obtained from $3D$-$2P$ transition energy measurements for nuclei of $A \geq 27$,[1] we see that the effect is very small.

The only scattering experiments available to compare with the values in Table I are the π-He^4 measurements of Block.[15] From the phase-shift

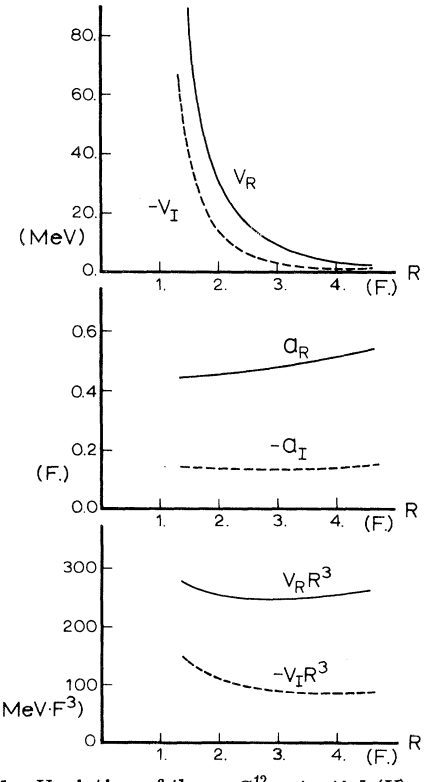

FIG. 1. Variation of the π-C^{12} potential (V), scattering length (a), and volume integral of the potential (VR^3) due to a change of the radius (R) from 1.5 to 4.5 F. Real (imaginary) parts of these quantities are suffixed as R (I) and shown in unbroken (broken) lines.

analyses of these data we obtain the best-fit scattering length $0.16 \pm 0.03 - i(0.13 \pm 0.08)$ F by assuming the usual effective range expansion. This value is in good agreement with our results from the π^--mesonic atom data.

A complete description of the work reported in this Letter will be presented elsewhere.

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