Note added in proof. – (i) We have been informed by J. D. Bjorken of a related paper by A. I. Vainshtein and B. L. Ioffe {Zh. Eksperim. i Teor. Fiz.–Pis'ma Redakt. 6, 917 (1967) [translation: Soviet Phys.–JETP Letters 6, 341 (1967)]}. We will discuss this work in our detailed paper. (ii) In the case when one current is an axial-vector current [the first two lines of Eq. (6)], we have omitted an SU(3)-singlet contribution to the Bjorken limit coming from the triangle diagram discussed by Johnson and Low. Addition of this piece does not alter any of our conclusions.

[1]J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[2]For references, see G. Preparata and W. I. Weisberger, Phys. Rev. 175, 1965 (1968).

[3]For a survey, see lectures by J. D. Bjorken, in Selected Topics in Particle Physics, Proceedings of the International School of Physics "Enrico Fermi," Course XLI, edited by J. Steinberger (Academic Press, Inc., New York, 1968).

[4]C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).

[5]K. Johnson and F. E. Low, Progr. Theoret. Phys. (Kyoto) Suppl. Nos. 37-38, 74 (1966).

[6]The renormalized vertex $\tilde{\Gamma}(C;p,p')$ is obtained from the unrenormalized vertex $\Gamma(C;p,p')$ by multiplying by the fermion wave-function renormalization constant Z_2, with no further finite rescalings.

[7]The term $\ln q_0{}^2$ is present when p_0 is finite and is not a result of the additional $p_0 \to \infty$ limit.

[8]See S. L. Adler and R. F. Dashen, Current Algebras (W. A. Benjamin, Inc., New York, 1968), Chap. 4.

[9]This connection was first noted by F. J. Gilman, Phys. Rev. 167, 1365 (1968).

VECTOR-MESON DOMINANCE AND HIGH-ENERGY ELECTRON-PROTON INELASTIC SCATTERING*

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A vector-meson dominance model for high-energy ep inelastic scattering is proposed. We predict that at high values of the missing mass (>2.5 BeV) the ratio of the longitudinal to the transverse cross section is given by $\xi(q^2/m_\rho{}^2)[1-(q^2/2m_p\nu)]^2$, where ξ is expected to be of the order of unity. Scale invariance (in the sense of Bjorken) is satisfied at very high q^2. The model may adequately account for the remarkably large cross section recently observed in the deep inelastic region.

The conjecture that the total (hadronic) photoabsorption cross section may be calculated from diffractive photoproduction of[1] ρ, ω, and φ has been brilliantly confirmed by recent experiments.[2] This implies that the hypothesis of vector-meson dominance can successfully account for virtual Compton scattering on protons of transversely polarized, $q^2 < 0$ (timelike in our metric) photons. It is generally conceded, however, that vector-meson dominance fails for spacelike values of q^2 because the photoabsorption cross section for spacelike photons deduced from electron-proton inelastic scattering[3] falls off with q^2 far more slowly than is expected from a $[m_\rho{}^2/(m_\rho{}^2+q^2)]^2$-type formula.[4] In this Letter we show that, when the longitudinal contribution is properly calculated, the hypothesis of vector-meson dominance is completely compatible with existing experimental data that do not separate the longitudinal from the transverse cross section. Our model makes a number of specific predictions; apart from its simplicity, its main virtue is that it can be readily destroyed by experiment.

We start by writing out the ep inelastic differential cross section in terms of W_1, W_2, σ_T, and σ_S:

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{E'}{E}\frac{4\pi\alpha^2}{q^4}\left[W_2(q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(q^2,\nu)\sin^2\frac{\theta}{2}\right], \tag{1}$$

$$W_2 = \frac{K}{4\pi^2\alpha}\frac{q^2}{q^2+\nu^2}(\sigma_T+\sigma_S), \quad W_1 = \frac{K}{4\pi^2\alpha}\sigma_T, \quad E-E'=\nu, \quad q=(\vec{q}, i\nu) = \text{virtual-photon momentum},$$

$$K = \nu-(q^2/2m_p) = (s-m_p{}^2)/2m_p,$$

where $\sqrt{s}$ = missing (hadronic) mass. The transverse and longitudinal cross sections σ_T and σ_S (in

Hand's notation) are given by

$$\sigma_T = (2\pi\alpha/K)(2\pi)^4 \sum_A |\langle A|j_\perp|p\rangle|^2 \delta^{(4)}(p+q-p^{(A)}),$$

$$\sigma_S = (2\pi\alpha/K)(q^2/\nu^2)(2\pi)^4 \sum_A |\langle A|j_{\parallel}|p\rangle|^2 \delta^{(4)}(p+q-p^{(A)}), \tag{2}$$

where $ej_\perp$ and $ej_\parallel$, respectively, stand for the components of the electromagnetic current density perpendicular or parallel to the virtual-photon three-momentum $\vec{q}$. The state vector $|p\rangle$ stands for the initial proton state of momentum $p=(0,0,0,im_p)$ and $|A\rangle$ stands for any hadronic state. The expression $\sum_A$ denotes averaging over the initial proton spin states as well as summing over the final states.

The hypothesis of vector-meson dominance relates an electromagnetic matrix element $\langle A|j_\mu|p\rangle$ to the corresponding hadronic matrix element $\langle A|J_\mu^{(\rho)}|p\rangle$ via[5]

$$\langle A|j_\mu|p\rangle = (m_\rho^2/f_\rho)(m_\rho^2+q^2)^{-1}\langle A|J_\mu^{(\rho)}|p\rangle, \tag{3}$$

where $J_\mu^{(\rho)}$ stands for the source density of the neutral-ρ-meson field.[6] Let us therefore study the ρp interactions on the ρ mass shell.

It is straightforward to calculate the total ρp cross section for transversely polarized ρ mesons:

$$\sigma_{\rho p}^{(\perp)} = (1/2|\vec{q}|)(2\pi)^4 \sum_A |\langle A|J_\perp^{(\rho)}|p\rangle|^2 \delta^{(4)}(p+q-p^{(A)}), \tag{4}$$

where $\vec{q}$ and q_0 are the three-momentum and energy, respectively, of the incident ρ in the laboratory system. To calculate the total ρp cross section for longitudinally polarized ρ mesons we must first note that (i) the polarization four-vector $\epsilon^{(L)}$ is given by $(1/m_\rho)(q_0\hat{q}, i|\vec{q}|)$, and (ii) the time-component matrix element is related to the longitudinal matrix element by the conservation equation

$$|\vec{q}|\langle A|J_\parallel^{(\rho)}|p\rangle = q_0\langle A|J_0^{(\rho)}|p\rangle. \tag{5}$$

Since the T matrix element for $\rho_{\text{longitudinal}}+p\to A$ is $\langle A|\epsilon^{(L)}J^{(\rho)}|p\rangle$, the longitudinal cross section is given by

$$\sigma_{\rho p}^{(\parallel)} = (1/2|\vec{q}|)(m_\rho^2/q_0^2)(2\pi)^4 \sum_A \langle A|J_\parallel^{(\rho)}|p\rangle|^2 \delta^{(4)}(p+q-p^{(A)}). \tag{6}$$

At high energies where diffraction scattering dominates, both $\sigma_{\rho p}^{(\perp)}$ and $\sigma_{\rho p}^{(\parallel)}$ are expected to become constant.

The fundamental hypothesis of vector-meson dominance is that

$$\sum_A |\langle A|J_{\perp,\parallel}^{(\rho)}|p\rangle|^2 \delta^{(4)}(p+q-p^{(A)}),$$

regarded as functions of q^2 and s, or equivalently K, vary very little with q^2. We can then write with the aid of (3)

$$(2\pi)^4 \sum_A |\langle A|j_\perp|p\rangle|^2 \delta^{(4)}(p+q-p^{(A)}) = 2(m_\rho^2/f_\rho)^2(m_\rho^2+q^2)^{-2}K\sigma_{\rho p}^{(\perp)}(K),$$

$$(2\pi)^4 \sum_A |\langle A|j_\parallel|p\rangle|^2 \delta^{(4)}(p+q-p^{(A)}) = 2(m_\rho^2/f_\rho)^2(m_\rho^2+q^2)^{-2}(K^3/m_\rho^2)\sigma_{\rho p}^{(\parallel)}(K), \tag{7}$$

where we have replaced $|\vec{q}|$ and q_0 appearing in (4) and (6) by[7] K (an approximation valid in the diffraction region of the ρp interaction). The transverse and longitudinal cross sections measurable in ep inelastic scattering are given by[8]

$$\sigma_T(q^2,\nu) = (e/f_\rho)^2[m_\rho^2/(m_\rho^2+q^2)]^2\sigma_{\rho p}^{(\perp)}(K),$$

$$\sigma_S(q^2,\nu) = \left(\frac{e}{f_\rho}\right)^2\left(\frac{m_\rho^2}{m_\rho^2+q^2}\right)^2\frac{q^2}{m_\rho^2}\left(\frac{K}{\nu}\right)^2\xi(K)\sigma_{\rho p}^{(\perp)}(K), \tag{8}$$

where the parameter ξ defined by

$$\xi(K) \equiv \sigma_{\rho p}^{(\parallel)}(K)/\sigma_{\rho p}^{(\perp)}(K) \tag{9}$$

is in principle measurable in electroproduction of the ρ meson.[9]

In our model the ratio σ_S/σ_T which we denote by R reads

$$R \equiv \frac{\sigma_S(q^2,\nu)}{\sigma_T(q^2,\nu)} = \xi(K)\frac{q^2}{m_\rho^2}\left[1-\frac{q^2}{2m_p\nu}\right]^2. \tag{10}$$

We therefore predict that R varies as q^2 at fixed ν/q^2 [as long as $\xi(K)$ is slowly varying]. It is amusing that precisely this behavior was conjectured by Callan and Gross[10] using the commutation relations of gauge field algebra.[11] If, on the other hand, we increase q^2 with the missing mass (or K) fixed, R first increases like q^2, goes through a maximum at $q^2 = s - m_p^2$, and decreases like $1/q^2$ as $q^2 \to \infty$.

As is clear from (1), ep inelastic scattering experiments with small θ (say, 6°, where measurements have been made) determine only the sum $\sigma_T + \epsilon\sigma_S$ with ϵ close to unity (typically 0.7-0.95). Equation (8) implies that, when ν/q^2 is not too low ($\nu/q^2 > 4$ BeV^{-1}) and ξ is the order of unity, $\sigma_T + \epsilon\sigma_S$ falls off with q^2 more like the <u>first</u> power of $m_\rho^2/(m_\rho^2+q^2)$ than like $[m_\rho^2/(m_\rho^2+q^2)]^2$, contrary to naive expectations based on vector-meson dominance. We therefore predict a relatively large number of events in high-q^2 regions as long as θ is kept small. As we shall see later, this agrees with experiment.

So far we have assumed that the photon is purely isovector. We can easily incorporate the ω and φ contributions in our formalism. For practical purposes we simply replace $(e/f_\rho)^2\sigma_{\rho p}(K)$ in (8) by the total photoabsorption cross section $\sigma_{\gamma p}(K)$. This is justified because $m_\omega \approx m_\rho$ and the φ contributions are relatively small.

Bjorken[12] has argued on the basis of current algebra that at high q^2 with q^2/ν fixed the dimensionless quantity $\nu W_2(q^2,\nu)$ becomes a universal function when plotted against the dimensionless variable $2m_p\nu/q^2$ ("scale invariance"). According to (8), we have

$$\nu W_2(q^2,\nu) = \frac{m_\rho^2}{4\pi^2\alpha}\frac{K}{\nu}\frac{1}{1+q^2/\nu^2}\left[\frac{1}{1+m_\rho^2/q^2}\right]^2\left[\xi(K)\left(\frac{K}{\nu}\right)^2+\frac{m_\rho^2}{q^2}\right]\sigma_{\gamma p}(K). \tag{11}$$

In the Bjorken limit (i.e., $q^2 \to \infty$ in such a way that q^2/ν is finite), νW_2 takes a remarkably simple form:

$$\nu W_2 \to [\xi(\infty)m_\rho^2/4\pi^2\alpha]\sigma_{\gamma p}(\infty)(1-q^2/2m_p\nu)^3, \tag{12}$$

which is indeed a "universal" function of ν/q^2. Thus vector-meson dominance accommodates Bjorken's scale invariance. If we further let $\nu/q^2 \to \infty$, then νW_2 approaches a constant.[13] For $\xi = 1$ and for a definite value of ν/q^2 , νW_2 increases monotonically as q^2 is increased, and even at $q^2 = 4$ BeV^2 it is lower than the Bjorken limit (12) by about 15 % for most values of ν/q^2 of practical interest. If ξ is raised (say, $\xi = 1.5$), the approach to the Bjorken limit is even slower. We therefore do not expect Bjorken's universality at $q^2 \approx 1$ BeV^2.

Since precise and extensive ep data at high q^2 and large angles are forthcoming, it may be fruitless at this moment to make a detailed comparison of our predictions with the 6° data of Ref. 3 that cover only a limited range of q^2 and ν. We have nevertheless attempted to test our theory by computing $\sigma_{\gamma p}(K)$ from

$$\sigma_{\gamma p}(K) = \left[1+\frac{q^2}{m_\rho^2}\right]^2(\sigma_T+\epsilon\sigma_S)_{\text{expt}}\left[1+\epsilon\xi(K)\frac{q^2}{m_\rho^2}\left(\frac{K}{\nu}\right)^2\right]^{-1}, \quad \epsilon \equiv \{1+2[1+\nu^2/q^2]\tan^2\tfrac{1}{2}\theta\}^{-1}, \tag{13}$$

and comparing it with the directly measured photoabsorption cross section of Ref. 2. Since the ρ photoproduction cross section appears to show some structure (indicative of possible nondiffractive mechanism) below $K = 3$ BeV ($\sqrt{s} \approx 2.6$ BeV), only the data points with $\sqrt{s} > 2.5$ BeV are used. To eliminate the only adjustable parameter we first set (somewhat arbitrarily) $\xi = 1$ (which may be reasonable if the

ρp interaction becomes spin independent at high energies). The resulting value of $\sigma_{\gamma p}(K)$ obtained from the data points in the range $\sqrt{s} = 2.5$-4.5 BeV and $q^2 = 0.3$-2.0 BeV^2 is 130-190 μb, a value somewhat higher than 110-140 μb indicated in Ref. 2. Since the higher q^2 points tend to result in higher values of $\sigma_{\gamma p}(K)$, the discrepancy can be significantly reduced by increasing ξ to the interval 1.3-1.6. However, in view of the much more extensive 10°, 18°, 26°, and 34° data (still being analyzed experimentally), a detailed fit to the 6° data alone does not seem to be warranted at the present moment.

In conclusion, contrary to widespread beliefs,[3] the observed large cross section in the deep inelastic region does not necessarily force us to the view that we are seeing pointlike structure ("partons," etc.[14]) within the proton. A relatively straightforward model based on vector-meson dominance can account for the slow decrease of $\sigma_T + \sigma_S$ with increasing q^2. Experiments to separate the longitudinal and transverse contributions are urgently needed to test vector-meson dominance.

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[1]L. Stodolsky, Phys. Rev. Letters 18, 135 (1967). See also P. G. O. Freund, Nuovo Cimento 44A, 411 (1966); J. J. Sakurai, Stanford Linear Accelerator Center Report No. SLAC-TN-68-11 (unpublished).

[2]Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Phys. Letters 27B, 474 (1968); J. Ballam et al., Phys. Rev. Letters 21, 1544 (1968). For a recent discussion see Z. G. T. Guiragossian and A. Levy, Stanford Linear Accelerator Center Report No. SLAC-PUB-535, 1968 (to be published).

[3]See W. K. H. Panofsky [in Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, Austria, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), pp. 36-37] based on the SLAC-MIT experiment of E. D. Bloom et al. (unpublished).

[4]To our best knowledge attempts to apply vector-meson dominance to the SLAC-MIT ep inelastic experiments of Ref. 3 were first made by S. Berman and W. Schmidt (unpublished).

[5]For the notation and normalization see J. J. Sakurai, Currents and Mesons (University of Chicago Press, Chicago, Ill., 1969), Chap. 3.

[6]For simplicity we ignore at this stage the isoscalar contributions.

[7]In the earlier version of this paper it was proposed that we replace $|\vec{q}|$ and q_0 appearing in (4) and (6) by ν rather than by K. We believe that the present procedure of treating K (or equivalently s) and q^2, rather than ν and q^2, as the basic variables is more reasonable for two reasons. First, if we fix ν and vary q^2, the missing mass of the final state A also varies (e.g., we move from the "diffraction" region to the resonance region), hence it is ridiculous to expect that $\sum_A |\langle A|J_{\perp,\parallel}^{(\rho)}|p\rangle|^2\delta^{(4)}(p+q-p^{(A)})$ regarded as a function of ν and q^2 varies little with q^2. Second, in the present version (but not in the earlier version) the quantity νW_2 automatically goes to zero as ν/q^2 is decreased to $1/2m_p$, in agreement with experiment [see Eq. (11)].

[8]There is some ambiguity as to whether we should apply the smoothness hypothesis to $\langle A|J_\parallel^{(\rho)}|p\rangle$ or to $\langle A|J_0^{(\rho)} \times|p\rangle$. In obtaining (8) we have eliminated $\langle A|J_0^{(\rho)}|p\rangle$ using (5) and applied the smoothness hypothesis to $\langle A|J_\parallel^{(\rho)} \times|p\rangle$. Alternatively one may prefer to eliminate $\langle A|J_\parallel^{(\rho)}|p\rangle$ and apply the smoothness hypothesis to $\langle A|J_0^{(\rho)}|p\rangle$, in which case our expression for σ_S in (8) must be multiplied by $\nu^2/(q^2+\nu^2)$. Fortunately the difference disappears for $\nu^2 \gg q^2$ (a region of practical interest).

[9]Investigations along this line are in progress in collaboration with C. F. Cho and G. J. Gounaris.

[10]C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).

[11]T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).

[12]J. D. Bjorken, Stanford Linear Accelerator Center Report No. SLAC-PUB-510, 1968 (to be published).

[13]After this work was completed, we learned of the paper of H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman [Phys. Rev. Letters 22, 500 (1969)] who also claim that $\nu W_2 \to$ const in the same limit using a Regge-type argument.

[14]R. P. Feynman (unpublished).