shed more light on the structure of ^{22}Ne.

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NUCLEAR HEAVY-ION–HEAVY-ION COLLISIONS AND THE INTERMEDIATE-STATE MODEL

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Five excitation functions from 50 to 90° (c.m. system) for O^{16}-O^{16} elastic scattering can be accounted for by a nuclear molecular potential. The parameters of this potential are estimated from the two-nucleon potential in a model of the transient nuclear matter state.

The classical optical model applied to O^{16}-O^{16} scattering has proven to be deficient in several respects.[1-3] These calculations fail to reproduce the extremely low cross sections seen in the valleys of the excitation function. The calculated peak-to-valley ratios of the cross section are much too small at energies above the Coulomb threshold. The optical model also fails to produce the first peak seen at about 17 MeV. In particular, the deep valleys and the large peak-to-valley ratios motivated an attempt to describe the scattering in terms of an average nonmonotonic potential containing a short-range repulsion, which is the lowest approximation to the general nonlocal potential originating from the Pauli principle. Applying the model of Kerlee, Blair, and Farwell,[4] the large peak-to-valley ratios of the 90° excitation function could be reproduced in the energy range 20-26 MeV.[5]

In this note, we shall present an effective nonmonotonic potential describing the excitation functions at 49.3°, 60°, 69.8°, 80.3°, and 90° (c.m.) in the energy range 10-22 MeV (c.m.), thus demonstrating that the low-energy data are compatible with the concept of a "core." Further, we develop a model to estimate important quan-

tities in this potential.

Figure 1 shows the experimental points of Siemssen et al.,[1] Maher,[3] and Bromley, Kuehner, and Almqvist,[6] along with our preliminary fit. The potential used is

$$\mathrm{Re}\,W = 100\exp(-0.6r) - 27\{1+\exp[(r-R)/a]\}^{-1},$$

$$\mathrm{Im}\,W = -1.613(1+0.02E+0.012E^2) \times \exp(-r^2/4.2),$$

with $R \simeq 2(O^{16}\ \text{radius}) = 7.0$ F (dashed curve in Fig. 1), and $a = 0.53$ F. In addition the usual Coulomb potential is added. (See Ref. 5 for a figure of the net real potential.)

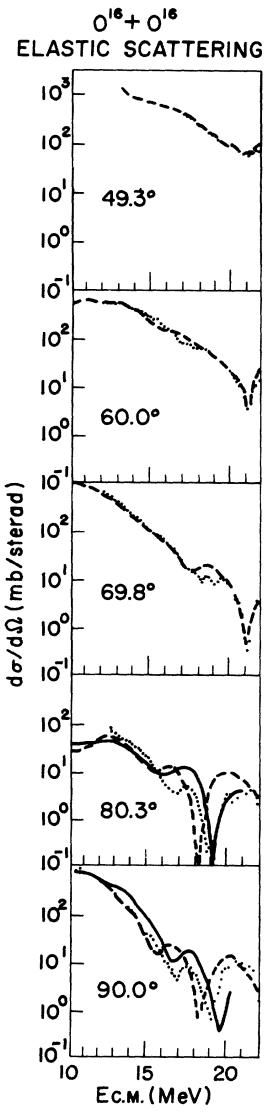

FIG. 1. Comparison of theoretical excitation functions with the experimental ones at various center-of-mass system angles. The data points are shown by dots. The dashed curve is the potential W. The solid curve is W with $R = 6.83$ F instead of 7.00 F.

The real part of the potential is very similar to the one used for fitting the higher energy data.[5]

Without an extensive search and with an energy-independent real part these preliminary results can produce the absolute cross section as well as important structural features. In particular, as regards the 90° data, the little dip and the peak around 17 MeV, followed by the deep valley around 19 MeV, are reproduced. The first dip and peak in other excitation functions are also accounted for. The first dip is extremely sensitive to the actual choice of the parameters, e.g., by changing R to 6.83 F (solid curve, Fig. 1) we obtain a better fit to the initial dip and valley in the 90° and the 80.3° excitation functions without disturbing the remainder of the fit at other energies and angles. This dip is also sensitive to the diffuseness parameter, e.g., using $a = 0.49$ F nearly washes out the 17-MeV dip. In fact, if this dip is overlooked, one can obtain a considerably improved fit of the 19-MeV dip and 20.5 MeV peak for the 90° excitation functions. The core radius B is also a critical parameter. The analysis indicates that many types of potentials with or without a core can fit the excitation functions between 10 and 13 MeV, provided they include a reasonable imaginary component. However, most of them cannot reproduce the first dip. The important point of our fit is that over a large energy region—about 12 MeV—one has a decent fit not only in terms of absolute magnitude, but also in terms of major features like deep valleys using energy-independent parameters for the real potential. It should be noted that at higher energies the shape of the imaginary potential near the ion-ion radius R will be important in giving the large experimental peak-to-valley ratios.

The 17-MeV dip in the 90° excitation function has an $L = 4$ resonance interfering destructively with other partial waves, in particular with $L = 8$. As the energy increases the $L = 4$ resonance undergoes a phase change so as to interfere constructively to reproduce the 17.5-MeV peak. The dip at ~19.0 MeV and the peak at 20.5 MeV are, respectively, destructive interferences and constructive interferences among a few partial waves; although partial waves up to $L = 14$ contribute and interfere, primarily $L = 2$ is out of phase with $L = 4$ and $L = 8$ at this dip, and $L = 2$ and $L = 14$ are in phase at the 20.5-MeV peak. These

interferences result in oscillatory angular distribution.[7]

In principle, the potential obtained empirically could be calculated within the framework of the Feshbach reaction theory[8-10] or the resonating-group method.[11] These reaction theories dictate that the ion-ion interaction should be composed of a nonlocal potential in the elastic channel with couplings to real and virtual excited channels. The empirical repulsive core is a rough manifestation of the nonlocalities originating from (a) the Pauli principle and (b) the nonlocality of the basic two-nucleon potential. The gross structure results from scattering by an average static potential which is the actual nonlocal potential suitably averaged over the energy. The fine structure is related to the excitation of various kinds of intermediate states. An optical-model potential, in this context, can be derived, if the effects of the coupling to inelastic channels are included in an average way in the elastic channel. However, the large number of particles involved in the heavy-ion collisions makes such an approach very difficult. On the other hand, for a system consisting of a large number of nucleons, a nuclear-matter approach can yield a rough estimate of the important parameters and considerable insight into the overall physical process. We take this approach, recognizing that a system of 32 nucleons is only a crude approximation to infinite nuclear matter.

In this spirit, let us visualize that at a particular instant, two O^{16} ions are in a configuration shown in Fig. 2. Nucleons near the surface interact first forming a "neck." The potential interacting between the centers of mass of two nuclei is generated by these nucleons at the neck.

For our purpose the density of this overlapping region is the important factor. Since the radius of the effective potential is twice the O^{16} radius, the average density of this blown-up S^{32} must be low. Moreover, the overlapping region involves the surface densities of the two O^{16}'s and this density is considerably lower than that of the O^{16} interior. Thus two points distinguish this situation from the state of usual nuclear matter: (a) The intermediate state configuration is far from that corresponding to equilibrium, i.e., far from the nuclear saturation for 32 nucleons, and (b) the average density is lower than that corresponding to equilibrium for nuclear matter.

The low density ρ of this "transient nuclear fluid state" (TNFS) (or the nuclear matter in the neck region) implies a lower Fermi momentum

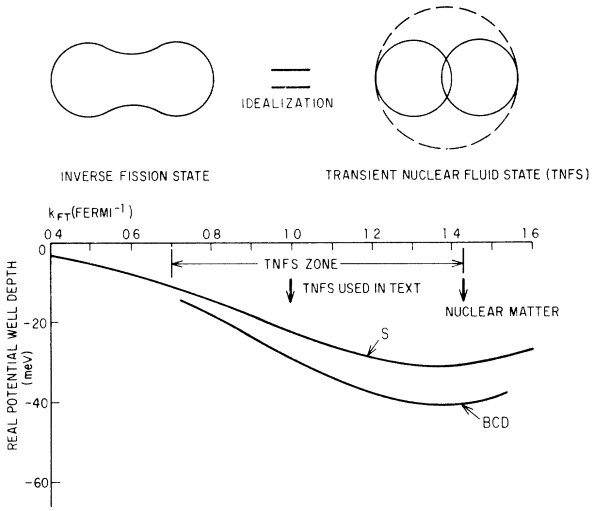

FIG. 2. Schematic diagram of the idealized configuration at the time of the reaction and the computed depth of the real part of the potential as a function of the k_{FT}. The curves S and BCD are calculated from D. W. L. Sprung [in Proceedings of the Third International Conference on Atomic Masses and Related Constants, Winnipeg, Canada, 1967, edited by R. C. Barber (University of Manitoba Press, Winnipeg, Canada, 1968)] and K. A. Brueckner, S. Coon, and J. Dabrowski (to be published), respectively.

p_F because

$$p_F = \hbar k_F \propto \rho^{1/3}. \tag{1}$$

The Fermi momentum k_{FT} of the TNFS therefore lies between

$$0.7\ F^{-1} \leq k_{FT} \leq k_F = 1.43\ F^{-1}. \tag{2}$$

For the purpose of a concrete estimate, we may take $k_{FT} \simeq 1\ F^{-1}$ which corresponds to a transient configuration of uniform density having exactly twice the radius of O^{16}.

Since the two-nucleon potential contains a strong short-range repulsion, the simple Hartree-Fock expression of the average potential seen by nucleons in state k,

$$W(k) = \sum_{k' \leq k_{FT}} \langle k'k|v|k'k\rangle_{\text{antisymmetrized}}, \tag{3}$$

is to be replaced by

$$W(k) = \sum_{k' \leq k_{FT}} \langle k'k|G|k'k\rangle_{\text{antisymmetrized}}, \tag{4}$$

where G is the reaction matrix related to v by

$$G = v - v(Q/e)G, \tag{5}$$

where Q and e are, respectively, the Pauli projection operator and the appropriate energy denominator.

A less arbitrary evaluation of the average potential seen by nucleons near the Fermi surface can be obtained also from the binding energy per nucleon using the Hugenholtz-Van Hove[12] theorem:

$$W(k_{\mathrm{FT}}) = [-T(\rho) - E(\rho) + \rho dE/d\rho]_{\rho=\rho_{\mathrm{FT}}}, \qquad (6)$$

where T, E, and ρ_{FT} are, respectively, the kinetic energy, the binding energy per nucleon and the density in the TNFS. At saturation, the last term in (6) vanishes.

We have evaluated $W(k_{\mathrm{FT}})$ using (6) and the results of the nuclear-matter calculations of Sprung[13] and Brueckner, Coon, and Dabrowski.[14] The former authors used the Reid soft-core potential[15] and obtained a saturation energy of -8.6 MeV/nucleon which is almost 7 MeV/nucleon less than the experimental one. The latter authors obtained -15.2 MeV/nucleon, using the older Brueckner-Gammel-Thaler potential. Figure 2 indicates that the depth of the potential seen by nucleons near the Fermi sea is very close to the phenomenological one. The potential is indeed a shallow one which has an upper limit of the potential seen by nucleons near the top of the Fermi sea in nuclear matter.

In principle it is difficult to separate the contribution of the nonlocality originating from the Pauli principle from the nonlocality of the nucleonic potential. However, we can make a very rough estimate of the effect of the Pauli principle on the energy shift if we look into the contribution to the energy shift originating from the Pauli operator in (5). According to Bhargava and Sprung,[16] this latter generates a positive energy shift of from 5 to 7 MeV for the density of our interest. The strength of the exponential repulsive core which has been used in our potential model to mock up this Pauli repulsion can be estimated roughly in the adiabatic approximation via

$$\Delta E \simeq \int_0^R \rho_{\mathrm{FT}}(r) A \times \exp(-Br) r^2 dr \left[\int_0^R \rho_{\mathrm{FT}}(r) r^2 dr\right]^{-1}. \qquad (7)$$

For constant density, $\Delta E \simeq 6$ MeV, and $B = 0.6$ F^{-1}, we find $A \simeq 74$ MeV—for $\Delta E \simeq 7$ MeV, we obtain instead $A \simeq 85$ MeV—both of which compare favorably with the value of 100 MeV used. In fact, for the exponential case, (7) implies $A \propto \Delta E(RB)^3$, and changing B to 0.67 F^{-1}, yields $A \simeq 100$ MeV. This simple but rough estimate tends to show that our empirical core strength is reasonable.

To estimate the strength of the imaginary potential in the context of this model, the "frivolous model" of Goldberger[17] may be used. In this model

$$\mathrm{Im}\, W = \textstyle\sum_i \frac{1}{2} v_{\mathrm{Av}}(i) \rho_{\mathrm{FT}} \langle \sigma_i \rangle, \qquad (8)$$

where $v_{\mathrm{Av}}(i)$ and $\langle \sigma_i \rangle$ are, respectively, the average velocity and collision cross section of the ith colliding nucleon with those of the target using the outgoing boundary condition with due regard to the Pauli principle. As yet no calculation of $\langle \sigma_i \rangle$ using the Reid potential has been carried out, although we are in the process of doing so. An evaluation of $\langle \sigma_i \rangle$ involves the average kinetic energy T of the incident nucleon. This is a sum of the Fermi kinetic energy T_{FT} of the nucleon in the neck, its binding energy in TNFS, and average external kinetic energy per nucleon, which for the O^{16}-O^{16} collision at 16 MeV (c.m.) is about 1 MeV. Thus T in the TFNS is somewhat lower than in the case of a 1-MeV nucleon incident on a chunk of nuclear matter. For the purpose of this crude estimate, we may overlook this difference and take the value of $\langle \sigma_i \rangle$ from the calculation of Gomes.[18] Noting that the density in the TNFS is about one-fourth that of nuclear matter, the velocity corresponding to the above T is about three quarters of that in nuclear matter and the summation i runs over 16 incident nucleons involved in a simultaneous collision. We obtain that $\mathrm{Im}W \simeq -6$ MeV, which compares favorably with the magnitude of our imaginary potential.

Since the TFNS is a nonequilibrium state, the compressibility modulus K is given by

$$K = [2\rho \partial E/\partial\rho + \rho^2 \partial^2 E/\partial\rho^2]_{\rho=\rho_{\mathrm{FT}}},$$

where E is the energy per nucleon at density ρ. Sprung's computation indicates $\partial^2 E/\partial\rho^2$ to be a sensitive function of k_{FT} between 0.7 F^{-1} and 1.1 F^{-1}. For $k_{\mathrm{FT}} = 1.0$ F^{-1}, K obtained from his calculation is about one-sixth that of nuclear matter at equilibrium.

The nonformation of S^{32} in its ground state, in spite of 17 MeV energy being available, may be attributed to the short collision time in comparison to the relaxation time. The former is about 10^{-22} sec, and the latter is of the order of the time taken by a nucleon to traverse a nucleus a few times, i.e., $\sim 10^{-21}$ sec. This characteristic

ratio τ prevents the formation of a nonequilibrium state.

This model is an inverse of the fission process. τ is the most critical parameter for reaching superheavy elements by an inverse fission process.

Obviously, for the O^{16}-O^{16} case our model is only a rough approximation. For heavier elements and higher bombarding energies the model should become better. The following general conclusions of this model hold for all heavy ion reactions: (i) The attractive part of the real potential is shallow. At higher bombarding energies its depth should increase somewhat because of larger penetration. (ii) The depth of the imaginary potential is also shallow, and (iii) the strength of the repulsive core is of the order of 100 MeV.

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PROTON-POLARIZATION MEASUREMENTS AND DISTORTED-WAVE CALCULATIONS FOR $^{52}Cr(d,p)^{53}Cr$ AT 11 MeV

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The proton polarization and the vector analyzing power for the reaction $^{52}Cr(d,p)^{53}Cr$(g.s.) were measured. From an analysis of the $^{52}Cr(d,d)^{52}Cr$ and $^{53}Cr(p,p)^{53}Cr$ elastic scattering cross sections and polarizations, potentials were obtained which, when used in a distorted-wave Born-approximation calculation, provided a good fit to the proton polarization and the deuteron analyzing power.

Stripping and pickup reactions have for some time been important tools in nuclear spectroscopy. In deuteron stripping reactions, $A(d,p)B$, a neutron is transferred from the incoming deuteron to the target nucleus A to form the final nucleus B in the ground state or in one of its excited states. The angular distribution of the outgoing protons depends primarily upon the orbital angular momentum l of the captured neutron.[1] The value of l is generally determined by a comparison between the angular distribution of the outgoing protons and cross sections calculated from distorted-wave Born approximation (DWBA). The DWBA has been very successful in reproducing