UNITARY SUM RULE FOR K_S-K_L DECAY*

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A new unitary sum rule for K_S-K_L decay is derived by use of a phenomenological S matrix.

The unitary sum rule derived by Bell and Steinberger[1] has played an important role in the analysis of CP violation in the neutral K system. The sum rule is

$$-i(M_L{}^*-M_S)\langle K_L|K_S\rangle = \sum_F \langle F|H|K_L\rangle^*\langle F|H|K_S\rangle, \qquad (1)$$

where M_S and M_L are the complex masses corresponding to the short- and long-lived particles and the summation is over all possible final states. Let us suppose the mass differences were large compared with the sum of the widths. The sum rule states that the orthogonality expressed in terms of K_0 and $\bar{K}_0$ (or the orthogonality of the decaying states with the energy variable suppressed) must be small. It is difficult to see why such a strong restriction on the decaying states is imposed if the masses differ by more than the sum of their widths.

In any event, the present authors thought it should be possible to derive a sum rule that results from unitarity by considering a phenomenological S matrix which has K_S and K_L as resonant states. Indeed it is possible but not with (1) as a result.

In a multichannel process in which the scattering is dominated by a single isolated resonance the approximate expression for the T matrix is

$$T=\tfrac{1}{2}\Gamma P/(M-E),$$

where M is the complex mass of the resonance and P is a one-dimensional projection operator, i.e., $P=\Phi\Phi^+$, with $\langle\Phi,\Phi\rangle=1$. For such a T, $S=1+2iT$. For two resonances characterized by P_L, M_L, and P_S, M_S one cannot in general define

$$T=\frac{\frac{1}{2}\Gamma_L P_L}{M_L-E}+\frac{\frac{1}{2}\Gamma_S P_S}{M_S-E}, \qquad (2)$$

for the resulting S matrix would not be unitary unless $P_SP_L=0$.

However, the K-matrix formalism does permit a handling of overlapping, nonorthogonal resonances.[2] If we define K such that

$$T=K(1-iK)^{-1},$$

then, for K Hermitian, the resulting S matrix is unitary. For isolated resonances we have

$$K=\frac{\frac{1}{2}\Gamma P}{m-E}, \quad T=\frac{\frac{1}{2}\Gamma P}{(m-\frac{1}{2}i\Gamma)-E}.$$

For two overlapping resonances one can write

$$K=\frac{\frac{1}{2}\Gamma_S P_S}{m_S-E}+\frac{\frac{1}{2}\Gamma_L P_L}{m_L-E}.$$

One then obtains for T

$$T=\frac{\frac{1}{2}\Gamma_S(M_L-E)P_S+\frac{1}{2}\Gamma_L(M_S-E)P_L+(i/4)\Gamma_S\Gamma_L(P_LP_S+P_SP_L)}{(M_L-E)(M_S-E)+(i/4)\Gamma_S\Gamma_L|\chi|^2}, \qquad (3)$$

where

$$\chi=\langle\Phi_S,\Phi_L\rangle, \quad M_L=m_L-\tfrac{1}{2}i\Gamma_L, \text{ and } M_S=m_S-\tfrac{1}{2}i\Gamma_S.$$

It has a two-pole structure and reduces to (2) if $P_1P_2=0$. If one assumes that the overlap χ is small,

$$T\simeq\frac{\frac{1}{2}\Gamma_L P_L}{M_L-E}+\frac{\frac{1}{2}\Gamma_S P_S}{M_S-E}+\frac{i}{4}\frac{\Gamma_L\Gamma_S(P_LP_S+P_SP_L)}{(M_L-E)(M_S-E)}. \qquad (4)$$

The residue matrix of the pole at $E=M_S$ is

$$R_S=\frac{1}{2}\Gamma_S P_S+\frac{i}{4}\frac{\Gamma_S\Gamma_L(P_L P_S+P_S P_L)}{M_L-M_S};\qquad(5)$$

and similarly at $E=M_L$:

$$R_L=\frac{1}{2}\Gamma_L P_L+\frac{i}{4}\frac{\Gamma_S\Gamma_L(P_L P_S+P_S P_L)}{M_S-M_L}.\qquad(6)$$

The transition amplitude of Φ_L to a state $|n\rangle$ is

$$\langle n|H|\Phi_L\rangle\equiv\langle n|(2R_L/\Gamma_L^{1/2})|\Phi_L\rangle,$$

and of Φ_S to a state $|n\rangle$ is

$$\langle n|H|\Phi_S\rangle\equiv\langle n|(2R_S/\Gamma_S^{1/2})|\Phi_S\rangle.$$

From these relations it is easy to derive the following sum rule:

$$(\Gamma_S\Gamma_L)^{1/2}\chi\left[\frac{\delta m}{\delta m+\frac{1}{2}i\Gamma_S}\right]=\sum_n\langle n|H|\Phi_S\rangle^*\langle n|H|\Phi_L\rangle,\qquad(7)$$

where $\delta m=m_L-m_S$. In (7) only terms linear in χ are retained and Γ_L was ignored compared with Γ_S.

This sum rule differs considerably from (1). It states, for instance, that the overlap can approach unity if $\delta m\gg\Gamma_S$. This statement still obtains if one keeps all orders of χ in (7). Thus there appears to be no restriction once the mass difference is large compared with the widths.

There is a further restriction on P_S and P_L, that is on $|\Phi_S\rangle$ and $|\Phi_L\rangle$, if one wants to interpret K_L and K_S resonant states as linear combinations of K_1 and K_2, the states of definite CP. Then $|\Phi_S\rangle$ and $|\Phi_L\rangle$ must be linear combinations of two states $|\Phi_1\rangle$ and $|\Phi_2\rangle$, where $|\Phi_1\rangle$ and $|\Phi_2\rangle$ have definite CP properties. This further assumption does not change the sum-rule result.

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[1]J. S. Bell and J. Steinberger, in Proceedings of the Oxford International Conference on Elementary Particles, September, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966), pp. 195-222.

[2]For instance, R. L. Warnock, in Conference on Elementary Particles, Annual Summer Institute for Theoretical Physics, Boulder, Colorado, 1966, edited by W. E. Brittin and A. O. Barut (Gordon and Breach Publishers, Inc., New York, 1967).

VENEZIANO MODEL FOR π-ρ SCATTERING*

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A Veneziano model for the odd-normality (π) contributions to $\pi\rho$ double-charge-exchange scattering is found. In the model, the leading trajectory is not parity doubled and its residues satisfy the factorization condition. An argument is given for the existence of high-mass exotic resonances.

We have investigated some of the problems which occur when one attempts to construct a model like Veneziano's[1] for scattering amplitudes in which more than one of the particles has spin. The new features are the possibility of parity doubling and the question of factorization of the residues (nondegeneracy) along the Regge trajectory. As an example, we have studied $\pi\rho$ double-charge-exchange scattering, $\pi^+ +\rho^- \to\pi^- +\rho^+$. Resonances occur only in the s and u channels, which are identical, and we are freed from the complications of dealing with channels which communicate with the ρ or the vacuum.

Let the pion momenta be p and p', the ρ momenta be q and q', and the final polarization vectors be ϵ and ϵ', with $\epsilon\cdot q=\epsilon'\cdot q'=0$. The primes refer to the final state. The scattering amplitude is $\epsilon'^{\nu}\epsilon^{\mu}T_{\mu\nu}$, and $T_{\mu\nu}$ can be expanded in terms of four scalar invariant amplitudes:

$$T_{\mu\nu}=A(s,u)P_\mu P_\nu+B(s,u)[P_\mu q_\nu+q_\mu'P_\nu]+C(s,u)q_\nu q_\mu'+D(s,u)g_{\mu\nu},\qquad(1)$$

where $P=\frac{1}{2}(p+p')$. Under exchange of s and u, A, C, and D are even and B is odd.

It is convenient to use the familiar "parity-con-