[2]I. J. Ashburner, R. C. Newman, and S. D. McLaughlan, Phys. Letters 27A, 212 (1968).

[3]J. M. Baker, E. R. Davies, and J. P. Hurrell, Phys. Letters 26A, 352 (1968).

[4]C. Froese, Proc. Cambridge Phil. Soc. 53, 206 (1957).

[5]C. L. Pekeris, Phys. Rev. 112, 1649 (1958).

[6]D. Kiro and W. Low, to be published.

[7]E. R. Davies, thesis, University of Oxford, 1968 (unpublished).

[8]R. E. Watson and A. J. Freeman, Phys. Rev. Letters 6, 277 (1961).

[9]We assume that $g_{\parallel}$ and $g_{\perp}$ are positive. $g_{\parallel}$ can be easily shown to be positive in order to explain the ground-state wave functions. If $g_{\perp}<0$, then if (a) $A_{\parallel}>0$ and $A_{\perp}<0$ we find $A_s \sim -0.6$ and $A_p \sim +5.7$ Mc/sec and if (b) $A_{\parallel}<0$ and $A_{\perp}>0$, we find $A_s \sim -11.0$ and $A_p \sim -12$ Mc/sec. The isotropic part in case (a) could fit the model suggested by Freeman and Watson. The small A_p then would mean that there is an anisotropic interaction of about -6 Mc/sec due to the higher configuration.

FINITE-NUCLEUS G MATRIX IN THE MICROSCOPIC DESCRIPTION OF INELASTIC PROTON SCATTERING

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The Kallio-Kolltveit interaction is used as the effective interaction in a distorted-wave–approximation analysis of a few transitions in the reactions $C^{12}(p,p')$ and $Ca^{40}(p,p')$ at energies ranging from 25 to 55 MeV, and a comparison is made with results obtained using Yukawa interactions parametrized on the basis of low-energy nucleon-nucleon and nuclear data. It is concluded that the Kallio-Kolltveit interaction gives a good account of the basic features of the reactions, whereas the other interactions considered lead to somewhat poorer overall agreement with experiment.

The G matrix approach to the nuclear shell model has enjoyed considerable success recently.[1] The purpose of this Letter is to present some results which have been obtained by assuming that the shell-model G matrix is a good approximation to the effective interaction for the inelastic scattering of protons from nuclei in the energy region 15-70 MeV. This assumption gave a good account of the real part of the optical potential for elastic proton scattering over the energy range 20-40 MeV.[2]

For the G matrix we have used the long-range part of the Kallio-Kolltveit (KK) interaction as defined via the Scott-Moszkowski method.[3] The separation distances, inside of which the interaction vanishes, are fixed as in Ref. 3. This gives an s-state interaction which is known to represent, reasonably well, the "effective" central part of the G matrix.[4]

Since all our calculations are carried out within the framework of the usual local, or zero-range, distorted-wave approximation (DWA)[5] it is convenient to relax the s-state condition and simply require that the interaction act only in even states. This may lead to errors of up to 20% in the strength of the interaction. Thus we have a local, state-independent, two-body interaction which is scalar separately in spin, isospin, and coordinate space. It can be written in standard form as

$$V(r_{12}) = V_{00}(r_{12}) + V_{10}(r_{12})\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_{01}(r_{12})\vec{\tau}_1\cdot\vec{\tau}_2 + V_{11}(r_{12})\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2. \quad (1)$$

In the above relations the double subscript on V is to be read as ST referring to the multipole components of the force in spin and isospin space, respectively.

A properly antisymmetrized formulation of the DWA gives rise to an exchange component in the

Table I. Strength of delta function in pseudopotential as a function of the lab energy for the spin and isospin multipoles of the KK interaction.

E (MeV)	$A(S=0;T=0)$	$A(S=1;T=0)$	$A(S=0;T=1)$	$A(S=1;T=1)$
20	−236.0	45.8	111.5	78.7
30	−183.0	32.8	89.2	61.0
40	−140.6	22.7	71.1	46.9
50	−106.5	14.9	56.2	35.5
60	−78.9	8.8	43.8	26.3
70	−56.4	4.0	33.6	18.8
80	−38.0	0.3	25.0	12.7

transition amplitude which is known to be important.[6] Existing DWA codes which include antisymmetrization are not set up to handle an interaction of the type being considered here. Therefore, we include effects due to antisymmetrization through the use of a pseudopotential in the usual DWA, thus maintaining the use of the simple, single-argument form factors (see Ref. 5) which are functions obtained by folding the appropriate multipole component of the effective interaction being used with the transition density which, for a given transition, contains all of the nuclear-structure information.

The definition of the pseudopotential is based on the approximation used by Perey[7] in reducing nonlocal interactions to equivalent local forms for use in evaluating reaction transition amplitudes. The definition is obtained by expressing the interaction in the exchange component of the transition amplitude in terms of its Fourier transform which is then expanded in a Taylor series about the point

$$\lambda_0^2=\frac{2ME_{\rm lab}}{\hbar^2},$$

where M is the nucleon mass and $E_{\rm lab}$ the lab energy. The choice of expansion point is due to a Born-approximation argument for the scattering of two identical nucleons which we treat using nucleon-nucleus kinematics. Keeping only the first term in this expansion allows the antisymmetrized transition amplitude to be reduced to the form encountered in the usual DWA with the pseudopotential $V(r_{12})+A(\lambda_0^2)\delta(\vec{r}_1-\vec{r}_2)$ appearing where the effective interaction $V(r_{12})$ is normally found. $A(\lambda_0^2)$ has an expansion in terms of spin and isospin operators which is equivalent in form to that given for $V(r_{12})$ in Eq. (1) with the coefficient of each term in this expansion, as a function of the lab energy, given in Table I for the case of the KK interaction.

Results obtained using this approximation have been compared with some of those previously given by two of the present authors using the properly antisymmetrized DWA for the case of a Serber interaction of Yukawa form. These give the ratio of the exchange total cross section to the direct total cross section for the $L=0, 2, 4, 6, 8$ transitions in the reaction $\mathrm{Zr}^{90}(p,p')$ at 18.8 MeV as a function of the range of the interaction.[8] For the $L=2$ transition an exact calculation of σ_E/σ_D as a function of energy has also been performed with the range of the force fixed at 1 F. It is found that the approximation improves with increasing L, where the exchange contribution is becoming more important, and also with increasing energy. Quantitatively the approximation overestimates the exchange contribution in all cases with the possible exception of the $L=8$ transition. The observed discrepancies correspond to an overestimate of the total cross sections (direct plus exchange) of not more than 30% and this limit is reached only for the case $L=0$.

Shell-model calculations using the G matrix unambiguously demonstrate the importance of taking into account long-range correlations near the Fermi surface.[1] It is these correlations that give rise to the strong pairing effects between like nucleons in nuclei. These same effects are expected to be important for the case of scattering. For closed or pseudoclosed shell nuclei, the random phase approximation (RPA) gives a good account of these correlations. In the present work we have confined ourselves to such cases, in particular, to nuclei which have equal proton-neutron numbers and to states whose RPA vectors have been shown to agree with inelastic electron scattering data.[9] This comparison demonstrates that the proton component of the transition density is approximately correct, and since neutrons and protons play equal roles in these nuclei, it is expected that this is sufficient to guarantee that both the neutron and proton components of the transition density are given correctly. Excitation of the 2^+ $T=0$ ($Q=-4.43$-MeV) and 3^- $T=0$ ($Q=-9.63$-MeV) levels of C^{12} by 28.05- and 45.5-MeV protons and of the 3^- $T=0$ ($Q=-3.73$-MeV) and 5^- $T=0$ ($Q=-4.48$-MeV) levels of Ca^{40} by 25- and 55-MeV protons are the cases actually considered in this paper. The experimental results for C^{12} at 45.5 MeV and Ca^{40} at 55 MeV have been published,[10] while the results for C^{12} at 28.05 MeV and Ca^{40} at 25 MeV are from the unpublished work of P. Locard and C. Gruhn, respectively. A vector due to Kuo was used for the 3^- level in Ca^{40},[11] while vectors due to Gillet and collaborators were used for the other cases.[12]

As a further check on the exchange-approximation comparison calculations using an even-state force of Yukawa form and 1-F range have been performed for these transitions. The range of 1 F has been selected as this has commonly been used in phenomenological analysis where antisymmetrization has been neglected. The strengths of the singlet even and triplet even components of this force are -73.5 and -119.5 MeV, respectively. These strengths have been found to reproduce, roughly, the cross sections obtained with the KK interaction in calculations which ignore antisymmetrization, and this will be referred to as the KK "equivalent" interaction.

The differential cross sections which have been obtained are compared with experiment on the left in Figs. 1 and 2. Here the dashed curves have been obtained with exchange treated approximately, while the solid curves show the results with exchange treated correctly. It is clear that no serious discrepancies are being introduced by the approximate treatment of exchange for the 1-F-range interaction. Differences between the approximate and exact results are almost negligible for the higher energies. We see that that this force produces results which are in agreement with the lower energy data but which significantly overstimate the experimental results for the higher energies.

The results shown on the right in these figures have been obtained with the KK interaction. The magnitudes of the differential cross sections are reproduced quite well by the KK force for all cases considered. No free parameters are involved in these calculations and the optical parameters which have been used were taken from standard

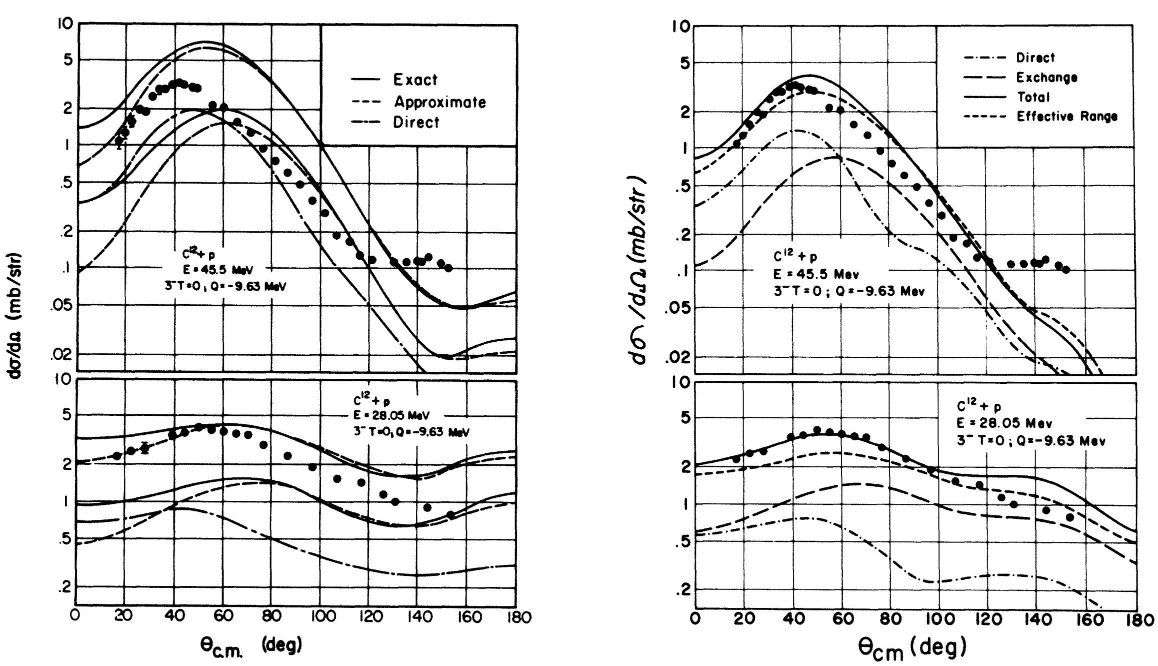

FIG. 1. Differential cross sections obtained with the 1-F-range interaction (left) and the KK and effective-range interaction (right) for the $L=3$ transition in the reaction $C^{12}(p,p')$ at 28.05 and 45.5 MeV. A decomposition of the cross sections into direct and exchange components is shown for the 1-F range and KK interaction and a comparison of approximate and exact results is given for the 1-F force. For the 1-F-range interaction the direct component which is not affected by the approximation is shown as a center line. The lower of the two sets of exact and approximate results shown is the exchange component and the upper set is the total differential cross section.

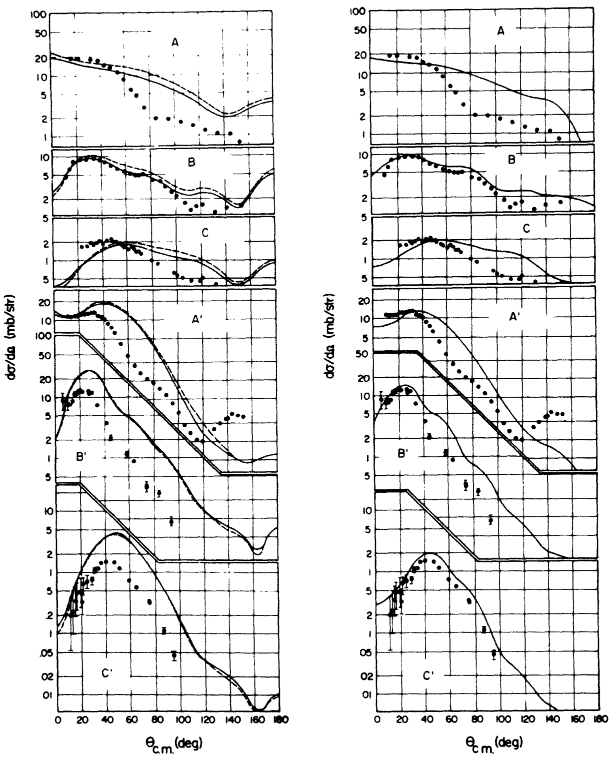

FIG. 2. Differential cross sections obtained with the 1-F–range interaction, computed approximately and exactly (left), and the KK force (right) for the excitation of the 2^+ $T=0$ ($Q=-4.43$-MeV) level of C^{12} at (A) 28.05 MeV and (A′) 45.5 MeV, of the 3^- $T=0$ ($Q=-3.73$-MeV) level of Ca^{40} at (B) 25 MeV and (B′) 55 MeV, and of the 5^- $T=0$ ($Q=-4.48$ MeV) level of Ca^{40} at (C) 25 MeV and (C′) 55 MeV.

compilations found in the literature.[13] Harmonic-oscillator wave functions have been used throughout with the constants fixed on the basis of elastic electron scattering, i.e., $\hbar\omega=10.5$ and 15 MeV for Ca^{40} and C^{12}, respectively.

The observed differences, i.e., the variation with energy, between the cross sections obtained with the KK force and those due to the 1-F–range "equivalent" can be attributed to the behavior with energy of their respective exchange components. This can be seen in Fig. 1 where the differential cross sections for the transition to the 3^- $T=0$ level in C^{12} have been decomposed into their direct and exchange components. It is to be noted that the exchange contribution for the KK force drops off faster with energy than does that for the 1-F–range force. For a Yukawa force the exchange contribution drops off with energy roughly as $(\lambda_0^2+m^2)^{-1}$, where m is the inverse range, thus using a range longer than 1 F would improve the results for this case. The KK force is a very short-range interaction but its exchange contribution is found to fall off relatively fast with energy. This can be attributed directly to the presence of the "hole" in this potential. These results are a nice illustration of the fact that the exchange contribution to the cross section places useful restrictions on the form that the effective interaction can take.

Differential cross sections have also been obtained for these transitions using an even state force of Yukawa form with a singlet-even strength of −47.6 MeV with a range of 1.17 F and a triplet-even strength of −41.5 MeV with a 1.58-F range. This interaction fits the nucleon-nucleon scattering lengths and effective ranges. Good agreement with experiment was achieved at the higher energies for all cases, but this force produced results which somewhat underestimated the lower energy data. Typical results are shown to the right in Fig. 1 and designated as effective range for the transition to the 3^- $T=0$ level in C^{12}. We have also found that this force overestimates the well depth and mean square radius of the real part of the optical potential for proton elastic scattering. Employing an RPA vector essentially equivalent to the one we have used, Agassi and Schaefer have recently obtained a good fit to the 55-MeV data for the 3^- $T=0$ level in Ca^{40} using an antisymmetrized DWA code and a Serber force of Yukawa form with $V=-40$ MeV and a range of 1.37 F.[14] This result is consistent with our findings for the similar effective-range force and with the remarks made earlier concerning increasing the range of the 1-F range KK "equivalent" force. They have found that the force *CAL*, used in the calculation of the state vectors, fails to reproduce the data for this case. We believe that Yukawa forces of this nature, which are based on low-energy phenomenology, while successful in particular applications will not reproduce experiment in general, whereas we conclude that the G-matrix interaction, which takes into account the high-energy behavior of the two-nucleon potential and gives a good account of the basic features of the bound-state problem, also gives a good account of the fundamental properties of the scattering problem such as the optical potential and the variation of inelastic cross sections with multipole and energy, the latter feature being sensitive to the shape of the interaction via the effects of exchange. It had previously been found that the impulse approximation, which also incorporates the high-energy features of the two-nucleon potential, gives a good account of the inelastic scattering cross sections discussed in this paper.[15]

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[1]T. T. S. Kuo and G. E. Brown, Nucl. Phys. 85, 40 (1966) and, A92, 481 (1967), and A114, 241 (1968); T. T. S. Kuo, Nucl. Phys. A90, 199 (1967), and A103, 71 (1967).

[2]D. Slanina and H. McManus, Nucl. Phys. A116, 271 (1968).

[3]A. Kallio and K. Kolltveit, Nucl. Phys. 53, 87 (1964).

[4]G. E. Brown, Unified Theory of Nuclear Models and Nucleon-Nucleon Forces (North-Holland Publishing Company, Amsterdam, The Netherlands, 1967), 2nd ed.

[5]N. K. Glendenning, Phys. Rev. 144, 839 (1966); G. R. Satchler, Nucl. Phys. 55, 1 (1964), and 77, 481 (1966); V. A. Madsen, Nucl. Phys. 80, 177 (1966).

[6]K. A. Amos, V. A. Madsen, and I. E. McCarthy, Nucl. Phys. A94, 103 (1967).

[7]F. G. J. Perey and D. S. Saxon, Phys. Letters 10, 107 (1964).

[8]Jay Atkinson and V. A. Madsen, Phys. Rev. Letters 21, 295 (1968).

[9]V. Gillet and M. A. Melkanoff, Phys. Rev. 133, B1190 (1964).

[10]K. Yagi et al., Phys. Letters 10, 186 (1964); E. L. Peterson et al., Nucl. Phys. A102, 145 (1967).

[11]T. T. S. Kuo, unpublished.

[12]V. Gillet and N. Vinh Mau, Nucl. Phys. 54, 321 (1964); V. Gillet, A. Green, and E. Sanderson, Nucl. Phys. 88, 321 (1966); V. Gillet, and E. Sanderson, Nucl. Phys. 91, 292 (1967).

[13]M. P. Fricke and G. R. Satchler, Phys. Rev. 139, B567 (1965); G. R. Satchler, Nucl. Phys. A92, 273 (1967).

[14]D. Agassi and R. Schaeffer, Phys. Letters 26B, 703 (1968).

[15]F. Petrovich, D. Slanina, and H. McManus, Bull. Am. Phys. Soc. 12, 12 (1967), and Michigan State University Technical Report No. MSPT-103, 1967 (unpublished).

ORIGIN OF MUON-POOR EXTENSIVE AIR SHOWERS

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Consideration is given to the relative intensities of muon-poor extensive air showers observed at mountain altitudes and at sea level. It is shown that the available results to date do not support the earlier hypothesis that primary gamma rays are responsible, and instead, a process involving charged primary cosmic rays is indicated.

Observations on extensive air showers (EAS) over a number of years at Lodz, Paris,[1-3] and Chacaltaya[4-6] have shown the existence of a distinct class of air showers containing a deficit of muons. Two alternative explanations have been put forward—that the showers are due to energetic primary gamma rays ($E_\gamma \approx 10^{14}$-10^{15} eV) or that they come from the interactions of primary charged particles.

The arguments so far[2] have favored the primary-gamma-ray hypothesis but we present in this note considerations which suggest the reverse,