*Work supported in part by the U. S. Atomic Energy Commission.

[1]Both experimental and theoretical developments are reviewed in J. Wilks, The Properties of Liquid and Solid Helium (Clarendon Press, Oxford, England, 1967), Chaps. 12 and 13; or E. L. Andronikashvili and Yu. G. Mamaladze, Rev. Mod. Phys. 38, 567 (1966).

[2]G. B. Hess, Phys. Rev. 161, 189 (1967).

[3]G. B. Hess and W. M. Fairbank, Phys. Rev. Letters 19, 216 (1967).

[4]G. Careri, W. D. McCormick, and F. Scaramuzzi, Phys. Letters 1, 61 (1962).

[5]R. J. Donnelly, Experimental Superfluidity (University of Chicago Press, Chicago, Ill., 1967), Chap. 6.

[6]R. L. Douglass, Phys. Rev. 141, 192 (1966).

[7]W. P. Pratt, Jr., and W. Zimmermann, Jr., Phys. Rev. 177, 412 (1969).

[8]L. Bruschi, B. Maraviglia, and F. E. Moss, Phys. Rev. Letters 17, 682 (1966).

[9]This is only an estimate. We have not made precise gain measurements.

NONLINEAR DECAY INSTABILITY AND PARAMETRIC AMPLIFICATION OF CYCLOTRON-HARMONIC PLASMA WAVES*

M. Porkolab and R. P. H. Chang
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540
(Received 4 March 1969)

Experimental observation of the resonant mode-mode coupling type of nonlinear decay instability and parametric amplification of cyclotron-harmonic waves is reported. The instability is due to the scattering of finite amplitude waves.

We report experimental observation of (1) nonlinear decay instability of cyclotron-harmonic plasma waves (CHW-or Bernstein modes[1]) when an rf signal (pump) above a threshold level is applied to a probe immersed in a plasma in a magnetic field and (2) parametric amplification of an additional, externally applied rf signal when the pump signal is below threshold level. By nonlinear decay instability we mean the scattering of a finite amplitude wave into other modes which grow exponentially as the initial wave decays. We recall that CHW are quasielectrostatic waves which propagate perpendicularly to a magnetic field,[1] with frequencies in the vicinity of the fundamental and harmonics of the cyclotron frequency. Detailed measurements of wave propagation suggest that the instability we observe is due to decay-type resonant mode-mode coupling of high-frequency traveling CHW, and thus it is different from experiments where the simultaneous emission of both high-frequency and low-frequency signals was observed when the plasma was subjected to intense microwave radiation.[2,3] Although parametric excitation of high-frequency signals has been reported recently when the plasma was subjected to microwave radiation, in these experiments the modes responsible for the interaction were believed to be, although not clearly identified, standing waves in regions with strong density gradients (Buchsbaum-Hasegawa modes).[4] We note that because of possible application to amplifiers, the traveling-wave type of parametric phenomenon is of considerable interest.[5]

The interaction we consider is the decay of a single CHW with frequency ω_0 into two (or more) other CHW with frequencies ω_1, ω_2 by the mechanism of resonant mode-mode coupling, namely,

$$\omega_0(\vec{k}_0) \rightarrow \omega_1(\vec{k}_1) + \omega_2(\vec{k}_2), \qquad (1)$$

where

$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2.$$

It has been recently predicted by theory that such a decay process of CHW can be unstable, i.e., the decay modes ω_1, ω_2 can grow exponentially.[6] The energy for the growth is supplied by the decaying pump wave. This process is essentially the inverse of the resonant scattering of CHW observed recently by the present authors[7] (although no instability in the present sense was involved in the latter case).

The experiments were carried out in a hot-cathode helium discharge in a magnetic field (PIG configuration) which was better than 1% uniform. The plasma parameters are as follows: background pressure $\sim 2\times10^{-3}$ Torr, electron density $n_e \sim 4\times10^9 - 1\times10^{10}$ cm^{-3}, electron temperature $T_e \sim 4.5$ eV, and collision frequency of electrons with electrons or with neutrals $\nu_e/\omega \lesssim 10^{-3}$. Three coaxial probes, which were movable radially, were employed for injecting or detecting signals. The probes were provided with 4.0-cm-long T-shaped antennas with the antenna bars aligned parallel to the magnetic field. It is well known that such probes can excite CHW.[7]

By applying a strong rf signal to one of the probes that was positioned near the center of the plasma column, we have detected signals at frequencies below the pump frequency on a second, radially displaced probe. The emitted signals always occurred in pairs such that the sum frequency of the decay modes was very nearly equal to the pump frequency. Depending on magnetic field, background pressure, and discharge current (density), we observed one, two, or three pairs of subfrequencies, with intensities 40-90 dB below pump signal levels. Furthermore, in some cases signals with frequencies equal to the pump frequency plus one of the subfrequencies were also observed. Such high-frequency emission can be expected to be present due to the passive coupling, or mixing, of the pump signal and one of the decay signals.[5,7] In particular, we have observed decay of waves for the pump frequency being in the second, third, or fourth passband (a passband is a range of frequencies such that $n \lesssim \omega/\omega_c \lesssim n+1$, n being a positve integer). Usually one of the decay modes had a frequency near an exact cyclotron harmonic and was thus strongly damped. Some typical frequencies observed are as follows: $(\omega_0/\omega_c, \omega_1/\omega_c, \omega_2/\omega_c) = (2.76, 1.02, 1.74)$; $(3.50, 1.37, 2.13)$; $(4.76, 2.13, 2.65)$, etc. The decay of the pump signal has been observed for a considerable range of magnetic fields as the latter was being varied, in some cases in the range $n+0.5 \lesssim \omega_0/\omega_c \lesssim n+0.9$. We note that usually the external pump power was varied between 0 and 20 W. However, measurements indicated that connection and probe losses amounted to approximately 15 dB, so that the actual power delivered to the probe tip was estimated to be below 0.5 W in most cases.

In order to study the decay processes in detail, we have used interferometer techniques to detect the waves. We recall that in such a receiving system a reference signal is taken from the transmitter, which is then mixed in a balanced mixer with the signal obtained from the receiving probe, and the detected signal is then displayed on an XY recorder as a function of receiving probe position. The important feature of this system is that the phase coherence (in time) between the reference and the received signals is required, and assured, during proper functioning. This property of the interferometer was used to show that the decay waves were due to decay of the pump signal.

In Fig. 1 we show a typical set of interferometer traces of the waves observed during decay instability, with $\omega_0/\omega_c = 3.72$, $\omega_1/\omega_c = 2.66$, and $\omega_2/\omega_c = 1.06$. In Fig. 1(a) we exhibit results of direct propagation at $f = 532$ MHz, $f_c = 200$ MHz, in order to determine the small signal wavelength associated with the $\omega_1/\omega_c = 532/200 = 2.66$ wave. We see that within experimental error, the wavelength obtained by direct propagation measurement agrees with the wavelength of the decay wave at the same frequency, magnetic field, and density as obtained in Fig. 1(b). Comparing this wavelength with the linear dispersion relation,[1] we obtain for the plasma frequency $\omega_p \simeq 3\omega_c$, or $n_e \simeq 4\times10^9$ cm^{-3}, which is within a factor of 2 of the density obtained by probe measurements. The second decay wave $f_2 = 212$ MHz, or $\omega_2/\omega_c = 1.06$, is severely damped since the frequency is so close to the cyclotron frequency. Such a damped wave is predicted by theory if collisions are taken into account.[8] Because of

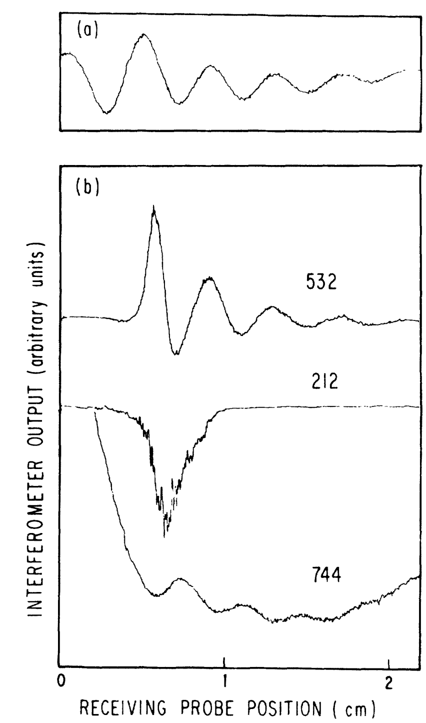

FIG. 1. (a) Direct, small-signal propagation measurements at $f = 532$ MHz; $f_c = 200$ MHz, $k = 15.6$ cm^{-1}. (b) Interferometer traces of the pump wave, $f = 744$ MHz, and the decay waves, $f_1 = 212$ MHz and $f_2 = 532$ MHz, at $f_c = 200$ MHz. $k(744) = 17.0$ cm^{-1}, $k(532) = 15.6$ cm^{-1}. The pump probe is at the origin. The receiver gain is varied from trace to trace for convenient display.

the large pump power, note the strong initial decay of the pump signal before wave propagation starts. Note that both decay waves appear to originate approximately 0.4 cm away from the pump transmitter probe, at about the same position where the pump waves begin to propagate. Thus, probe effects can be clearly excluded to be responsible for the decay.

Comparing the wave numbers of the pump and decay waves, we see that the interference condition given by Eq. (1) is satisfied only if we assume that the real part of the wave number for the $\omega_2/\omega_c = 1.06$ signal is nearly zero. In particular, $Rk_0 \simeq 1.21$, $Rk_1 \simeq 1.12$, and $R|k_0-k_1| \simeq 0.09$ (where R is the Larmor radius). However, since in the present case the second scattered wave is so heavily damped, no clear conclusion can be drawn about wave number conservation.

We note that the reference signals for the interferometer traces of ω_1 and ω_2 were obtained by using a third probe which was positioned approximately 0.5-1.0 cm away from the pump probe (on the opposite side radially from the receiving probe). A signal with frequency $\omega_2 = \omega_0 - \omega_1$ was then selected from this probe, mixed externally in a balanced mixer with a small fraction of the pump signal ω_0, and amplified in a tuned amplifier at the difference frequency, namely, ω_1. This signal was then used as reference for the ω_1 signal obtained from the receiving probe after proper filtering. This process was afterward repeated by exchanging ω_1 and ω_2. Furthermore, we note that the high-frequency background thermal emission from the plasma was negligibly small, typically 40-50 dB below the levels of the decay signals. Thus, by these arguments wave mixing of the pump signal with the background emission can be excluded as being the responsible mechanism for the generation of the decay waves. Finally, we remark that no noticeable change in the low-frequency spectrum (ion modes) was observed when the pump was turned on.

In Fig. 2 we exhibit a plot of the relative signal powers as a function of pump power, and a frequency spectrum up to 300 MHz. Note that the instability threshold is near 7 W of pump power (measured at the transmitter). Similar data were obtained by measuring the amplitudes of the decay waves from interferometer traces, showing that the increase in the signal levels measured by the receiver was associated with increased wave amplitudes.

In a second set of experiments we obtained data which showed that traveling-wave parametric amplification of CHW could also be achieved under the foregoing conditions.[5] In these experiments the pump power ($f = 744$ MHz) was reduced below threshold level so that no emission above thermal noise level was observed. A signal with frequency $f = 532$ MHz was injected by the third probe, which was positioned about 1.0 cm radially from the pump probe, on the side 180° away from the receiving probe. As the pump power was increased, we observed increasing signal levels at $f = 532$ MHz, as shown in Fig. 3. In this figure we note that as the pump power was increased from 1 to 6 W, the signal wave amplitude increased by an order of magnitude at a given receiving probe position, namely, at 0.5 cm away from the pump probe (we remark that in the present case, due to smaller pump powers, the pump waves began to propagate within a fraction of a wavelength from the exciting probe). The background emission was at least 50 dB below the signal amplitude. Due to cylindrical geometry, the pump and signal waves decreased in amplitude as a function of radial distance. Thus, the spatial amplification factor has not been determined. By using plane grids or a cylindrical grid system it may be possible to determine the rate of such growth, so as to determine whether a parametric amplifier using CHW is practical.

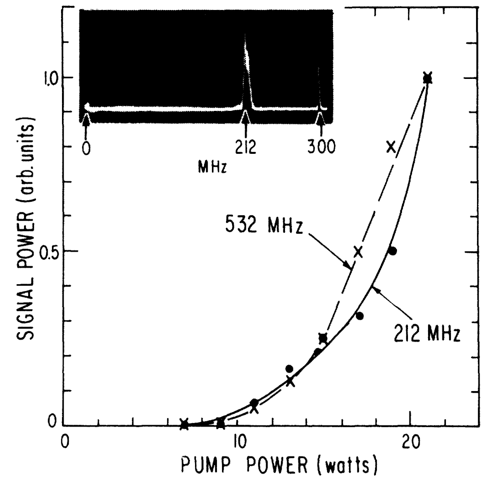

FIG. 2. Relative signal power versus pump power (the latter measured at the transmitter). Each signal power is normalized to that at $P_{pump} = 21$ W. The inset shows the frequency spectrum of the $f = 212$ MHz signal. The arrows at $f = 0$ and 300 MHz are external markers. The baseline width is that of the spectrum analyzer.

In summary, we have presented experimental

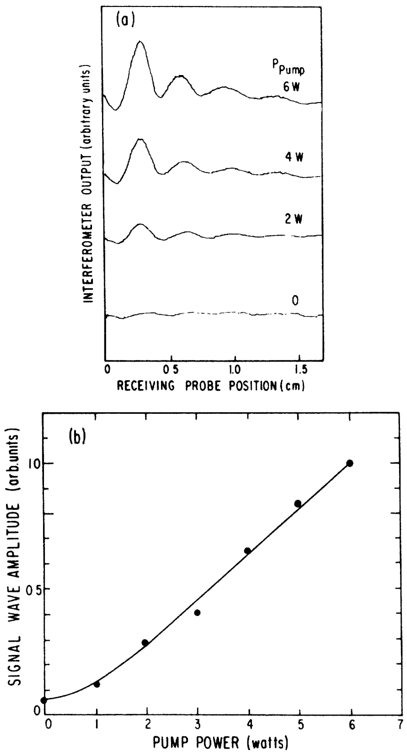

FIG. 3. Parametric amplification of CHW. Pump power below threshold. (a) Interferometer traces of "signal" waves for different pump powers. $f_s = 532$ MHz, $f_0 = 744$ MHz, $f_c = 200$ MHz. (b) "Signal" wave amplitude versus pump power, as obtained from traces similar to (a) at 0.5 cm from the pump probe. $f = 532$ MHz.

results which show the existence of the nonlinear decay instability of CHW when the amplitude of such a wave becomes sufficiently large. It is believed that the instability is due to decay-type resonant mode-mode coupling. By reducing the pump level below threshold and injecting an additional small-amplitude signal with the same frequency as the propagating decay wave, we observed parametric amplification of this signal. Possible application of this phenomenon to practical amplifiers depends on the results of further investigation using more favorable probe geometries. A detailed study of the present phenomena is in progress.

We thank Dr. R. A. Ellis, Jr., for his assistance in the early phases of this work and for valuable discussions, and Professor T. H. Stix for his interest and encouragement.

---

*Research sponsored by the U. S. Atomic Energy Commission, Contract No. AT (30-1)-1238.

[1]I. B. Bernstein, Phys. Rev. 109, 10 (1958).

[2]R. A. Stern and N. Tzoar, Phys. Rev. Letters 17, 903 (1966).

[3]S. Hiroe and H. Ikegami, Phys. Rev. Letters 19, 1414 (1967).

[4]R. A. Stern, Bull. Am. Phys. Soc. 13, 1563 (1968), and private communication.

[5]W. H. Louisell, Coupled Mode and Parametric Electronics (John Wiley & Sons, Inc., New York, 1960).

[6]K. J. Harker and F. W. Crawford, Bull. Am. Phys. Soc. 13, 887 (1968); N. Tzoar, to be published.

[7]R. A. Ellis, Jr., and M. Porkolab, Phys. Rev. Letters 21, 529 (1968); M. Porkolab, Bull. Am. Phys. Soc. 13, 1514 (1968); M. Porkolab and R. P. H. Chang, Princeton Plasma Physics Laboratory Report No. MATT-663, 1969 (unpublished).

[8]J. A. Tataronis, thesis, Stanford University, 1967 (unpublished).

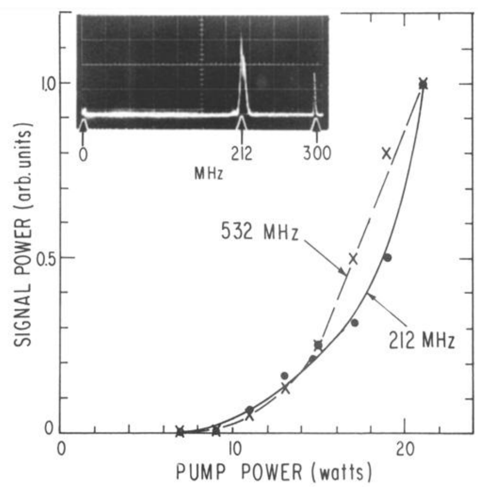

FIG. 2. Relative signal power versus pump power (the latter measured at the transmitter). Each signal power is normalized to that at $P_{\text{pump}} = 21$ W. The inset shows the frequency spectrum of the $f = 212$ MHz signal. The arrows at $f = 0$ and 300 MHz are external markers. The baseline width is that of the spectrum analyzer.