incorporates detailed information about the two-body interaction.

The authors are grateful to the Lawrence Radiation Laboratory for hospitality and for use of computer facilities. They are also indebted to Dieter Tuerpe for his assistance.

---

*Much of this work was performed at the Lawrence Radiation Laboratory, Livermore, Calif.

†Work supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-2098.

[1]B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Skrifter 1, No. 8 (1959).

[2]S. G. Nilsson, in Nucleonic Structure of Equilibrium and Fission Deformations, Proceedings of the International School of Physics "Enrico Fermi," Course XL, 1968, edited by M. Jean (Academic Press, Inc., New York, 1968).

[3]V. M. Strutinsky, Nucl. Phys. A95, 420 (1967).

[4]A. Lande, Phys. Rev. Letters 18, 496 (1967).

[5]A. K. Kerman, Ann. Phys. (N.Y.) 12, 300 (1961).

[6]L. Wilets, Theories of Nuclear Fission (Oxford University Press, London, England, 1964), p. 55, and in Proceedings of the Summer Study Group on the Physics of the Emperor Tandem Van de Graaff Region (Brookhaven National Laboratory, Upton, N.Y., 1965), p. 929.

[7]F. Tabakin Ann. Phys. (N.Y.) 30, 51 (1964).

[8]W. H. Bassichis, B. A. Pohl, and A. K. Kerman, Nucl. Phys. A112, 360 (1968).

[9]C. F. M. Baranger and K. Kumar, Nucl. Phys. A110, 490 (1968).

A_1-NUCLEON CROSS SECTION FROM COHERENT PRODUCTION IN NUCLEI

Alfred S. Goldhaber
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790

and

Charles J. Joachain
Service de Physique Théorique et Mathématique,
Université Libre de Bruxelles, Faculté des Sciences, Bruxelles, Belgium

and

H. J. Lubatti
Department of Physics and Laboratory for Nuclear Science,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

J. J. Veillet
Faculté des Sciences, Ecole Normale Supérieure, Laboratoire de l'Accélérateur Linéaire, 91 Orsay, France

(Received 26 December 1968)

Comparison of A_1 coherent production in Freon with production in H_2 implies a total A_1-nucleon cross section $\sigma(A_1N) \leq (0.5^{+0.3}_{-0.2})\sigma(\pi N)$ with a further theoretical uncertainty of about 10 %. The result appears to preclude the possibility, suggested by the Deck model, that the ρ and π in the A_1 are produced close together without appreciable interaction between them.

Controversy surrounds the "A_1" bump[1] in the $(\rho\pi)$ mass spectrum observed in the reaction

$$\pi + N \to (\rho\pi) + N. \tag{1}$$

It is difficult to establish that this bump is a particle, or resonance, for the following reasons:

(1) With the usually accepted quantum numbers $J^{PG} = 1^{+-}$, $\rho\pi$ is the only available decay mode for the A_1. Thus a resonance cannot be confirmed by observing alternative decay modes.[2]

(2) A_1 has not been observed with convincing statistics in other reactions (e.g., $\pi + p \to A_1 + \pi + N$)[3].

(3) There is a theoretical model, the Deck or "diffraction dissociation" model, which predicts a low-mass enhancement in Reaction (1). In this model the outgoing ρ and π may be thought of as close to one another in position and velocity because of the production mechanism and not because of any strong interaction between them.[4]

If we are given two alternatives: (i) The A_1 is a resonance, or (ii) it is a kinematic enhancement with no strong $\rho\pi$ interaction, then we might settle the question by a measurement of the A_1-nucleon total cross section $\sigma(A_1N)$. We shall see below that (ii) implies $\sigma(A_1N) \geq 1.7\sigma(\pi N)$. Thus a $\sigma(A_1N)$ substantially smaller than this limit would impose alternative (i). The only way to measure $\sigma(A_1N)$ is to place a target nucleon within a few fermis of the A_1 production point. In practice this means to look at A_1 production in an atomic nucleus; for high incident π energies ($\gtrsim 5$ GeV/c) such processes may be described simply by the "high-energy" model.[5] We have used this model to extract $\sigma(A_1N)$ from available data on coherent A_1 production by 16-GeV/c pions.

The amplitude for A_1 production on a single nucleon, without charge exchange, is

$$g(\vec{q}) = a + \vec{b}\cdot\vec{\sigma} + c\tau_3 + \vec{d}\cdot\vec{\sigma}\tau_3, \tag{2}$$

where $\vec{\sigma}$ and $\vec{\tau}$ are the nucleon spin and isospin, and dependence on A_1 spin is left implicit. Averaging the differential cross section $d\sigma/dt_{\text{free}} = |g|^2$ over nucleon spin and charge gives

$$(d\sigma/dt)_{\text{free}} = |a|^2 + |\vec{b}|^2 + |c|^2 + |\vec{d}|^2 \geq |a|^2, \tag{3}$$

where $|a|^2 \equiv d\sigma^0/dt$ is the isospin-0 exchange, spin-nonflip production cross section. In the high-energy model the amplitude for coherent production of an A_1 on an $I=0$ nucleus of mass number A is given to order $1/A$ by

$$G(\vec{q}) = a\int d\vec{r}\,\rho(r)\exp[i\delta_i(\vec{r})]\exp[i\delta_f(\vec{r})]e^{i\vec{q}\cdot\vec{r}} \equiv aN(\vec{q}) \tag{4}$$

where $\rho(r)$ is the nucleon density,[6] and

$$\delta_i(\vec{r}) = \int_{-\infty}^{z} dz' f_\pi \lambda_\pi \rho(x,y,z'),$$
$$\delta_f(\vec{r}) = \int_{z}^{\infty} dz' f_A \lambda_A \rho(x,y,z'), \tag{5}$$

with f_π (f_A) the isospin-averaged πN (A_1N) forward scattering amplitude,[7] λ_π (λ_A) the de Broglie wavelength of the π (A_1), and $\vec{q}$ the momentum transfer from π to A_1. The above formula neglects correlations between nucleons and effects arising from the finite range of the meson-nucleon interaction, as well as other intermediate states of the fast-meson system. The differential cross section for production is then

$$\left(\frac{d\sigma}{dt}\right)_{\text{coh}}(\vec{q}) = \frac{d\sigma^0}{dt}|N(\vec{q})|^2 \leq \left(\frac{d\sigma}{dt}\right)_{\text{free}}|N(\vec{q})|^2, \tag{6}$$

and $N(\vec{q})$ may be thought of as a nuclear form factor reduced by attenuation of the incoming and outgoing wave. The quantity which is well determined by experiment is the total coherent cross section $\sigma_{\text{coh}} = \int dt (d\sigma/dt)_{\text{coh}}$. For the nuclei constituting Freon, we have found that σ_{coh}, as calculated from Eq. (6), is insensitive to variations in the shape of the nucleus, so that uncertainties in that shape resulting from uncertainties in electromagnetic form factors and smearing effects due to finite nucleon size do not alter our conclusions. This insensitivity may be understood qualitatively by noting that two compensating effects are at work. For fixed mass number A, the bigger the nucleus, the smaller are the absorption effects, and therefore the bigger is $|N(0)|^2$. On the other hand, a bigger nucleus means a more rapid fall of $|N(\vec{q})|^2$ with q. Thus σ_{coh} changes much less than $(d\sigma/dt)_{\text{coh}}(0°)$ as the assumed nuclear shape varies.

Possible attractive and repulsive correlation effects are the major sources of uncertainty in the theoretical predictions, and that uncertainty is about 10%. Thus the high-energy model gives a theoretical upper limit on the ratio

$$\eta \equiv \left.\frac{\sigma_{\text{coh}}}{(d\sigma/dt)_{\text{free}}}\right|_{\vec{q}_\perp = 0} = \frac{\sigma_{\text{coh}}}{\Lambda\sigma_{\text{free}}}, \tag{7}$$

where $\vec{q}_\perp$ is the momentum transfer perpendicular to the beam, and we have assumed $(d\sigma/dt)_{\text{free}} = \Lambda\sigma_{\text{free}}\exp(-\Lambda q_\perp^2)$.[8] Since all other parameters are well-enough known, measuring η is tantamount to setting an upper limit on $\sigma(A_1N)$.

In order to understand the implications of alternative (ii) for $\sigma(A_1N)$, we consider a simple model of the A_1 interaction with nucleons after formation. First, we suppose the ρ and π are produced at the same point in coordinate space. They will not separate appreciably on passing through the nucleus, since their maximum relative transverse velocity in the A_1 rest frame is about c; so relativistic time dilation implies that they separate by less than 1 F for every 16 F traveled by the A_1. A lower limit on the attenuation of the ($\rho\pi$) system can be obtained by assuming that the ρ and π remain exactly coincident during passage through the nucleus and that they have the same elastic scattering amplitude[9] on a nucleon (taken as an imaginary Gaussian function of $q_\perp$). At a given impact parameter, a ρ or a π wave passing a nucleon is attenuated by a factor $e^{2i\delta(b)}$. We assume that the coincident but noninteracting $\rho + \pi$ will be attenuated as $e^{4i\delta(b)}$.[10] The result is $\sigma(A_1N) = 1.72\sigma(\pi N)$. It is less than $2\sigma(\pi N)$ because of the partial opacity of the π for a nucleon passing through it.

One might argue that the π and ρ will be characteristically produced with a separation of about 1 F, since they form a system with orbital angular momentum zero and relative momentum $\approx 1.25\ \mathrm{F}^{-1}$.[11] Thus, sometimes the π (ρ) would be produced inside the nucleus, and attenuated, while the π (ρ) would be just on the edge and suffer negligible attenuation. For such a production configuration the apparent $\sigma(A_1N)$ would be simply $\sigma(\pi N)$. However, in order to obtain the full amplitude one must average over all configurations.

To do this, we first compute the total cross section on a nucleon for such a spread-out A_1. For a Gaussian $\rho\pi$ wave function, our previous result is modified to read

$$\sigma(A_1N)=\sigma(\pi N)[2-0.28/(1+2a^2\lambda^{-1})]. \tag{8}$$

Here, λ is the slope of the elastic $\pi\rho$ differential cross section, $\lambda\approx 0.36\ \mathrm{F}^2$,[12] and $3a^2$ is the mean square radius of the $\rho\pi$ system. In uniform nuclear matter, the above number, bigger than $1.7\sigma(\pi N)$, would give the attenuation of the A_1 beam. In a finite nucleus, however, the A_1 size gives a blurred edge to the effective nucleon density distribution in Eqs. (4) and (5).[13] Offhand, one might suppose that this would lead to enhanced A_1 production, but the total cross section σ_{coh} is nearly independent of nuclear surface diffuseness, as mentioned earlier. Hence, if the phase shift of an A_1 on passing a nucleon is obtained simply by adding the phase shifts due to ρ and π, then analysis of coherent nuclear production of A_1 using the high-energy model must yield $\sigma(A_1N)\gtrsim 1.7\sigma(\pi N)$.

The experimental value of η for $\rho^0\pi^-$ production at 16 GeV/c is obtained by comparing production in a Freon bubble chamber[14] with a weighted average of A_1^+ and A_1^- production in hydrogen.[15] Defining the A_1 enhancement to be all events observed in the mass interval 0.96-1.20 GeV,[16] we obtain[17] $\eta=1.32\pm0.25\ \mathrm{GeV}^2$ and deduce the upper limit[18]

$$\sigma(A_1N)\leq(0.5_{-0.2}^{+0.3})\sigma(\pi N). \tag{9}$$

We conclude that the A_1 is not merely a ρ and a π close together. Whether it is a conventional resonance or a type of phenomenon yet unclassified, we leave to the reader.

We have profited from discussions with many colleagues, on both theory and experiment. Special thanks are due to H. H. Bingham, F. R. Huson, A. Lagarrigue, I. Pless, and N. P. Samios for their interest and encouragement.

[1]Particle Data Group, Lawrence Radiation Laboratory Report No. UCRL-8030, 1968 (unpublished); A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, P. Söding, L. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. 39, 1 (1967).

[2]The modes 3π or 5π without ρ are possible in principle, but strongly inhibited by centrifugal barriers and phase space.

[3]G. Ascoli, H. B. Crawley, U. Kruse, D. W. Mortara, E. Schafer, A. Shapiro, and B. Terreault, Phys. Rev. Letters 21, 113 (1968).

[4]R. T. Deck, Phys. Rev. Letters 13, 169 (1964); E. L. Berger, Lawrence Radiation Laboratory Report No. UCRL-17825 (unpublished); G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968). The last authors argue that there need not be a contradiction between the Deck picture and the notion of a resonant A_1.

[5]R. J. Glauber, Lectures in Theoretical Physics (Interscience Publishers, Inc., New York, 1959), Vol. I, p. 315; L. Stodolsky, Phys. Rev. 144, 1145 (1966); A. S. Goldhaber and C. J. Joachain, Phys. Rev. 171, 1566 (1968).

[6]In comparing with data on Freon (C_2F_2Cl) [Orsay-Saclay-Milan-Berkeley Collaboration, Nuovo Cimento 46A, 737 (1966), and unpublished (OSMB, π^-)], we have used a density distribution $\rho=\rho_0/\{1+\exp[(r-R)/a]\}$ with $R=2.3$ F for C^{12}, 2.8 F for F^{19}, 3.3 F for Cl^{35}, and $a\approx 0.5$ F.

[7]We take f_π as pure imaginary, corresponding to $\sigma(\pi N)=25$ mb. See K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters 19, 193, 622(E), 330 (1967), and earlier references therein. The neglect of the measured real parts of f_π leads to insignificant error ~1%.

[8]In fact, the data of the following references are consistent with a $\vec{q}_\perp$ dependence of $d\sigma/dt_{\mathrm{free}}$ identical to that for elastic πN scattering: Orsay-Saclay-Milan-Berkeley Collaboration, Ref. 6; J. Ballam, A. D. Brody, G. B. Chadwick, D. Fries, Z. G. T. Guiragossián, W. B. Johnson, R. R. Larsen, D. W. G. S. Leith, F. Martin, M. Perl, E. Pickup, and T. H. Tan, Phys. Rev. Letters 21, 934 (1968) (SLAC, π^-); W. B. Johnson, private communication (SLAC, π^+); Aachen-Berlin-Bonn-CERN-Heidelberg Collaboration, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (unpublished); D. R. O. Morrison, private communication (ABBCH, π^-).

[9]Indications from coherent nuclear photoproduction of ρ^0 support this assumption and are inconsistent with $\sigma(\rho N)\ll\sigma(\pi N)$. M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966); S. D. Drell and J. S. Trefil, Phys. Rev. Letters 16, 552, 832(E) (1966); B. Margolis, Phys. Letters 26B, 524 (1968); S. C. C. Ting, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 60. See Foley, Jones, Lindenbaum, Love, Ozaki, Platner, Quarles, and Willen, Ref. 7, for πp parameters.

[10]This is a natural definition of "noninteraction." It means that the phase shift of a proton beam on passing through a $\rho\pi$ pair is independent of the separation of ρ and π in the beam direction.

[11]Alternatively, in a Deck model, ρ and π would be separated by $\sim 1/m_\pi = 1.4$ F, a number similar to that in the text.

[12]For elastic πp data, see Foley, Jones, Lindenbaum, Love, Ozaki, Platner, Quarles, and Willen, Ref. 7.

[13]See Glauber, or Goldhaber and Joachain, Ref. 5.

[14]Orsay-Saclay-Milan-Berkeley Collaboration, Ref. 8.

[15]Ballam, Brody, Chadwick, Fries, Guiragossián, Johnson, Larsen, Leith, Martin, Perl, Pickup, and Tan; Johnson; Aachen-Berlin-Bonn-CERN-Heidelberg Collaboration; and Morrison, Ref. 8.

[16]In comparing the data, a ρ cut was imposed [at least one $m(\pi^+\pi^-)$ in the interval 0.66-0.90 BeV]. Also N^*'s [$m(\pi p)$ in 1.0-1.4 BeV] were removed from the H_2 data. As a check on sensitivity of $\eta_{\rm exp}$ to choice of mass cut, we note that the fraction φ of events in this interval that are contained also in the smaller interval 1.00-1.16 GeV is consistent for H_2 and Freon (Ref. 8): $\varphi(\text{SLAC}, \pi^+) = 0.72$, $\varphi(\text{SLAC}, \pi^-) = 0.77$, $\varphi(\text{ABBCH}, \pi^-) = 0.74$, and $\varphi(\text{OSMB}, \pi^-) = 0.71$, all with statistical errors of $\sim 10\%$.

[17]The values used were $\sigma_{\rm coh}(\text{Freon}) = 1.58 \pm 0.19$ mb, $\sigma_{\rm free}(H_2) = 0.133 \pm 0.011$ mb, and $\Lambda = 9.0 \pm 0.6$ $(\text{GeV}/c)^{-2}$. The last number was obtained by combining the fitted SLAC π^- slope (9.1 ± 0.7, Ref. 5) with the fitted slope for incoherent A_1 production in Freon (8.8 ± 0.9).

[18]For the "3-standard-deviation point" $\eta = 0.70$, one has $\sigma(A_1N) \le 1.4\sigma(\pi N)$; the 4-standard-deviation point is $\eta = 0.46$, $\sigma(A_1N) \le 2\sigma(\pi N)$. This should be compared with the lower limit $\sigma(A_1N) \le 1.7\sigma(\pi N)$ if A_1 is a noninteracting $\rho\pi$, keeping in mind an estimated additional theoretical uncertainty of 10% in $\sigma(A_1N)$. In guessing the effect of systematic errors on $\eta_{\rm exp}$, one should remember that η is a ratio and not a small difference between large numbers. Our result for $\sigma(A_1N)$ supersedes that of H. H. Bingham, W. B. Fretter, and K. Moffeit, Bull. Am. Phys. Soc. 13, 1641 (1968).