RADIATION-INDUCED DECAY INSTABILITY OF BERNSTEIN MODES

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Intense excitation of Bernstein modes with $\vec{k}\perp\vec{B}$ by means of nonlinear coupling to a higher frequency mode at the sum frequency has been observed.

Microinstabilities can arise in plasmas with sufficiently anisotropic velocity distribution functions.[1] Recent reports indicate[2] that stimulating radiation at frequencies close to the electron cyclotron frequency ω_c can increase the anisotropy and excite instabilities of higher frequency electrostatic waves near $n\omega_c$. A different mechanism whereby radiation causes instabilities is provided by nonlinear decay mode coupling; this is effective even in plasmas with stable distribution functions.[3] This communication describes the observation and the principal features of an intense instability in which decay mode coupling is the dominant process. The instability is fed by the "decay" of a single, radiation-driven electrostatic mode of frequency much above ω_c into several lower frequency modes.

The dispersion relation for Bernstein modes with $\vec{k}\perp\vec{B}$ consists of passbands in the frequency ranges $(n+1)\omega_c > \omega > n\omega_c$, where n is an integer.[4] The case considered here is the nonlinear excitation of modes at ω in the lowest frequency band $n=1$ $(2\omega_c \gtrsim \omega > \omega_c)$ by means of a "pump" mode at approximately twice their frequency, $\omega_0 \sim 2\omega$, lying in the $n=3$ band $(4\omega_c > \omega_0 > 3\omega_c)$. By choosing a pump frequency much higher than ω_c $(\omega_0 \sim 4\omega_c)$, particle-wave interactions were reduced, and mode-coupling processes were allowed full scope in the generation of instabilities.

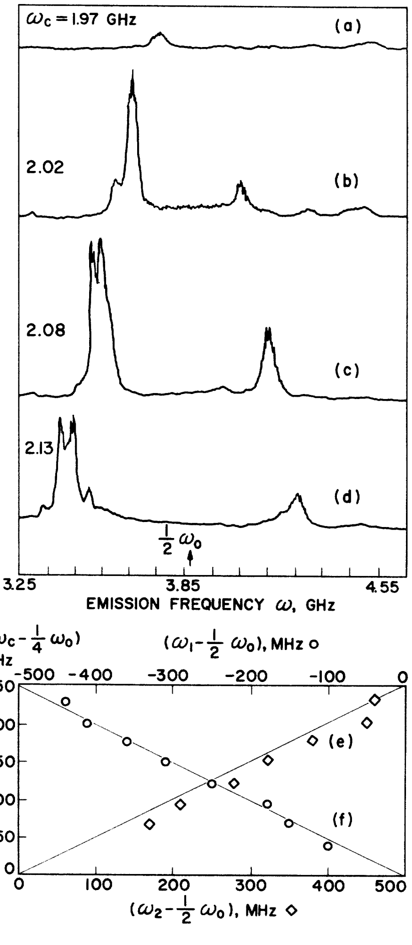

FIG. 1. (a)-(d) Spectrum of stimulated emission near half the pump frequency. Pump intensity and electron density are slightly above threshold. Plot shows shift of stimulated emission frequencies with changes in cyclotron frequency. Ar discharge, $\omega_0/2\pi = 7.72$ GHz. (e), (f) Dependence of emission band-center frequencies on electron cyclotron frequency.

In the experiment, microwave signals of frequency ω_0 were beamed at a plasma column aligned coaxially with the static magnetic field $\vec{B}$ of a pair of large Helmholtz coils.[5] The electric field and propagation vectors $\vec{E}, \vec{k}$ of the pump and stimulated radiation were normal to the column axis and to the static field $\vec{B}$. The electron density of the plasma column could be increased so that the upper hybrid frequency $\omega_{uh} = (\bar{\omega}_p^2 + \omega_c^2)^{1/2} \geq \omega_0$ (where $\bar{\omega}_p$ is the average plasma frequency) even for $\omega_c < \omega_0$. Under these conditions, pump signals propagated in the extraordinary mode in the plasma and could linearly excite[6] Bernstein modes at ω_0. The nonlinear coupling between pump and lower frequency modes manifested itself as stimulated radiation from the plasma at microwave frequencies below ω_0, occurring above a sharp threshold (discussed below). A typical spectrum of the stimulated emission in the vicinity of $\frac{1}{2}\omega_0$ is shown in Fig. 1. Each of the traces (a)-(d) corresponds to a different, increasing value of ω_c, but to the same pump frequency and slightly above-threshold intensity. At $\omega_c \sim \frac{1}{4}\omega_0$, emission from the plasma occurs weakly and in a single band near $\frac{1}{2}\omega_0$. As the magnetic field is increased so that $\omega_c > \frac{1}{4}\omega_0$, the pump is caused to drive successively the Bernstein modes in the band $n=3$; the stimulated emission spectrum now changes progressively into two intense bands splitting off to either side of $\frac{1}{2}\omega_0$, as shown in traces (a)-(d).

The dependence of the band center frequencies $\omega_1 < \frac{1}{2}\omega_0$ and $\omega_2 > \frac{1}{2}\omega_0$ on ω_c is plotted in Figs. 1(f) and 1(e). We find that ω_1 decreases, and ω_2 increases, approximately twice as fast as ω_c. Repeating these measurements at several values of the pump frequency ω_0, it is found that the bands occur at the frequencies $\omega_1 \cong \omega_0 - 2\omega_c$ and $\omega_2 \approx 2\omega_c$, respectively. Since $\omega_1 < 4\omega_c$, it follows that $\omega_1 < 2\omega_c$ represents radiation from Bernstein modes in the $n=1$ band, which propagate in the core of the plasma column (where $\omega_1 < \omega_{uh}$). The stimulated emission in the other band $\omega_2 \gtrsim 2\omega_c$ corresponds to radiation from Bernstein modes in the "cutoff" band near $n=1$. These modes propagate in the outer region of the plasma column, where $\omega_2 > \omega_{uh}$. Cutoff modes have been linearly excited with appreciable amplitude, e.g., as the so-called "*A*" lines appearing on the low-cyclotron frequency edge of Buchsbaum-Hasegawa modes.[4] They are the subject of a recent detailed study, and their nonlinear excitation via a particle-wave instability has also been reported.[7]

Figure 1 shows that the "cutoff" mode ω_2 grows in amplitude with increasing ω_c much less than the mode $\omega_1 < 2\omega_c$. In addition, its linewidth (~30 MHz) does not change appreciably, whereas the ω_1 mode acquires a fine structure, some of the lines within which are as narrow as the spectral resolution, 1 MHz. This is consistent with the fact that the cutoff mode is more heavily damped in the linear regime than the ω_1 modes. For very large pump strengths "cutoff"-mode radiation becomes almost as large as that from the ω_1 mode and also exhibits line narrowing. We note also that the conservation relation $\omega_1 + \omega_2 = \omega_0$ is satisfied very closely by the two bands, as shown in Table I. From the preceding, it can be concluded that the process described here represents the instability of a pair of Bernstein modes in the $n=1$ band, induced by a radiation-driven Bernstein "pump" mode in the $n=3$ band.

Table I. Data corresponding to Fig. 1; $\omega_0/2\pi = 7.72$ GHz.

$\omega_1/2\pi$ (GHz)	$\omega_2/2\pi$ (GHz)	$(\omega_1+\omega_2)/2\pi$ (GHz)
3.71	4.01	7.72
3.68	4.07	7.75
3.61	4.14	7.75
3.55	4.18	7.73
3.50	4.24	7.74
3.45	4.31	7.76
3.42	4.32	7.74

The detailed growth of the modes from "noise," i.e., spontaneous oscillations, is followed by monitoring the monochromatic emission at a frequency $\omega_1 < \frac{1}{2}\omega_0$ while continuously varying ω_c. In Fig. 2, trace (a) shows the spontaneous emission at ω_1 in the total absence of a pump signal. The structure in the region $\omega_c > \frac{1}{2}\omega_1$ represents emission from thermally excited Bernstein modes in the $n=1$ band. The succeeding traces (b)-(e) correspond to increasing pump intensities at $\omega_0 > 2\omega_1$ and $>2\omega_c$. In the region $\frac{1}{2}\omega_1 < \omega_c < \frac{1}{4}\omega_0$ the spontaneous emission from the Bernstein modes in the $n=1$ band remains unchanged, even though very strong above-threshold pump signals are incident on the plasma. It is only when ω_c exceeds the critical value $\frac{1}{4}\omega_0$ that stimulated emission at ω_1

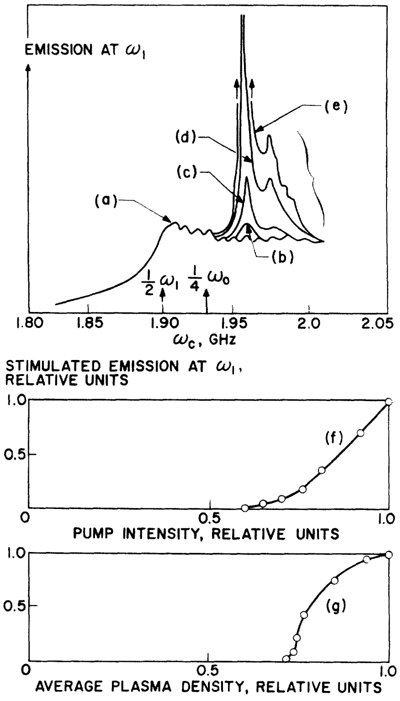

FIG. 2. (a)-(e) Emission at a single frequency $\omega_1 < \frac{1}{2}\omega_0$ as a function of the cyclotron frequency ω_c. Pump intensities below threshold yield trace (a). Above-threshold pump intensities produce stimulated emission, traces (b)-(e), but leave trace (a) unchanged in the region $\omega_c < \frac{1}{4}\omega_0$. (f) and (g) show dependence of stimulated emission, for $\omega_c > \frac{1}{4}\omega_0$, on pump intensity and electron density. Argon discharge, $\omega_0/2\pi = 7.72$ GHz. Unit pump intensity is 130 mW; unit electron density is 5×10^{11} cm^{-3}.

sets in. Throughout, however, the pump is able to feed energy to particles, to which it is coupled at the upper hybrid frequency.[6] Since $\overline{\omega}_p \approx 4\omega_c$, absorption at the upper hybrid frequency can be expected to change very little in the limited region $\frac{1}{2}\omega_1 < \omega_c < \frac{1}{4}\omega_0$, over which ω_c changes by only 2.5%. Therefore any particle-wave instabilities fed by pump heating would have set in well before $\omega_c = \frac{1}{4}\omega_0$. In conclusion, with the pump frequency well above cyclotron resonance ($\omega_0 \sim 4\omega_c$), particle-wave instabilities are negligible in comparison with mode-coupling, the dominant process in our experiment.

Figure 2(f) shows the dependence of the stimulated (i.e., excess) emission at ω_1 on the pump intensity. This exhibits a sharp threshold, followed by strong nonlinear growth. Figure 2(g) gives the dependence of the stimulated emission on the electron density. Again a sharp threshold is observed, which we estimate to correspond to the point at which the maximum plasma frequency ω_{p_0} in the column is such that $(\omega_{p_0}{}^2 + \omega_c{}^2)^{1/2} \geqslant \omega_0$. This indicates that the mechanism coupling the pump at ω_0 to the modes at ω_1, ω_2 depends critically on the efficient linear driving of a Bernstein mode by the pump.

When the pump intensity was suddenly switched on and off, the stimulated emission rose and decayed as fast as the pump, independently of background gas pressure, within experimental resolution (rise time $\geqslant 3$ nsec). Since electron-neutral collision times could be as long as 1 μsec at low neutral-gas pressures, it follows that the coupling process itself does not involve collisions. As pressure was decreased, however, the threshold became lower, consistent with the fact that collisions are the principal damping mechanism for Bernstein modes at our conditions. The intensity of the stimulated emission within a frequency band around ω_1 was found to undergo large-amplitude fluctuations in time. As ω_c was varied and the spectrally observed linewidth narrowed [as in Figs. 1(a)-1(d)], the fluctuations became more regular and widely spaced. This may indicate that the fluctuations represent beats between spectral components of the stimulated emission line, which appear to have an appreciable degree of coherence.

The preceding discussion has been limited only to the simplest and most prominent set of observed instabilities. Other unstable modes were found and will be discussed elsewhere. Among other features, at pump levels much above threshold the simple mode structure appearing in Fig. 1 does not obtain any longer. Instead, a quasicontinuum fills in the spectrum between ω_1 and ω_2, representing pairs of Bernstein modes whose frequencies add up to ω_0. The efficiency of the coupling is so high that as much as 20 mW of radiation was emitted by the plasma over a 1-GHz bandwidth about $\frac{1}{2}\omega_0$ for a pump intensity of 400 mW.

In conclusion, instabilities due to "decay" coupling between Bernstein modes are shown to be responsible for very strong interactions between electromagnetic waves and plasmas. Since the process described here represents down conversion of frequencies, the instabilities are close to each other so that beats between them may give rise to ion heating (as has been lately surmised[8]), with important consequences for the stability and heating of the plasma. It is of interest also that a good measure of the cyclotron frequency can be obtained from observations of the stimulated frequencies; this may be useful as a high-frequency diagnostic of the local magnetic field in a plasma.

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DIFFUSION IN TOROIDAL PLASMAS WITH RADIAL ELECTRIC FIELD

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The equilibrium of an axisymmetric toroidal plasma is analyzed. In the resistive regime the diffusion rate derived by Pfirsch and Schlüter is modified when the plasma rotates about its minor axis because of electric drift. In the weakly collisional regime the electron-diffusion rate is greater than that derived by Galeev and Sagdeev, except in the ambipolar condition where the diffusion rate is comparable with theirs.

The drift motions of ions and electrons in a toroidal magnetic field lead to a charge separation in a toroidal plasma which must be neutralized by current flow along the magnetic field. When finite resistivity is included this neutralization is not complete. Pfirsch and Schlüter[1] have shown that the residual electrostatic field can lead to a large enhancement of the diffusion rate compared with that in a comparable straight plasma column. In this and other papers[2,3] on resistive diffusion in tori any density variation over magnetic surfaces due to finite ion inertia is neglected. The weak collisional case, where collisions are too infrequent to produce an effective resistivity but are enough to prevent toroidally trapped particle orbits, was investigated recently by Galeev and Sagdeev.[4] Here Landau damping replaces resistivity. These authors include the density perturbation, but neglect any potential variation, over magnetic surfaces.

This note considers the equilibrium of resistive and weakly collisional toroidal plasmas, including both density and potential variation over magnetic surfaces. Radial electrostatic fields producing rotation of the toroidal plasma around its minor axis, which are observed in most toroidal confinement experiments, are included in the equilibrium.

The coordinate system, illustrated in Fig. 1, and magnetic field are the same as discussed in Ref. 2. Relative to the (r,θ,φ) coordinates the magnetic field is taken to be

$$\vec{B}=(R_0/R)[0,B_{0\theta}(r),B_{0\varphi}], \tag{1}$$

where R_0 is the radius of the magnetic axis, $R=R_0(1+\epsilon\cos\theta)$, and $\epsilon=r/R_0$. The displacement of magnetic surfaces relative to the magnetic axis in a realistic toroidal field will be neglected for simplicity. Since the aspect ratio will be assumed large, the equilibrium density and potential may be expanded as a series in ϵ:

$$n(r,\theta)=n_0(r)+n_1(r,\theta)+\cdots,$$
$$\Phi(r,\theta)=\Phi_0(r)+\Phi_1(r,\theta)+\cdots. \tag{2}$$

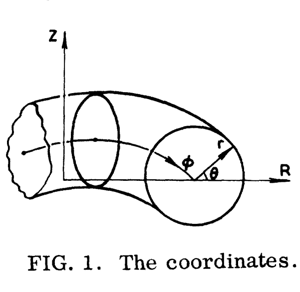

FIG. 1. The coordinates.