and the linearized Boltzmann equation solved for the perturbed velocity distribution. The current density can then be calculated and related back to the electric field by means of Maxwell's equations so determining the coefficients α_n. For $\omega \ll \omega_p$ and a Maxwellian velocity distribution in the unperturbed state,

$$\alpha_n = \frac{-8ia\omega B(a)}{(n\pi)^2} \sin\frac{n\pi}{2}\left[1+8\left(\frac{a}{\delta_0}\right)^2 \frac{a\omega}{c_0}\frac{Z(\xi)}{(n\pi)^3}\right],$$

where $B(a)$ is the amplitude of the magnetic field at $x=\pm a$, c_0 is the most probable speed of electrons, $Z(\xi)$ is the plasma dispersion function,[7] and $\xi = 2a\nu(i-\omega/\nu)(\pi c_0 n)^{-1}$. The electromagnetic field and the current distributions are then determined, leading to a value for R. In making these calculations we put $\nu = \nu_{en} + \nu_{ei}$, where the electron-ion collision frequency ν_{ei} was obtained from an expression for the resistivity of a fully ionized plasma given by Spitzer.[8] The theory confirms the presence of a critical frequency and density for which R is much greater than predicted by cold plasma theory. Theoretical values of R are shown in Fig. 1 for a frequency near the critical value. Despite the plane geometry used in the theoretical treatment, the theory gives a good description of the observed field penetration. On the other hand, the cold-plasma theory in plane geometry gives

$$R = \left|\cosh\left[\frac{a}{\delta_0}\left(\frac{i\omega}{i\omega+\nu}\right)^{1/2}\right]\right|,$$

which is plotted in Fig. 1 for the same value of ω/ν_{en}. The results show a monotonic increase of R with increasing N (similar to the Bessel-function expression given earlier) and do not agree with the experiment.

For densities greater than that corresponding to the maximum of R the theory shows that in general $|B|$ does not decrease monotonically from the plasma surface to the central plane. This has been observed by Demirkhanov, Kadysh, and Khodyrev.[4] The theory of Weibel[3] for the anomalous skin effect in a semi-infinite plasma also predicts analogous behavior in that the amplitudes of the field vectors do not decrease steadily from the plasma boundary.

Other experiments have been carried out with plasmas of different radii and also with steady state plasmas for which the plasma parameters could be determined more accurately. These results will be described in a later publication.

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SECOND SOUND IN SOLID HELIUM-3*

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An experiment on temperature pulse propagation in a bcc crystal of ^{3}He grown at 95.9 atm shows the existence of second sound in the solid below about 0.58°K. In the temperature range 0.42°K < T < 0.58°K, reflections of the received pulses were observed. In the range 0.58°K < T < 1.2°K, the temperature pulses were observed to propagate diffusively.

The phenomenon of second sound in solids has been observed previously[1] in experiments on temperature pulse propagation in crystals of hcp ^{4}He. The purpose of this Letter is to report preliminary results of an experiment in which temperature pulses were observed to propagate as second sound in a crystal grown from high-purity ^{3}He.

In theories of second sound in solids,[2-5] changes in the density of phonon excitations are supposed to propagate as damped waves through the crystal. Damping should be small enough to permit observation of second sound when the frequency Ω of the injected temperature oscillation falls in the "window"[4,5] $\tau_N \ll \Omega^{-1} \ll \tau_R$, where τ_N and τ_R are relaxation times for normal and resistive phonon-phonon scattering processes, respectively. For our purposes, these times are best regarded as parameters to be extracted from the experimental thermal conductivity curve. Conditions favorable to second-sound propagation occur in the vicinity of the low-temperature maximum of this curve in the temperature range called the "Poiseuille flow" region.[5] If one injects unmodulated temperature pulses instead of oscillations, the phenomenon manifests itself in the ability of the solid to propagate the pulses as wave packets and to reflect them at interfaces. The ^{4}He experimental results,[1] analyzed in terms of τ_N and τ_R, are confirmed by the theoretical predictions rather well.[6] In the ^{3}He experiments reported here, second sound occurs in a lower range of temperatures than for ^{4}He but further details, analyzed in the manner of Ref. 6, will appear elsewhere.

The subject crystal for these experiments was grown from ^{3}He with a concentration ratio ^{4}He/^{3}He $<2\times10^{-6}$. This was obtained by purification of normal grade ^{3}He (99.8%) by Dr. R. H. Sherman of this laboratory using a technique reported previously.[7] The crystal was grown at a constant pressure of 95.9 atm (molar volume $\approx$21.05 cm^3/mole; melting temperature $\approx$2.25°K).

Carbon film transducers used for generation and detection of heat pulses were spaced 0.125 cm, had areas of 0.25 and 0.1 cm^2, and had resistances at 0.6°K of 2400 and 4500 Ω, respectively. The values of $R^{-1}dR/dT$ at 0.6°K were about 0.65. Otherwise, the equipment was similar to that used previously.[1]

Temperature-pulse propagation was studied during slow cooling between 1.2 and 0.4°K. Input pulse durations were 60 to 100 nsec, amplitudes 5 to 15 V, and pulsing was done in the one-shot or manual mode. The input energy per pulse was always less than about 1 erg. Sample chamber heating due to the detector bias current was between 30 and 120 erg/sec. Signals generated by temperature excursions at the detector were amplified by a 17-MHz bandwidth amplifier and displayed as one-shot traces on a broadband oscilloscope.

Photographs of representative oscilloscope traces in Fig. 1 are as follows: Fig. 1(a), temperature pulse propagation by diffusion; 1(c), second-sound signal superimposed upon a diffusion ramp; 1(e), second-sound main arrival at the detector followed by two subsequent discernible reflections; 1(b) and 1(d), traces with the detector bias removed. The negative-going pulse shown for each trace was due to pickup by capacitive coupling. This decayed to negligible amplitude in about 2 μsec and did not interfere with signals arriving at the detector later.

The signal arrival times denoted in Fig. 1 are as follows: t_0, starting time of input pulse; t_1,

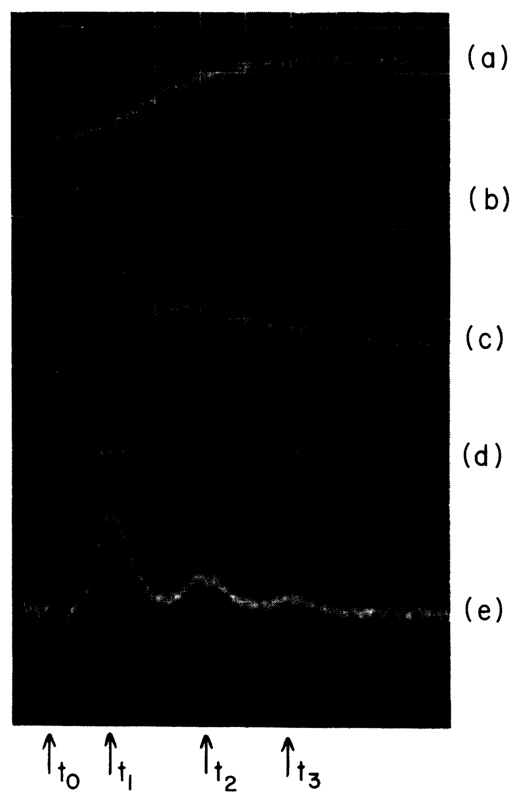

FIG. 1. Oscilloscope traces of typical receiver signals obtained with a horizontal sweep of 10 μsec/cm. Traces are (a) signal which propagated diffusively at 0.68°K, in the Ziman region of the thermal conductivity curve; (b), (d) traces with zero receiver bias current; (c) first evidence of second sound, at 0.58°K, a well-defined first arrival packet plus the hint of an echo, both superimposed upon the diffusion pulse ramp; (e) well-developed second-sound signal at 0.50°K showing the main arrival peaking at t_1 and two reflections peaking at t_2 and t_3. The receiver signal amplitude at t_1 in (e), 2.2 cm, corresponds to a signal peak of 110 μV, a received power of 7×10^{-10} W/μsec and a temperature excursion at the receiver film of about 0.51 mdeg.

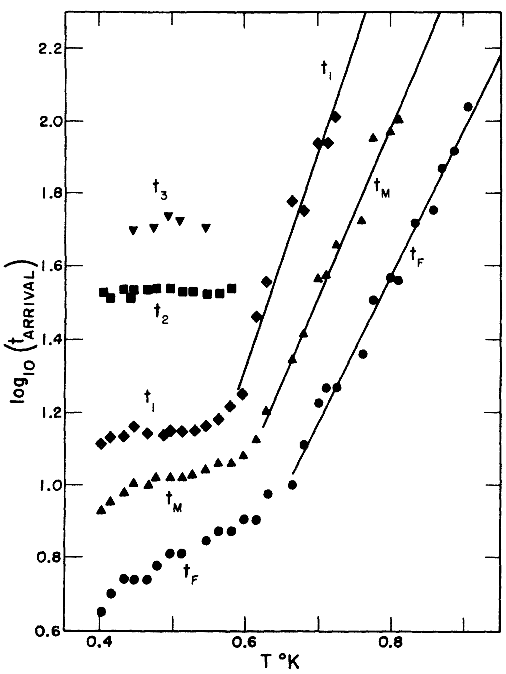

FIG. 2. Plot of the logarithm of various arrival times in microseconds versus temperature. The first discernible signal arrives at t_F, t_M is its half-amplitude point, t_1 its peak. Peak arrival times of reflected signals are t_2 and t_3. Portions fitted with a straight line define approximately the Ziman region where propagation is diffusive. The second-sound region is conservatively defined where the packet peaking at t_2 is clearly discernible.

peak of diffusion signal as in 1(a), or of the wave packet as in 1(e); t_2, t_3, etc., peaks of subsequent reflected packets. In Fig. 2 we show also t_F, the first discernible onset of the signal, and t_M, the half-amplitude point for either the ramp as in Fig. 1(a) or the first arrival as in Fig. 1(e). The transition from diffusive propagation to second sound is epitomized in Fig. 1(c).

In Fig. 2 we plot the logarithm of the various arrival times in microseconds versus temperature. Above about 0.6°K the t_F, t_M, and t_1 data are well fitted by the solid curves shown and they imply the rapidly decreasing velocity with increasing temperature characteristic of diffusive propagation.[6] The arrival data for t_1, t_2, and t_3 indicate that between 0.42 and 0.58°K the time differences t_2-t_1 and t_3-t_2 are the same, within experimental error, and nearly constant in this range at 20.0 μsec. The corresponding second-sound velocity is 125 ± 7 m/sec but this begins to increase significantly below about 0.42°K.

The corresponding theoretical first sound velocity is $\sqrt{3}\times125=217$ m/sec. We now compare this with 260 m/sec = mean Debye velocity for $\Theta_D=23.7$°K at 0.5°K and 21.05 cm^3/mole[8]; 254 and 202 m/sec, shear-mode velocities interpreted from theory[9]; 290 m/sec = experimental shear velocity in ^3He below 1°K and at 44 atm.[10] Only the theoretical low-mode velocity is consistent with our experimental results. Otherwise, there is an apparent discrepancy for which we have two plausible explanations: bcc ^3He is known to have a large elastic anisotropy so that if our crystal had a considerably different orientation from that of Ref. 10, the shear and second-sound velocities would be somewhat different; there is the possibility of time delays in converting temperature pulses into electrical signals at the transducer interface. Space limitations preclude further consideration of these points here.

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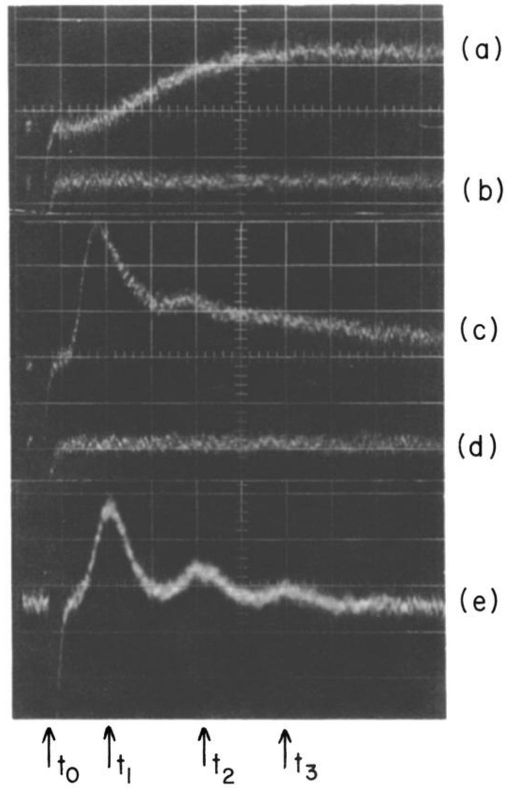

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