true $2m\pi$ pulse solutions for any values of m except 1 and ∞. A large pulse is unstable against evolution toward a long chain of 2π pulses. This would seem to be the origin of the large-pulse breakup observed in computer solutions and discussed by Lamb.[9]

We have not discussed the very interesting "fast" solutions; and can only mention without proof that, for a wide range of initial inversions, pulse propagation seems possible at both of two distinct velocities. In addition, there are solutions for which the field envelope oscillates but is never zero. One of these appears to have been found by Arecchi, DeGiorgio, and Someda.[8] Initial conditions appropriate to a photon echo pulse, with $v_0 \neq 0$, may also be investigated. A detailed discussion of these and other related topics is in preparation.

It is a pleasure to acknowledge numerous helpful discussions with L. Allen, L. Mandel, and N. E. Rehler.

Note added in proof.—While this paper was in preparation, E. T. Jaynes called our attention to a similar investigation being undertaken by M. D. Crisp.

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[5]The equations are the same as those referred to by McCall and Hahn (Ref. 2) except for insignificant sign changes. We have also assumed that phenomenological relaxation times are longer than the pulse duration and may be ignored.

[6]It is interesting to observe that there is a 2π pulse solution to the amplifier equations of Arecchi and Bonifacio which is conjugate in the sense of the theorem to the McCall-Hahn solution. We are happy to thank several persons for pointing out that this amplifier solution is new, and does not coincide with one of the Arecchi-Bonifacio steady-state pulse solutions.

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ANOMALOUS SKIN EFFECT IN BOUNDED PLASMAS

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The penetration of an electromagnetic field into a cylindrical plasma has been measured under conditions where the electron mean free path is comparable with the plasma diameter. The experimental results are in qualitative agreement with a theoretical model in which electron thermal motion is included.

An electromagnetic wave incident on a bounded plasma with no static magnetic field present is strongly attenuated when $\omega \ll \omega_p$ (ω is the wave frequency, ω_p the plasma frequency). The skin depth is usually calculated using the complex conductivity $\sigma = Ne^2/m(i\omega + \nu)$, where N, e, m, and ν are the electron density, charge, mass, and collision frequency, respectively. However, when electrons travel a distance comparable with the skin depth during a wave period (due to their thermal motion), a local relation between the electric field and the current density no longer provides a valid description of wave attenuation.[1-3] In this communication we describe an experiment which demonstrates the importance of electron thermal motion and compare the results with a simplified theory.

The plasma was produced by a pulsed discharge through low-pressure mercury vapor in a cylindrical glass tube 150 cm long and 8.1 cm diam. After a 20-μsec current pulse of 250 A, a steady current of 13.7 A was passed through the plasma, maintaining the electron temperature at about 2 eV. The electron-neutral collision frequency ν_{en}

was ~7×10^6 $\sec^{-1}$. Wave-penetration measurements were carried out during the decaying phase of the discharge with $10^{12} > N > 8\times10^{10}$ cm^{-3} as measured by Langmuir probes. An axial magnetic field, alternating at frequencies from 100 kHz to 10 MHz, was produced by a signal generator feeding into a 80-cm-long screened solenoid. The amplitude of the magnetic field was ~0.5 G and had negligible influence on the plasma parameters. These conditions differ from those described in earlier published work.[4-6]

The ratio R of the amplitude of the alternating magnetic field at the plasma boundary to the amplitude at the plasma axis was measured using two screened magnetic probes, the central probe being contained in a glass tube placed along the axis of the plasma. For each pulsed discharge the frequency of the field was constant, giving R as a function of the electron density. The results were reproducible; so by changing the frequency after each pulse it was possible to determine R over the ranges of frequency and electron density indicated above.

For a uniform cylindrical plasma, the theory which neglects electron thermal motion gives $R = |I_0((a/\delta_0)[i\omega/(i\omega+\nu)]^{1/2})|$, where I_0 is a modified Bessel function; a the plasma radius; $\delta_0 = c/\omega_p$, the collisionless skin depth; and c the velocity of light. For a fixed frequency this expression gives values of R which decrease monotonically as the electron density decreases, even allowing for the dependence of ν on electron density. Our experimental results do not show this behavior. In the frequency range 3-6 MHz, R remains almost independent of electron density from 10^{12} to 4×10^{11} cm^{-3}, increases to a maximum at $N \sim 2.5\times10^{11}$ cm^{-3}, and then decreases steadily as the density decreases further. The maximum in R is particularly marked for a critical frequency near 4.5 MHz, and Fig. 1 shows experimental values of R as a function of $\ln a/\delta_0$ (a/δ_0 is proportional to $N^{1/2}$) for this frequency.

This behavior is attributed to the thermal motion of electrons, the critical frequency being related to the transit time for electrons crossing the tube. Resonance phenomena would be expected to become particularly noticeable when the electron transit time is of the same order as a half-period of the field. This view is confirmed by calculations of the field penetration into a uniform plasma slab.

Consider a plasma which extends from $x=-a$ to $x=+a$. In the absence of plasma a magnetic field, represented as $Be^{i\omega t}$, exists in the z direction. For specular reflection of electrons at $x=\pm a$ the induced electric field can be represented by

$$E_y(x) = \sum_n \alpha_n \sin(n\pi x/2a)e^{i\omega t}$$

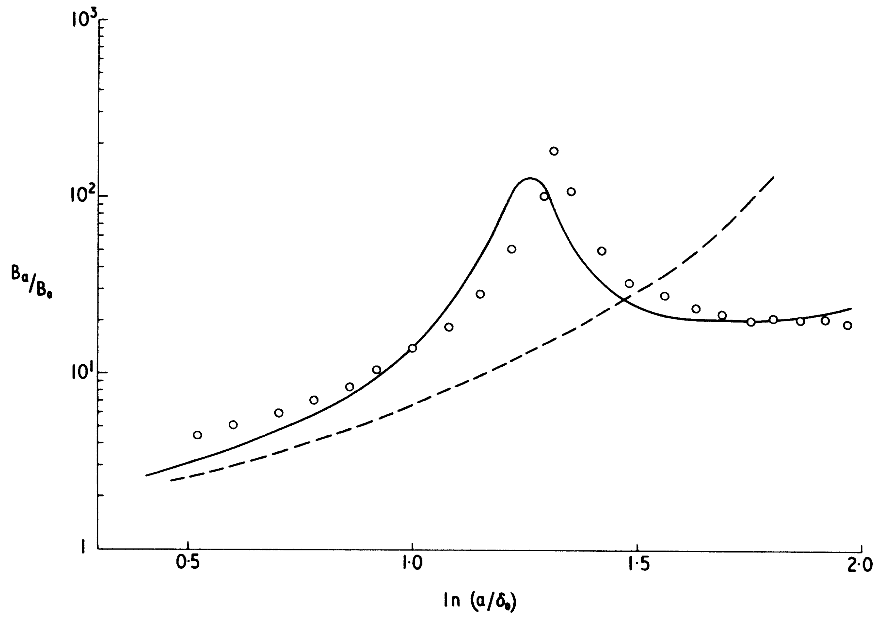

FIG. 1. Values of $R = B_a/B_0$, where B_a is the amplitude of the magnetic field at the plasma boundary and B_0 is the amplitude at the plasma axis in cylindrical geometry or at the center plane in plane geometry. Circles, experimental values of R for a cylindrical plasma; wave frequency = 4.5 MHz. Solid line, hot-plasma theory for a plane plasma slab; $\omega/\nu_{en} = 5.0$, $2a\nu_{en}/\pi c_0 = 0.3$. Dashed line, cold-plasma theory for a plane plasma slab; $\omega/\nu_{en} = 5.0$.

and the linearized Boltzmann equation solved for the perturbed velocity distribution. The current density can then be calculated and related back to the electric field by means of Maxwell's equations so determining the coefficients α_n. For $\omega \ll \omega_p$ and a Maxwellian velocity distribution in the unperturbed state,

$$\alpha_n = \frac{-8ia\omega B(a)}{(n\pi)^2} \sin\frac{n\pi}{2}\left[1+8\left(\frac{a}{\delta_0}\right)^2 \frac{a\omega}{c_0}\frac{Z(\xi)}{(n\pi)^3}\right],$$

where $B(a)$ is the amplitude of the magnetic field at $x=\pm a$, c_0 is the most probable speed of electrons, $Z(\xi)$ is the plasma dispersion function,[7] and $\xi = 2a\nu(i-\omega/\nu)(\pi c_0 n)^{-1}$. The electromagnetic field and the current distributions are then determined, leading to a value for R. In making these calculations we put $\nu = \nu_{\text{en}} + \nu_{\text{ei}}$, where the electron-ion collision frequency ν_{ei} was obtained from an expression for the resistivity of a fully ionized plasma given by Spitzer.[8] The theory confirms the presence of a critical frequency and density for which R is much greater than predicted by cold plasma theory. Theoretical values of R are shown in Fig. 1 for a frequency near the critical value. Despite the plane geometry used in the theoretical treatment, the theory gives a good description of the observed field penetration. On the other hand, the cold-plasma theory in plane geometry gives

$$R = \left|\cosh\left[\frac{a}{\delta_0}\left(\frac{i\omega}{i\omega+\nu}\right)^{1/2}\right]\right|,$$

which is plotted in Fig. 1 for the same value of ω/ν_{en}. The results show a monotonic increase of R with increasing N (similar to the Bessel-function expression given earlier) and do not agree with the experiment.

For densities greater than that corresponding to the maximum of R the theory shows that in general $|B|$ does not decrease monotonically from the plasma surface to the central plane. This has been observed by Demirkhanov, Kadysh, and Khodyrev.[4] The theory of Weibel[3] for the anomalous skin effect in a semi-infinite plasma also predicts analogous behavior in that the amplitudes of the field vectors do not decrease steadily from the plasma boundary.

Other experiments have been carried out with plasmas of different radii and also with steady state plasmas for which the plasma parameters could be determined more accurately. These results will be described in a later publication.

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SECOND SOUND IN SOLID HELIUM-3*

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An experiment on temperature pulse propagation in a bcc crystal of ^{3}He grown at 95.9 atm shows the existence of second sound in the solid below about 0.58°K. In the temperature range 0.42°K < T < 0.58°K, reflections of the received pulses were observed. In the range 0.58°K < T < 1.2°K, the temperature pulses were observed to propagate diffusively.

The phenomenon of second sound in solids has been observed previously[1] in experiments on temperature pulse propagation in crystals of hcp ^{4}He. The purpose of this Letter is to report preliminary results of an experiment in which temperature pulses were observed to propagate as second sound in a crystal grown from high-purity ^{3}He.