[6]C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).

[7]Use of an infinite-momentum frame simplifies calculations [S. Weinberg, Phys. Rev. 150, 1313 (1966)]. However, Weinberg's result that in φ^3 and φ^4 theory all "Z diagrams" disappear in such a frame cannot be generalized to a theory involving particles with spin. As to be discussed in the text later, we can omit these diagrams only in the large-w region for our present calculations because they do not give leading contributions.

[8]Weinberg, Ref. 7.

[9]We can also make the important observation from this relation that the cutoff $k_{\perp\,\max}^2$ appearing in our calculation corresponds to a Lorentz-invariant momentum cutoff for $w \gg 1$.

DECAYS OF THE ω, η, AND A_2 MESONS IN THE VENEZIANO MODEL

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By considering the decay reactions $\omega \to 3\pi$ and $\eta \to \pi + \pi + \gamma$ within the framework of the Veneziano model and the ρ dominance of the isovector electric current, we fix certain coupling constants and are able to obtain zero parameter relations among others.

Veneziano[1] has recently proposed a representation of a crossing-symmetric four-point function whose appeal lies in its ability to accommodate an infinite number of resonances in each channel, retain some semblance of Regge behavior at high energies, and at the same time comply with the low-energy criteria inherent in the hypothesis of partially conserved axial-vector current.[2,3] The detailed behavior of the formula has also been tested by Lovelace[2] in certain models of $K \to 3\pi$ and $\eta \to 3\pi$ decay and of the annihilation $N + \bar{N} \to 3\pi$. It is our aim in this communication to present several further studies and experimentally observable predictions of the Veneziano formula which demand no postulates outside the simplest version of the model except for the use of vector dominance for the photon in the decays $\omega^0 \to \pi^0 + \gamma$, $\eta \to \pi^+ + \pi^- + \gamma$, and $\eta \to \gamma + \gamma$.

(i) $\omega \to 3\pi$ and $\omega \to \pi^0 + \gamma$.—The decay amplitude for $\omega \to 3\pi$ may be obtained via crossing from the scattering amplitude for $\pi^a + \pi^b \to \pi^c + \omega$ as given by Veneziano[1]:

$$T(\pi^a(p_1) + \pi^b(p_2) \to \pi^c(p_3) + \omega(p_4)) = \epsilon^{abc}\epsilon_{\mu\nu\lambda\sigma}e^\mu p_1^\nu p_2^\lambda p_3^\sigma \beta_{\pi\pi\pi\omega}(B_{st} + B_{tu} + B_{us}) \tag{1}$$

with $B_{xy} = \Gamma(1-\alpha(x))\Gamma(1-\alpha(y))/\Gamma(2-\alpha(x)-\alpha(y))$ and $\alpha = \rho$-meson trajectory function. Defining the $\omega\rho\pi$ and $\rho\pi\pi$ couplings

$$\mathcal{L}(\omega_\mu(p+q) \to \pi^a(p) + p_\nu^b(q)) = \delta^{ab}G_{\omega\rho\pi}\epsilon_{\mu\nu\lambda\sigma}\epsilon^\mu\epsilon^\nu p^\lambda q^\sigma, \tag{2}$$

$$\mathcal{L}(\rho_\mu^a(p+q) \to \pi^b(p) + \pi^b(q)) = i\epsilon^{abc}G_{\rho\pi\pi}e^\mu(p-q)_\mu, \tag{3}$$

one can easily make the identification

$$\beta_{\pi\pi\pi\omega} = \alpha' G_{\omega\rho\pi}G_{\rho\pi\pi}, \tag{4}$$

where all three particles in $G_{\omega\rho\pi}$ are on shell. After performing the integration over the 3π phase space and setting $\alpha' = (2m_\rho^2 - 2m_\pi^2)^{-1} \cong 0.9$ GeV^{-2}, we find the relation

$$\Gamma_{\omega\to 3\pi} = 13.27 \times 10^{-5}\frac{G_{\omega\rho\pi}^2}{4\pi}\frac{G_{\rho\pi\pi}^2}{4\pi} \tag{5}$$

with all quantities in units of GeV. Setting[4] $\Gamma_{\omega\to 3\pi} \cong 10.9$ and $\Gamma_\rho = 115$ MeV, we find

$$G_{\omega\rho\pi} \simeq 21.5 \text{ GeV}^{-1} \tag{6}$$

in very good agreement with the values obtained via superconvergent sum rules.[5] It should be noted that the value (6) of $G_{\omega\rho\pi}$, when used in the simplest pole calculation of $\omega \to 3\pi$, leads to $\Gamma_{\omega\to 3\pi} + 13.5$ MeV.

The pleasing aspect of the result (6), however, is that when it is used to calculate the rate of $\omega \to \pi^0 + \gamma$ by the use of ρ dominance for the electric current (with a $\rho\gamma$ coupling $G_{\rho\gamma} = em_\rho^2/G_{\rho\pi\pi}$), the width obtained is

$$\Gamma_{\omega\to\pi^0+\gamma}=1.27\text{ MeV}. \tag{7}$$

This lies within the experimental limits 1.13 ± 0.15 MeV; otherwise said, the Gell-Mann–Sharp–Wagner model for the ratio $\Gamma_{\omega\to\pi^0+\gamma}/\Gamma_{\omega\to3\pi}$ works if the Veneziano amplitude for $\omega\to3\pi$ is used. The reconciliation of this result with the current-algebraic need for rapid variation of $G_{\omega\rho\pi}$ with ρ mass[6] is still to be made.

(ii) $\eta\to\pi^++\pi^-+\gamma$ and $\eta\to\gamma+\gamma$. —The process $\pi^a(p_1)+\eta(p_2)\to\pi^b(p_3)+\rho^c(p_4)$ is described by the amplitude $T^{abc}(s,t,u)=\epsilon^{abc}\epsilon_{\mu\nu\lambda\kappa}e^\mu p_1{}^\nu p_2{}^\lambda\times p_3{}^\kappa A(s,t,u)$, where A permits the Veneziano representation[1] $A(s,t,u)=\beta_{\pi\pi\eta\rho}(B_{st}+B_{tu}-B_{us})$, where B_{xy} was defined above. The exchange degeneracy of the ρ and A_2 trajectories allows the use of a single linear trajectory function α,[7] while the (satisfied) constraint $\alpha(s)+\alpha(t)+\alpha(u)=2$ eliminates the wrong-signature resonances.

Defining the $\eta\rho\rho$ coupling

$$\mathcal{L}(\rho_\mu{}^a(p+q)\to\rho_\nu{}^b(q)+\eta(p))$$
$$=\delta^{ab}G_{\eta\rho\rho}\epsilon_{\mu\nu\kappa\lambda}e_1{}^\mu e_2{}^\nu p^\kappa q^\lambda$$

we can establish (as with the ω) that

$$\beta_{\pi\pi\eta\rho}=\alpha'G_{\eta\rho\rho}G_{\rho\pi\pi}. \tag{8}$$

The use of crossing and ρ dominance permits us to relate the amplitude for $\eta\to\pi^++\pi^-+\gamma$ to the amplitude for $\pi+\eta\to\pi+\rho$. The constant $G_{\eta\rho\rho}$ is then fixed by demanding that $\Gamma_{\eta\to\pi^++\pi^-+\gamma}=126\pm39$ eV.[4] The result is

$$G_{\eta\rho\rho}=18.6\pm4\text{ GeV}^{-1}. \tag{9}$$

As in $\omega\to\pi^0+\gamma$, we have approximated $G_{\rho\gamma}\simeq em_\rho{}^2/G_{\rho\pi\pi}$ and have allowed for a 5% error in this approximation. The value (9) is in good agreement with our value (6) for $G_{\omega\rho\pi}$; the two should equal each other in the limit of SU(3) symmetry.

We can also perform a calculation of the ratio $\Gamma_{\eta\to\pi^++\pi^-+\gamma}/\Gamma_{\eta\to\gamma+\gamma}$ using the simplest SU(3)-symmetric interaction model[8]:

$$\mathcal{L}(V_{1\mu}{}^a(p+q)\to V_{2\nu}{}^b(q)+P^c(p))$$
$$=\sqrt{3}G_{vvp}d^{abc}\epsilon_{\mu\nu\kappa\lambda}e_1{}^\mu e_2{}^\nu p^\kappa q^\lambda, \tag{10}$$

where $G_{vvp}=G_{\eta\rho\rho}=-G_{\eta\omega_8\omega_8}$. Using vector dominance with $G_{\rho\gamma}=\sqrt{3}G_{\omega_8\gamma}=em_\rho{}^2/G_{\rho\pi\pi}$ we obtain the result

$$\Gamma_{\eta\to\pi^++\pi^-+\gamma}/\Gamma_{\eta\to\gamma+\gamma}=0.10 \tag{11}$$

in good agreement with the experimental value 0.12 ± 0.02.[4]

We conclude this section by remarking that the Veneziano model for $\eta\to\pi^++\pi^-+\gamma$ differs considerably from standard pole models in that the contributions of the A_2 (and its daughters and recurrences) are included. They can be included because the Veneziano model automatically supplies the necessary relationships between the strengths of the $A_2\eta\pi$-$A_2\rho\pi$ and the $\eta\rho\rho$-$\rho\pi\pi$ couplings. It is to these relations that we now turn.

(iii) Decays of the A_2 mesons. —The Veneziano formula for $\pi+\eta\to\pi+\rho$ scattering given above predicts both $J^P=2^+$ and 1^- particles at $\alpha=2$ in the s (or u) channel. If we call these A_{22} and A_{21}, respectively, and define the couplings

$$\mathcal{L}(A_{22,\,\mu\nu}{}^a(p+q)\to\eta(q)+\pi^b(p))$$
$$=\delta^{ab}G_{A_{22}\eta\pi}e_{\mu\nu}q^\mu p^\nu,$$

$$\Gamma(A_{22,\,\mu\nu}{}^a(p+q)\to\rho_\lambda{}^b(q)+\pi^c(p))$$
$$=i\epsilon^{abc}G_{A_{22}\rho\pi}e_{\mu\nu}e_\lambda\epsilon^{\lambda\mu\sigma\kappa}P_\kappa q^\nu q_\sigma,$$

$$\Gamma(A_{21,\,\mu}{}^a(p+q)\to\eta(q)+\pi^b(p))$$
$$=i\delta^{ab}G_{A_{21}\eta\pi}e_\mu(q-p)^\mu,$$

$$\Gamma(A_{21,\,\mu}{}^a(p+q)\to\rho_\nu{}^b(q)+\pi^c(p))$$
$$=\epsilon^{abc}G_{A_{21}\rho\pi}e_\mu e_\nu\epsilon^{\mu\nu\kappa\lambda}q_\kappa p_\lambda,$$

then the following identifications can be made[9]:

$$\beta_{\pi\pi\eta\rho}=\tfrac{1}{4}G_{A_{22}\eta\pi}G_{A_{22}\rho\pi}$$
$$=[2m_{A_2}{}^2/(m_\rho{}^2-m_\pi{}^2)(m_\eta{}^2-m_\pi{}^2)]\times G_{A_{21}\eta\pi}G_{A_{21}\rho\pi} \tag{12}$$
$$=\alpha'G_{\eta\rho\rho}G_{\rho\pi\pi}. \tag{8}$$

Using Eq. (9) and converting to decay widths [with

$\alpha' = (2m_\rho^2 - 2m_\pi^2)^{-1}$], we find

$$\Gamma_{A_{22}\to\eta+\pi}\Gamma_{A_{22}\to\rho\pi} \simeq 12\Gamma_{A_{21}\to\eta+\pi}\Gamma_{A_{21}\to\rho+\pi} \qquad (12')$$

$$\simeq (2700\pm1100)\ \mathrm{MeV}^2. \qquad (8')$$

Equation (8′) has a solution only if[10]

$$\Gamma_{A_{22}} \simeq \Gamma_{A_{22}\to\eta+\pi} + \Gamma_{A_{21}\to\rho+\pi} \geqslant 102\pm25\ \mathrm{MeV},$$

the lower limit being obtained in the case that $\Gamma_{A_{22}\to\eta+\pi} \simeq \Gamma_{A_{22}\to\rho+\pi}$. If, as seems more likely, $\Gamma_{A_{22}\to\eta+\pi} \sim (1/7)\Gamma_{A_{22}\to\rho+\pi}$,[4] the solution is

$$\Gamma_{A_{22}} \simeq \Gamma_{A_{22}\to\eta+\pi} + \Gamma_{A_{22}\to\rho+\pi} \simeq 150\pm75\ \mathrm{MeV}. \qquad (13)$$

The errors in (13) are probably even higher than indicated; for instance, interference effects among the ρ bands in the $A_2\to3\pi$ decay probably are the source of at least 20% uncertainty in the relation between $G_{A_{22}\rho\pi}$ and $\Gamma_{A_{22}\to\rho+\pi}$. In view of the uncertainty in the data and in the result (13), the most that can be said about this result is that it does not favor a very narrow A_{22}.

The relation (12′) will generally predict that $\Gamma_{A_{22}} > \Gamma_{A_{21}}$, unless the $\eta\pi/\rho\pi$ branching ratio of A_{21} is small compared with that of A_{22}. For instance, if $\Gamma_{A_{22}} \approx \Gamma_{A_{21}}$, then Eq. (12′) predicts $\Gamma_{A_{21}\to\eta+\pi}/\Gamma_{A_{21}\to\rho+\pi} \lesssim 1/50$. If the branching ratios are equal, then Eq. (12′) predicts $\Gamma_{A_{22}} \simeq 3.5\Gamma_{A_{21}}$. The A_2-region data are rather uncertain at the moment[11]; especially uncertain is the key question concerning the presence of any $J^P = 1^-$ component in the A_2 region. We stress again that if there is a 1^- component with total width comparable with the 2^+ component, the relation (12′) predicts that the $\eta\pi$ decay mode of this 1^- component would be highly suppressed. This conclusion also follows in an entirely unrelated way from SU(3) arguments.[12]

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[9]Our result $G_{A_{22}\rho\pi}G_{A_{22}\eta\pi} = 4\alpha' G_{\eta\rho\rho}G_{\rho\pi\pi}$ differs considerably from the finite-energy sum-rule relation $G_{A_{22}\rho\pi}G_{A_{22}\eta\pi} \cong (8/5)G_{\eta\rho\rho}G_{\rho\pi\pi}$ [M. Ademollo, H. R. Rubenstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. Letters 19, 1402 (1967); we have normalized their result to our definitions of the couplings]. The finite-energy sum-rule relation, using our value (9) for $G_{\eta\rho\rho}$, predicts a much narrower A_{22}.

[10]We neglect here the contribution of the $K\bar{K}$ mode to the total decay width. This seems to be justified [see footnote (n) of Table S in Ref. 4].

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