symmetry breaking.

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†A. P. Sloan Foundation Fellow.

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A FIELD-THEORETIC MODEL FOR ELECTRON-NUCLEON DEEP INELASTIC SCATTERING*

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The structure functions for inelastic electron-nucleon scattering are studied in the Bjorken limit in the framework of canonical field theory. The "parton" model of Feynman is derived and the structure functions' asymptotic behavior for large energy transfers is computed.

In this note we report some preliminary results derived from a field-theoretical study of the structure functions W_1 and W_2 summarizing inelastic electron-nucleon scattering, as defined by

$$W_{\mu\nu} = 4\pi^2 \frac{E_p}{M} \int dx\, e^{+iqx} \langle P | J_\mu(x) J_\nu(0) | P \rangle$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2, \nu) + \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right) W_2(q^2, \nu), \quad (1)$$

where $|P\rangle$ is a one-nucleon state with four-momentum P_μ, $J_\mu(x)$ is the total hadronic electromagnetic current operator, q_μ is the four-momentum of the virtual photon, $q^2 \equiv -Q^2 < 0$ is the square of the virtual photon's mass, and $M\nu \equiv P\cdot q$ is the energy transfer to the photon in the laboratory system. An average over the nucleon spin is understood in the definition of $W_{\mu\nu}$.

Bjorken[1] has shown on general grounds that W_1 and W_2 become experimentally important probes of small-distance nucleon structure in the experimentally accessible limit $Q^2, M\nu \to \infty$ and $w = 2M\nu/Q^2$ fixed. In particular, if they are nonvanishing in this asymptotic limit, W_1 and νW_2 should become nontrivial functions of w. We have studied the structure functions in the Bjorken limit on the basis of canonical quantum field theory, starting from the familiar interaction Lagrangian and current operator for the charge-symmetric theory of pseudoscalar pions and nucleons with γ_5 coupling.

First, we will present our results and interpretation and then sketch the formal calculations from which they emerge. A more detailed description of the formalism will appear in a forthcoming paper. Working in an infinite-momentum frame of the nucleon, we find that only certain classes of diagrams or amplitudes survive in this limit. To identify them we find it useful to "undress" the current operator and go into the interaction picture with the U matrix,

$$U(t)=\{\exp[-i\int_{-\infty}^{t} dt' H_I(t')]\}_+. \quad (2)$$

The fully interacting current and the corresponding free or "undressed" current are connected in the well-known manner by the perturbation series of time-ordered products of the interaction, or the U matrix,

$$J_\mu(x)=U^{-1}(t)j_\mu(x)U(t),$$

where $j_\mu(x)$ has the same form in terms of in fields as does $J_\mu(x)$ in terms of fully interacting Heisenberg fields. Our first result states

$$\lim_{\substack{q^2, M\nu\to\infty \\ w>1}} W_{\mu\nu}=4\pi^2\int dx\, e^{iqx}\left[\frac{Ep}{M}\langle UP|j_\mu(x)j_\nu(0)|UP\rangle\right]_{P\to\infty};\quad UP\rangle\equiv U(0)|P\rangle. \quad (3)$$

Equation (3) permits a very intuitive physical interpretation. In the Bjorken limit, the nucleon can be regarded as composed of free constituents described by the infinite sum of terms generated by $U|P\rangle$. These interact with the photon as free point particles and the current operator describing this interaction has the form $j_\mu=\bar\psi_p\gamma_\mu\psi_p+i(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+)$ in terms of free proton and charged-pion fields as indicated. We view this result as a field-theoretic derivation of the "parton model" of independent point constituents, proposed by Feynman.[2] We believe that this result is more general than the simple model implies.

A basic ingredient required to prove (3) is the existence of an asymptotic region in which Q^2 can be made greater than the transverse momenta of all the particles involved, i.e., of the pions and nucleons that are the (virtual) constituents of $UP\rangle$. The present high-energy scattering data strongly indicate that the transverse momenta of the final particles are indeed very limited in magnitude. Accordingly, we will take the existence of a transverse momentum cutoff as our fundamental assumption. This cutoff is a parameter in the theory and is invariant under Lorentz transformations along the $P\to\infty$ direction.

We have computed the structure functions W in the large-w region ignoring the renormalization loops, a procedure which will be partially justified later. Then, order by order in g^2, the pion-nucleon coupling constant, a simple class of diagrams dominates, namely, the ladder diagrams with pion lines as the rungs as shown in Fig. 1 which pictures the square of the matrix element; the vertical line signifies that we are computing only the absorptive part describing the production of real multipion states. Contributions of all other diagrams in the $P\to\infty$ system are smaller by at least one power of $\ln w$. Examples are diagrams with crossing pion lines, diagrams with currents interacting with pions, diagrams with a Z in the nucleon line describing nucleon-antinucleon pair formation, and those with final states involving pairs.[3] To leading order in $\ln w \gg 1$ for each order in g^2, the series of contributions can be summed for the proton and neutron to

$$\nu W_2^{(p)}=\nu W_2^{(n)}=\tfrac{1}{2}c\xi w^{\xi-1},$$
$$\nu W_2^{(p)}-\nu W_2^{(n)}=-\tfrac{1}{3}c\xi w^{-(\frac{1}{3}\xi+1)}, \quad (4)$$

and

$$W_1/\nu W_2=+w/2M,$$

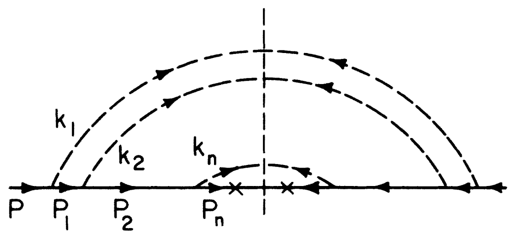
where

$$\xi\equiv\frac{3}{4\pi}\frac{g^2}{4\pi}\ln\left[1+\frac{k_{\perp\max}^2}{M^2}\right]. \quad (5)$$

$k_{\perp\max}$ is a cutoff for the transverse momentum to be described below, and the constant c is not fixed by summing an exponential series of powers of $\ln w$. Several remarks can be made:

(1) The recent Stanford Linear Accelerator Center data[4] indicate that $\nu W_2^{(p)}$ for the proton approaches a constant for large w. This demands

$$\xi\approx 1,\text{ or } k_{\perp\max}^2/M^2\approx 0.3, \quad (6)$$



$$\underset{\sim}{P}_i = \eta_1 \cdots \eta_i \underset{\sim}{P} + \underset{\sim}{k}_{i\perp}, \qquad \underset{\sim}{k}_i = \eta_1 \cdots \eta_{i-1}(1-\eta_i)\underset{\sim}{P} + \underset{\sim}{k}_{i-1\perp} - \underset{\sim}{k}_{i\perp}$$

$$E_i = \sqrt{\underset{\sim}{P}_i^2 + M^2}, \qquad \omega_i = \sqrt{\underset{\sim}{k}_i^2 + \mu^2}$$

$$0 \le \eta_i \le 1, \qquad k_{i\perp} \cdot P = 0$$

FIG. 1. Dominant ladder diagrams for large w. Solid nucleon and dashed pion lines are labeled with indicated momenta.

which is consistent with the indication from other high-energy collision data, $k_{\perp\,\max} \approx 400 \sim 500$ MeV.

(2) The fact that $W^{(p)}$ and $W^{(n)}$ approach each other rapidly makes it desirable to estimate the next leading contributions in $\ln w$ since the difference $W^{(p)} - W^{(n)}$ is very important in calculating the proton-neutron mass difference. Notice that the sign of $W^{(p)} - W^{(n)}$ is negative. We also need these next leading contributions to evaluate c and verify that the series of leading terms yields the dominant sum.[5]

(3) The ratio of structure functions in this limit, $W_1/\nu W_2 = +w/2M$, corresponds to a vanishing of the ratio of "scalar" to "transverse" photoabsorption cross sections for virtual photons of mass Q^2 on protons. It corresponds to the Callan and Gross[6] result for a spin-$\frac{1}{2}$ quark current because as we have seen the current interacts with the spin-$\frac{1}{2}$ nucleon in the high-w limit.

(4) The multiplicity of pions produced is given by $\bar{n}_\pi \approx \xi \ln w$.

We can only sketch here the proof of the assertions we have made above and reserve details for our forthcoming paper. In the infinite-momentum center-of-mass frame of the electron and the nucleon in which we carry out our calculation,[7] we have

$$q^0 = \frac{2M\nu - Q^2}{4P}, \quad q_3 = -\frac{2M\nu + Q^2}{4P},$$
$$|\vec{q}_\perp| = (Q^2)^{1/2} + O(P^{-2}), \tag{7}$$

where the nucleon momentum $\vec{P}$ is along the 3 axis. We now rewrite Eq. (1) as

$$W_{\mu\nu} = 4\pi^2(E_p/M)\textstyle\sum_n \langle UP | j_\mu(0) U(0) | n \rangle \langle n | U^{-1}(0) j_\nu(0) | UP \rangle (2\pi)^4 \delta^4(q + P - P_n). \tag{8}$$

If a final state $|n\rangle$ is not directly attached to the current because of the intervention of $U(0)$, the energy denominators introduced by the time integrals in the expansion of the time-ordered products in (2) connect states differing by $\vec{q}$ in general. This comes about because each order of U describes the absorption or emission of a pion and the associated energy denominators in this case connect states before and after the current has introduced a momentum $q \approx (Q^2)^{1/2}$. Thus the contributions of these diagrams vanish in the limit $Q^2 \to \infty$. To illustrate what we mean, consider the diagram in Fig. 2. Using the momentum labels of the particles denoted in the diagram, Weinberg's argument[8] implies that both η and $1-\eta$, the fraction of momentum carried by the nucleon and pion lines, respectively, along the $\vec{P}$ direction must be positive. Hence, the energy denominator in question is

$$\frac{1}{2E_{p+q}[E_{p+q} - E_{\eta p + k_\perp + q} - \omega_{(1-\eta)p - k_\perp}]} \approx \frac{-\eta(1-\eta)}{q^2(1-\eta)^2 + k_\perp^{\,2} + 2(1-\eta)k_\perp q + M^2(1-\eta)^2} \to O\left(\frac{1}{q^2}\right), \tag{9}$$

and such terms can consistently be dropped so long as the pion acquires a finite fraction of the momentum in the $P \to \infty$ frame when $(Q^2)^{1/2} \gg |k_{\max}|$. A corollary of this discussion is that diagrams with currents attached to two different lines are excluded by the same reasoning. We have now shown therefore that the $U(0)$'s adjacent to the final states $|n\rangle\langle n|$ may be effectively replaced by unity.

By similar arguments we can show formally, order by order, that the transformed proton state $UP\rangle$ can be treated as an eigenstate of the total Hamiltonian with eigenvalue E_p. To show this let E_{up} symbolically denote the energy of one of the multipion+nucleon states in the perturbation expansion of $|UP\rangle$. In the infinite momentum frame, $E_p - E_{up}$ is of the order of $1/P$ multiplied by the sum of squares of some characteristic transverse momentum and some characteristic mass. However, q_0 given by (7) appears in the energy delta function in (8) and thus for $Q^2, M\nu \to \infty$ and $w \ne 1$, $E_p - E_{up}$ can be neglected relative to q_0. One can then make use of the translation operators to restore the space-time dependence to obtain (3). This establishes the "parton model" by allowing us to work with free point currents and the superposition of essentially free (i.e., long-lived) constituents in describing the proton's ground state in the infinite-momentum frame and in the Bjorken limit.

We can now formally sum the series of the leading contributions from the class of diagrams we have claimed to dominate in the large-w region. We shall first ignore the charged pions completely, since they can be taken into account later by a simple consideration, and consider the proton. The contribution from a diagram as in Fig. (1) involving $n\pi^0$'s in the final state is obtained by introducing

$$U_{[n\pi^0]}(0)|P\rangle = \left[\frac{g}{(2\pi)^{3/2}}\right]^n (n!)^{1/2} \frac{2M}{(2E_p)^{1/2}} \int \frac{d^3P_n}{(2E_n)^{1/2}} \prod_{i=1}^{n} \frac{d^3k_i}{(2\omega_i)^{1/2}} \delta^3(\vec{P}-\vec{P}_n-\vec{k}_1-\vec{k}_2\cdots-\vec{k}_n)|P_n k_1\cdots k_n\rangle$$
$$\times \frac{\bar{u}_{p_n}\gamma_5(M+\gamma P_{n-1})\gamma_5\cdots\gamma_5(M+\gamma P_1)\gamma_5 u_p}{(2E_1)\cdots(2E_{n-1})(E_p-E_1-\omega_1)\cdots(E_p-E_n-\omega_1-\omega_2-\cdots-\omega_n)}$$

into (3), evaluating the bare-current matrix element

$$\int dx\, e^{iqx}\langle P_n|j_\mu(x)j_\nu(0)|P_n\rangle = \frac{1}{4\pi^2}\frac{M}{E_{p_n}}\bar{u}_{p_n}\gamma_\mu[M+\gamma(P_n+q)]\gamma_\nu u_{p_n}\delta(q^2+2p_n\cdot q),$$

and doing the standard traces. For large w the integrand simplifies to

$$\nu W_{2[n\pi^0]}\xrightarrow[w\to\infty]{}\frac{1}{w}(\xi_0)^n\int_0^1 d\eta_1\cdots d\eta_n\,\delta\left(\eta_1\cdots\eta_n-\frac{1}{w}\right)=\frac{\xi_0}{w}\frac{1}{(n-1)!}(\xi_0\ln w)^{n-1},$$

$$\frac{W_1}{\nu W_2}=\frac{w}{2M}, \tag{10}$$

where

$$\xi_0=\frac{g^2}{16\pi^3}\int d^2k_\perp\frac{1}{k_\perp^2+M^2}=\frac{1}{4\pi}\left(\frac{g^2}{4\pi}\right)\ln\left[1+\frac{k_{\perp\max}^2}{M^2}\right], \tag{11}$$

and $k_{\perp\max}$ is cutoff introduced for the transverse momentum integrals in accordance with our fundamental assumption. Summing over all numbers of π^0's, we find

$$\nu W_{2[\pi^0]}=(\xi^0/w)\exp[\xi_0\ln w]=\xi_0 w^{\xi_0-1}. \tag{12}$$

To include the charged pions in the calculation, we observe that an initial proton can emit a π^0 and remain as a proton with coupling constant g or it can emit a π^+ and become a neutron with coupling constant $\sqrt{2}g$. An analogous situation applies to a neutron. Let the contribution from a final state with $n\pi^0$'s and a proton be taken as the basis unit, and denote the total numbers of contributions from all possible final states with n charged plus neutral pions by P_n and N_n for the proton and neutron, respectively. They satisfy the recursion relations

$$P_n=P_{n-1}+2N_{n-1};\quad N_n=2P_{n-1}+N_{n-1}.$$

These give

$$P_n+N_n=3^n,\quad P_n-N_n=(-1)^n, \tag{13}$$

which convert (12) to (4) and (5).

To discuss the contributions of all other diagrams, we need only recall that at each nucleon vertex with spin-0 pions, the nucleon likes to give up most of its momentum to the pion. In fact, the $(\ln w)^{n-1}$ behavior in (10) comes simply from this fact that each segment of the nucleon

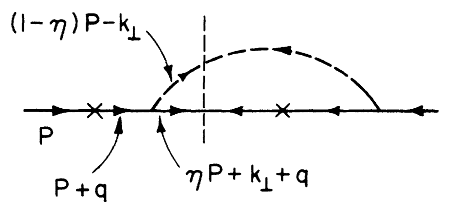


FIG. 2. Example of diagram describing pion emission between currents which vanishes for $Q^2\to\infty$.

line has but a small fraction $\eta \ll 1$ of the longitudinal momentum of the one preceding it in Fig. 1. Moreover, the delta function in (10) tells us that the η's measuring the fraction of energy retained by the nucleons is small for $w \gg 1$. However, when the currents are attached to a pion line, the delta function would dictate that a pion and not the nucleon pick up a small fraction $\sim 1/w$ of the longitudinal momentum from the initial nucleon, in the large-w region. This is not favored by the vertex, and hence at least one power of $\ln w$ is lost. If two pion lines get crossed in a diagram, the two virtual nucleons which connect the two pion lines on each side have a momentum mismatch. If a nucleon on one side picks up a small fraction of the available longitudinal momentum, the nucleon on the other side has to pick up a large fraction by momentum conservation. For a diagram with final state involving nucleon-antinucleon pairs, the virtual pion creating the pair is favored to have a large fraction of the available longitudinal momentum. Finally, for a Z diagram (an antinucleon or nucleon moving backward in time) the vertex favors a high-momentum virtual nucleon (or antinucleon). In all these cases at least one virtual particle has a large fraction of the longitudinal momentum available; thus at least one power of $\ln w$ is lost. Having exhausted all possible classes of diagrams except the diagrams involving renormalization loops, we now have derived the ladder approximation for the leading term order by order for $w \gg 1$.

As we have just shown, in the large-w region all the dominant contributions come from diagrams such as Fig. 1. The invariant momentum transfers to the virtual intermediate nucleon lines are of minimum magnitudes in this region, i.e.,[9]

$$(E_p - \omega_{(1-\eta)p - k_\perp})^2 - [\vec{P} - (1-\eta)\vec{P} + \vec{k}_\perp]^2 \approx -k_\perp^{\,2}/(1-\eta) + \eta M^2 \approx -k_\perp^{\,2} \text{ for } \eta \sim \ll 1.$$

We expect then that the neglect of renormalization effects and loops that are responsible for the structure at the pion-nucleon vertices may not be crucial in this region. To study them we must be able to master the series expansion in g^2 rather than relying on a larger parameter $g^2 \ln w$ as we have done. We conjecture that the cutoff $k_{\max}$ introduced in (5) reflects this structure, measuring the rate of decrease in the pion-nucleon vertices with increasing momentum transfer to the nucleon. There is no analogous cutoff at the electromagnetic vertex which is "bare" as in (3) once the $U(0)$ operating on final states $|n\rangle$ has been removed from (8). Once again it will be necessary to master a series in powers of g^2 in order to clarify the vanishing of the elastic form factors at large Q^2. We suspect that the origin of this behavior will be found in the vanishing of the wave function renormalization. Moreover, it will also be necessary to evaluate terms of order unity relative to $\ln w$ in order to determine the scale constant c appearing in (4). Our analysis also suggests that for $w \approx 1$, detailed dynamics will play an important role since the structure of the interaction for very large momentum transfers at the vertices is being probed in this region. Thus it is here near $w \sim 1$ that the small-distance structure will be most clearly probed. For large $w \gg 1$ the structure functions behave in the same manner as the very high-energy total hadron cross sections and are dominated by the "Pomeranchukon" or the ladder series.

The prediction of pion multiplicities follows from (10) and (12) if each term in the series is weighted by n, the number of pions. Our result that $\bar{n}_\pi \approx \xi \ln w$ for $\ln w \gg 1$ is a consequence of the fact that the pions, or rungs of the ladder in Fig. 1, do not interact with each other—i.e., in (3) we have made the replacement $U(0)|n\rangle \to n\rangle$. Our model also predicts that the pions are focused in momentum space with transverse momentum less than $k_{\perp\max}$ about the incident electron direction while the nucleon recoils with the large transverse momentum $q_\perp = (Q^2)^{1/2}$.

We thank J. D. Bjorken and R. P. Feynman for discussions of their ideas and work in advance of publication.

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[2]R. Feynman (unpublished).

[3]Diagrams with pion lines landing on the nucleon line between the two currents have already been disposed of in deriving Eq. (3).

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[6]C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).

[7]Use of an infinite-momentum frame simplifies calculations [S. Weinberg, Phys. Rev. 150, 1313 (1966)]. However, Weinberg's result that in φ^3 and φ^4 theory all "Z diagrams" disappear in such a frame cannot be generalized to a theory involving particles with spin. As to be discussed in the text later, we can omit these diagrams only in the large-w region for our present calculations because they do not give leading contributions.

[8]Weinberg, Ref. 7.

[9]We can also make the important observation from this relation that the cutoff ${k_{\perp}}_{\max}^2$ appearing in our calculation corresponds to a Lorentz-invariant momentum cutoff for $w \gg 1$.

DECAYS OF THE ω, η, AND A_2 MESONS IN THE VENEZIANO MODEL

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By considering the decay reactions $\omega \to 3\pi$ and $\eta \to \pi + \pi + \gamma$ within the framework of the Veneziano model and the ρ dominance of the isovector electric current, we fix certain coupling constants and are able to obtain zero parameter relations among others.

Veneziano[1] has recently proposed a representation of a crossing-symmetric four-point function whose appeal lies in its ability to accommodate an infinite number of resonances in each channel, retain some semblance of Regge behavior at high energies, and at the same time comply with the low-energy criteria inherent in the hypothesis of partially conserved axial-vector current.[2,3] The detailed behavior of the formula has also been tested by Lovelace[2] in certain models of $K \to 3\pi$ and $\eta \to 3\pi$ decay and of the annihilation $N + \bar{N} \to 3\pi$. It is our aim in this communication to present several further studies and experimentally observable predictions of the Veneziano formula which demand no postulates outside the simplest version of the model except for the use of vector dominance for the photon in the decays $\omega^0 \to \pi^0 + \gamma$, $\eta \to \pi^+ + \pi^- + \gamma$, and $\eta \to \gamma + \gamma$.

(i) $\omega \to 3\pi$ and $\omega \to \pi^0 + \gamma$.—The decay amplitude for $\omega \to 3\pi$ may be obtained via crossing from the scattering amplitude for $\pi^a + \pi^b \to \pi^c + \omega$ as given by Veneziano[1]:

$$T(\pi^a(p_1) + \pi^b(p_2) \to \pi^c(p_3) + \omega(p_4)) = \epsilon^{abc}\epsilon_{\mu\nu\lambda\sigma}e^{\mu}p_1^{\nu}p_2^{\lambda}p_3^{\sigma}\beta_{\pi\pi\pi\omega}(B_{st} + B_{tu} + B_{us}) \tag{1}$$

with $B_{xy} = \Gamma(1-\alpha(x))\Gamma(1-\alpha(y))/\Gamma(2-\alpha(x)-\alpha(y))$ and $\alpha = \rho$-meson trajectory function. Defining the $\omega\rho\pi$ and $\rho\pi\pi$ couplings

$$\mathcal{L}(\omega_{\mu}(p+q) \to \pi^a(p) + p_{\nu}^{\ b}(q)) = \delta^{ab}G_{\omega\rho\pi}\epsilon_{\mu\nu\lambda\sigma}\epsilon^{\mu}\epsilon^{\nu}p^{\lambda}q^{\sigma}, \tag{2}$$

$$\mathcal{L}(\rho_{\mu}^{\ a}(p+q) \to \pi^b(p) + \pi^b(q)) = i\epsilon^{abc}G_{\rho\pi\pi}e^{\mu}(p-q)_{\mu}, \tag{3}$$

one can easily make the identification

$$\beta_{\pi\pi\pi\omega} = \alpha' G_{\omega\rho\pi}G_{\rho\pi\pi}, \tag{4}$$

where all three particles in $G_{\omega\rho\pi}$ are on shell. After performing the integration over the 3π phase space and setting $\alpha' = (2m_{\rho}^2 - 2m_{\pi}^2)^{-1} \cong 0.9$ GeV^{-2}, we find the relation

$$\Gamma_{\omega\to 3\pi} = 13.27 \times 10^{-5}\frac{G_{\omega\rho\pi}^{\ 2}}{4\pi}\frac{G_{\rho\pi\pi}^{\ 2}}{4\pi} \tag{5}$$

with all quantities in units of GeV. Setting[4] $\Gamma_{\omega\to 3\pi} \cong 10.9$ and $\Gamma_{\rho} = 115$ MeV, we find

$$G_{\omega\rho\pi} \simeq 21.5 \text{ GeV}^{-1} \tag{6}$$

in very good agreement with the values obtained via superconvergent sum rules.[5] It should be noted that the value (6) of $G_{\omega\rho\pi}$, when used in the simplest pole calculation of $\omega \to 3\pi$, leads to $\Gamma_{\omega\to 3\pi} + 13.5$ MeV.

The pleasing aspect of the result (6), however, is that when it is used to calculate the rate of $\omega \to \pi^0 + \gamma$ by the use of ρ dominance for the electric current (with a $\rho\gamma$ coupling $G_{\rho\gamma} = em_{\rho}^2/G_{\rho\pi\pi}$), the