near $-\sqrt{2}$ or our value near -1.0.

Similar considerations are applicable to πN and KN even scattering amplitudes. For the KN case, in fact, one may avoid the difficulty of comparing two small numbers since the small factor $\sqrt{2}+c$ does not occur.

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[1]S. Weinberg, Phys. Rev. Letters 17, 616 (1966); K. Kawarabayashi and W. Wada, Phys. Rev. 146, 1209 (1966); K. Raman, Phys. Rev. 164, 1736 (1967); D. N. Levin, Phys. Rev. 174, 1759 (1968).

[2]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[3]J. K. Kim, Phys. Rev. Letters 19, 1079 (1967).

[4]C. H. Chan and F. T. Meiere, Phys. Rev. Letters 20, 568 (1968).

[5]Unless otherwise specified, all our quantities are expressed in units of fermis.

[6]The singularity structure for $T_\Lambda(\nu)$ is very similar to that of the crossing even πN amplitude. Our normalization is $\mathrm{Im}T_\Lambda(\nu)=k\sigma_{\pi\Lambda}(\nu)$.

[7]C. H. Chan and F. T. Meiere, Phys. Letters 28B, 125 (1968).

[8]W. Weisberger, Phys. Rev. 143, 1302 (1966).

[9]N. Fuchs, Phys. Rev. 149, 1145 (1966).

[10]We use the notation and normalization of Ref. 2 with $1/(\sqrt{2}f_\pi)=0.94\mu$.

[11]S. L. Adler, Phys. Rev. 140, B736 (1965).

[12]F. von Hippel and J. K. Kim, Phys. Rev. Letters 20, 1303 (1968).

SYSTEMATIC TEST OF THE SOFT-MESON THEOREMS AND OF A THEORY OF SU(3)⊗SU(3) SYMMETRY BREAKING*

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We make a systematic test of soft-meson theorem predictions for both elastic and inelastic P-B threshold scattering amplitudes. The predictions are obtained by using an extrapolation procedure developed by Fubini and Furlan and by ourselves. Our results are in reasonable agreement with a theory of SU(3)⊗SU(3)-symmetry breaking proposed recently by Gell-Mann, Oakes, and Renner.

In this paper we use the experimental values of the real parts of 13 elastic and inelastic pseudoscalar meson scattering amplitudes,

$$P_\alpha+B_i\to P_\beta+B_f \qquad (1)$$

evaluated at initial threshold, to test soft-meson theorems. The details of our analysis will be presented elsewhere.

The soft-meson theorems. – These are, as is well known, deduced from the hypotheses of partial conservation of axial-vector currents (PCAC) and SU(3)⊗SU(3) charge algebra.[1] In their exact form they depend, not only upon the matrix element of the equal-time commutator

$$\mathfrak{M}_Q\equiv-\langle f|[Q_\beta{}^5,A_\alpha{}^0(0)]|i\rangle, \qquad (2)$$

but also on the customarily neglected matrix element of the symmetrized equal-time commutator

$$\mathfrak{M}_\sigma=+\tfrac{1}{2}i\langle f|[Q_\beta{}^5,\partial_\mu A_\alpha{}^\mu(0)] +[Q_\alpha{}^5,\partial_\nu A_\beta{}^\nu(0)]|i\rangle. \qquad (3)$$

Here $Q_\beta{}^5$ is the axial charge with the SU(3) quantum numbers of P_β, and $A_\alpha{}^\mu$ is the axial current associated with P_α. In the SU(3)⊗SU(3)-symmetry limit the axial-vector currents are divergenceless and $\mathfrak{M}_\sigma=0$. Therefore, if one is to predict the values of $\mathfrak{M}_\sigma$, it is obviously necessary to have a theory of the symmetry breaking. Recently Gell-Mann, Oakes, and Renner have shown[2] that, if one makes the assumption that the symmetry-breaking term in the hadron energy density transforms like a member of the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$, one may use the PCAC hypothesis to calculate all the $\mathfrak{M}_\sigma$ up to a single parameter in terms of the square of the mass of the pseudoscalar mesons and the baryon mass splittings. Following their argument we obtain, when B_i, B_f are both at rest[3] (and neither P_α nor P_β is an η),

$$\mathfrak{M}_\sigma = +[(\tfrac{2}{3})^{1/2} + \tfrac{1}{2}c(d_{8\alpha\alpha} + d_{8\beta\beta})]\{(\tfrac{2}{3})^{1/2}\mu_0\delta_{\beta\alpha}\delta_{fi} + (1/c)d_{\beta\alpha\gamma}[iFf_{f\gamma i} + Dd_{f\gamma i}]\}. \quad (4a)$$

Here c is the ratio of the strength of the singlet and octet parts in the symmetry breaking term and F and D are the coefficients of the SU(3) structure constants (if_{f8i}) and (d_{f8i}), respectively, in the baryon octet mass splitting formula:

$$c = -2\sqrt{2}[\mu_K^2 - \mu_\pi^2]/[2\mu_K^2 + \mu_\pi^2],$$

$$F = (M_N - M_\Xi)/\sqrt{3}, \quad D = (\tfrac{1}{2}\sqrt{3})[M_\Sigma - M_\Lambda]. \quad (4b)$$

The parameter in (4a) which remains to be determined is μ_0, the change in the average mass of the baryon octet due to SU(3)⊗SU(3)-symmetry breaking.

The soft-meson theorems fix the scattering amplitudes (1) off the meson mass shells at points where one or both of the mesons has zero four-momentum. We will be concerned in particular with the values of the amplitudes at the "soft-meson points" on the surface $t \equiv (p_f - p_i)^2 = (\Delta M)^2$ (where $\Delta M \equiv M_f - M_i$). On this surface we may take the invariant amplitude, $\mathfrak{M}(\nu, q_\alpha^2, q_\beta^2)$, to be a function of $(q_\alpha)^2$, $(q_\beta)^2$, and $\nu = (p_i \cdot q_\alpha)/M_i$, the lab energy of P_α.

When B_i, B_f are not in the same isospin multiplet, $\Delta M \neq 0$ and we have two soft-meson points. At the point corresponding to $q_\alpha = 0$ the soft-meson theorems give us

$$\mathfrak{M}(0, 0, (\Delta M)^2) = \mathfrak{M}_\alpha \equiv [1 - (\Delta M/\mu_\beta)^2]\{\mathfrak{M}_\sigma + \tfrac{1}{2}\Delta M \mathfrak{M}_Q\}/(F_\alpha F_\beta) \quad (5a)$$

and, when $q_\beta = 0$ we have

$$\mathfrak{M}(\Delta M, (\Delta M)^2, 0) = \mathfrak{M}_\beta \equiv [1 - (\Delta M/\mu_\alpha)^2]\{\mathfrak{M}_\sigma + \tfrac{1}{2}\Delta M \mathfrak{M}_Q\}/(F_\alpha F_\beta). \quad (5b)$$

Here F_α, F_β are the leptonic decay constants[4] which appear in the PCAC identification,

$$\varphi_\alpha = \partial_\mu A_\alpha^{\ \mu}/(F_\alpha \mu_\alpha^{\ 2}), \quad (6)$$

when the α's are the indices of a self-conjugate basis of SU(3). The familiar case when B_i, B_f are in the same isospin multiplet can be regained from (5) in the limit $\Delta M \cong 0$.

Extrapolation procedure. – In order to test the predictions of the soft-meson theorems for the off-mass-shell amplitude by using on-mass-shell data, one must have an extrapolation procedure. We use the prescription arrived at independently by us[5] and, in a more systematic treatment, by Fubini and Furlan.[6] We follow Fubini and Furlan here, extending their argument to inelastic amplitudes.

The extrapolation is accomplished by using the Low equation in the lab frame to write down a dispersion relation in ν for Re$\mathfrak{M}$ along the curve

$$q_\alpha^{\ 2} = \nu^2, \quad q_\beta^{\ 2} = (\nu - \Delta M)^2, \quad (7)$$

on which both mesons are at rest in the lab frame. This curve passes through both the soft-meson points. The dispersion relation may therefore be subtracted at each of these points with the two subtraction constants $\mathfrak{M}_\alpha$, $\mathfrak{M}_\beta$ being fixed by (5),

$$\mathrm{Re}\mathfrak{M}(\nu, \nu^2, (\nu - \Delta M)^2) = \mathfrak{M}_\alpha + (\mathfrak{M}_\beta - \mathfrak{M}_\alpha)\left(\frac{\nu}{\Delta M}\right) + \nu(\nu - \Delta M)\frac{P}{\pi}\int \frac{d\nu' \, \mathrm{Im}\mathfrak{M}(\nu', (\nu')^2, (\nu' - \Delta M)^2)}{\nu'(\nu' - \Delta M)(\nu' - \nu)}. \quad (8)$$

If $\mathfrak{M}_\alpha$ and $\mathfrak{M}_\beta$ were approximated in (5) by setting $\mathfrak{M}_\sigma$ equal to zero and if the integral in (8) were entirely neglected at $\nu = \mu_\alpha$ we would get back the conventional soft-meson approximation for the threshold amplitude.[1] Our purpose here is to get the best possible estimate we can of the integral "rescattering" correction and to look for evidence of a nonzero $\mathfrak{M}_\sigma$.

At P_α-B_i threshold we have, according to (7),

$$\nu = \mu_\alpha, \quad q_\alpha^{\ 2} = \mu_\alpha^{\ 2}, \quad q_\beta^{\ 2} = (\mu_\alpha - \Delta M)^2. \quad (9)$$

For elastic scattering amplitudes, both mesons are on their mass shells and (8) may be used to predict the values of Re$\mathfrak{M}$ at threshold in terms of the theoretical subtraction constants (5). For inelastic scattering amplitudes P_β is not on its mass shell. We will ignore this fact here, however, as P_β is not far off its mass shell in our applications. As a partial test of this approximation, one may compare the success of our predictions for elastic and inelastic amplitudes below.

Estimate of Im$\mathfrak{M}$. –As pointed out by Fubini and Furlan,[6] the cuts in the off-mass-shell amplitude $\mathfrak{M}$ come from two types of intermediate states in the lab-frame Low equation: (i) real intermediate states, i.e., states accessible in the Reaction (1) and the crossed reaction, $\bar{P}_\beta + B_i \to \bar{P}_\alpha + B_f$, with both P_α, P_β on their mass shells and (ii) virtual intermediate states which are accessible only because P_α, P_β are off their mass shells. The cuts of type (ii) nearest to the soft-meson points are associated with the dissociation of P_α or P_β into three mesons at $\nu = \mu_\alpha + 2\mu_\pi$ and $\nu = \mu_\beta + 2\mu_\pi + \Delta M$, respectively.

Of necessity we will neglect cuts of type (ii). Our experience below with cuts of type (i) tends to justify this approximation. We find for type (i) cuts that the integral in (10) has converged within a distance of μ_π of P_α–B_i threshold. Because cuts of type (ii) will be suppressed near their thresholds by three-body phase space we expect them to contribute only at a considerably greater distance.

A feature of the curve (7) is that along it $\vec{q}_\alpha = \vec{q}_\beta = 0$ in the lab frame, i.e., Im$\mathfrak{M}$ is entirely due to s-wave scattering of the off-mass-shell mesons. Our estimate of Im$\mathfrak{M}$ is therefore obtained by approximating the cuts of the off-mass-shell scattering amplitude (there are no s-wave poles) by the corresponding s-wave cuts of the on-mass-shell amplitude at the same c.m. energy,

$$\mathrm{Im}\mathfrak{M}(\nu', (\nu')^2, (\nu'-\Delta M)^2) \cong \mathrm{Im}\mathfrak{M}_{l=0}(\omega(\nu'), \mu_\alpha^{\,2}, \mu_\beta^{\,2}), \qquad (10a)$$

where

$$\omega(\nu') = \nu' + [(\nu')^2 - \mu_\alpha^{\,2}]/2M_i \qquad (10b)$$

for the direct-channel cut and

$$\omega(\nu') = \nu' - [(\nu'-\Delta M)^2 - \mu_\beta^{\,2}]/2M_i \qquad (10c)$$

for the crossed-channel cut.

There are two ways in which we justify the approximation (10): (i) The approximation is self-consistent in the sense that the dispersion integral in (10) converges near threshold where the mesons are near their mass shells; (ii) as independent evidence of the unimportance of faraway singularities in $\mathfrak{M}$, we find that, in those cases (π-N, K-N) where s-wave scattering is small near threshold, the experimental scattering lengths are close to their soft-meson approximation values.

Results and conclusions. –Our results are presented in Table I. The T_{exp} are the experimental amplitudes[7] at initial threshold normalized according to the prescription

$$T = -[(M_iM_f)^{1/2}/4\pi(M_i + \mu_\alpha)]\mathfrak{M}. \qquad (11)$$

For elastic scattering T_{exp} is just the scattering length.

The T_Q are theoretical values of the same amplitudes calculated in soft-meson approximation, i.e., by setting $\mathfrak{M}_\sigma = 0$ in (5) and neglecting the integrals in (8) at $\nu = \mu_\alpha$.

The ReT_C are the contributions of integrals in (8) to the theoretical threshold amplitudes evaluated using the approximation (10). It may be seen that in many cases the value of ReT_C is larger than T_Q, i.e., the conventional soft-meson approximation fails badly. In evaluating ReT_C we have found that the integrals over the direct cuts converge within the interval where Im$\mathfrak{M}_{l=0}$ is well established. The integrals over the crossed cuts are unimportant in general and, in all cases where we have no experimental information [such as the contribution of $(\pi\text{-}\Sigma)_{I=2}$ scattering to the crossed cut in the $(\pi\text{-}\Sigma)_{I=0,1}$ amplitudes[5]], we have found that we can argue that the neglected terms are unimportant.

In column 4 we give

$$\Delta\,\mathrm{Re}T \equiv \mathrm{Re}T_{\mathrm{exp}} - T_Q - \mathrm{Re}T_C. \qquad (12)$$

This is the discrepancy of theory with experiment when we set $\mathfrak{M}_\sigma = 0$. (The reader may notice that, in some cases, the errors on Δ ReT are smaller than those on either ReT_{exp} or ReT_C. This is because the errors are correlated.) Finally, in the last column we list (T_σ) the contribution associated with $\mathfrak{M}_\sigma$ to the theoretical threshold amplitude. If in every case T_σ were equal to Δ ReT within experimental error, we would have confirmation of both the soft-meson theorems and our extrapolation procedure.

We first compare the last two columns for the inelastic amplitudes where T_σ is independent of μ_0. We believe that the correspondence, in particular the agreement in sign in every case, provides strong support for the theory of SU(3)⊗SU(3) symmetry breaking used here.[2] However, the quantitative agreement is not complete. The fact

Table I. Threshold amplitudes at initial threshold in fermis: $T_{\rm exp}$ are the experimental amplitudes; T_Q are the contributions to the theoretical amplitudes due to the (charge)-(charge density) commutator [Eq. (2)]; $\mathrm{Re}T_C$ is the theoretical rescattering correction [Eqs. (8) and (10)]; $\Delta\mathrm{Re}T_C$ is $\mathrm{Re}T_{\rm exp}-T_Q-\mathrm{Re}T_C$; and T_σ is the contribution to the theoretical threshold amplitude due to the (charge)-(current divergence) commutators.

$(\text{Reactions})_I$	Experimental Values Re $T_{\rm exp}$	Im $T_{\rm exp}$	T_Q	Re T_C	Δ Re T	T_σ(μ_0 in BeV)
$(\pi\Sigma\to\pi\Sigma)_0$	1.09 ±0.23	---	0.442	0.44 ±0.14	0.22 ±0.09	0.003-0.126 μ_0
$(\pi\Sigma\to\pi\Sigma)_1$	0.39 ±0.07	0.15 ±0.02	0.221	0.00 ±0.10	0.17 ±0.09	0.003-0.126 μ_0
$(\bar{K}N\to\bar{K}N)_0$	-1.65 ±0.04	0.73 ±0.02	0.519	-2.89 ±0.18	0.73 ±0.19	-0.031-0.65 μ_0
$(\bar{K}N\to\bar{K}N)_1$	-0.12 ±0.02	0.51 ±0.03	0.173	-0.27 ±0.04	-0.03 ±0.04	0.061-0.65 μ_0
$(KN\to KN)_0$	0.00 ±0.04	---	0.00	0.01 ±0.02	-0.01 ±0.02	0.107-0.65 μ_0
$(KN\to KN)_1$	-0.29 ±0.02	---	0.346	0.16 ±0.01	-0.10 ±0.02	0.015-0.65 μ_0
$(\pi N\to\pi N)_{1/2}$	0.28 (0.04)[a]	---	0.216	0.016±0.01	0.05 (0.04)[a]	-0.014-0.126 μ_0
$(\pi N\to\pi N)_{3/2}$	-0.09 (0.03)[a]	---	-0.108	0.01 ±0.01	0.01 (0.03)[a]	-0.014-0.126 μ_0
$(\bar{K}N\to\pi\Sigma)_0$	+0.90 ±0.01	0.00 ±0.01	0.066	1.50 ±0.11	-0.67 ±0.10	-0.336
$(\bar{K}N\to\pi\Sigma)_1$	-0.39 ±0.01	-0.45 ± .02	0.054	-0.24 ±0.07	-0.21 ±0.07	-0.274
$(\pi\Sigma\to\bar{K}N)_0$	-1.50 ±0.38	---	0.013	-0.84 ±0.23	-0.65 ±0.15	-0.236
$(\pi\Sigma\to\bar{K}N)_1$	-0.50 ±0.07	0.22 ±0.01	0.011	-0.31 ±0.13	-0.21 ±0.06	-0.193
$(\bar{K}N\to\pi\Lambda)_1$	0.28 ±0.01	-0.26 ± .03	0.069	0.59 ±0.11	-0.39 ±0.10	-0.210

[a]As experimental errors are not available, we quote the difference between two experimental values (see text).

that the discrepancies occur in those cases for which $\mathrm{Re}T_C$ is largest may indicate that the discrepancies are more likely to be originating from fractionally small systematic errors in the extrapolation procedure than in the soft meson theorems.

Next we turn to the elastic amplitudes. It will be noticed that, according to theory, the σ-term contributions to the π-N and π-Σ scattering lengths are much smaller than they are to the K-N and $\bar{K}$-N scattering lengths. This prediction is consistent with our results for the π-N scattering lengths but is not for the π-Σ scattering lengths. We do not think that this latter result constitutes strong evidence against the theory, however. Once again a fractionally small systematic error in $\mathrm{Re}T_C$ could account for the discrepancy in the $(\pi\text{-}\Sigma)_0$ case. In the $(\pi\text{-}\Sigma)_1$ case we expect systematic error due Kim's parametrization[7] to be maximal, because (unlike the $I=0$ case) a principal-value integral centered at π-Σ threshold and running down to π-Λ threshold is involved and because the deduction of the π-Σ scattering data depends entirely on the coupled-channel effective-range parametrization.

We therefore turn finally to the K-N and $\bar{K}$-N scattering lengths. Here the situation is nearly optimal for three of the scattering lengths. We are dealing with directly observed reactions in the neighborhood of physical threshold and, with the exception of the $(\bar{K}\text{-}N)_0$ case, the values of $\mathrm{Re}T_C$ are relatively small. In fact the differences between the $(\bar{K}\text{-}N)_1$, $(K\text{-}N)_0$, and $(K\text{-}N)_1$ scattering lengths are in excellent agreement with the predictions T_σ. If these three scattering lengths are used to obtain μ_0 we arrive at the estimate

$$\mu_0\approx 175\ \mathrm{MeV} \tag{14}$$

with about 15% statistical error.

In conclusion, we have seen that the theoretical and experimental values of $\mathrm{Re}T$ are in quite good agreement. The agreement is good in the sense that the discrepancies when large are considerably smaller than the "rescattering" correction $\mathrm{Re}T_C$. If we take these results as a confirmation of the theory of symmetry breaking used here,[2] then it appears that approximately 175 MeV of the average baryon mass is due to SU(3)⊗SU(3)-

symmetry breaking.

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[1]A. P. Balachandran, M. Gundzik, and F. Nicodemi, Nuovo Cimento 44A, 1257 (1966); Y. Tomozawa, Nuovo Cimento 46A, 707 (1967); S. Weinberg, Phys. Rev. Letters 17, 616 (1966).

[2]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968). See also R. W. Griffith, to be published.

[3]Our amplitudes are normalized according to the conventions of J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields (McGraw-Hill Book Company, Inc., New York, 1965).

[4]We have used the values $F_\pi = 0.69\mu_\pi$, $F_K = 0.87\mu_\pi$ corresponding to a Cabibbo angle with $\tan\theta_C = 0.21$.

[5]F. von Hippel and J. K. Kim, Phys. Rev. Letters 20, 1303 (1968).

[6]S. Fubini and G. Furlan, Ann. Phys. (N.Y.) 48, 322 (1968). See also A. de Alfaro and C. Rossetti, Nuovo Cimento Suppl. 6, 575 (1968).

[7]For $\pi\Sigma \to \pi\Sigma$, $\bar{K}N \to \bar{K}N$, and the inelastic amplitudes see Ref. 5 and J. K. Kim, Phys. Rev. Letters 19, 1074 (1967). The signs of the inelastic amplitudes were determined from their interference with the P_{13} amplitudes on the assumption that the signs of the P_{13} amplitudes at resonance are appropriate to the $Y_1^*(1385)$ being predominantly a member of an SU(3) decuplet. Our SU(3) phase conventions are those of J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963). For the $(KN)_1$ amplitude see S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halloran, T. F. Stubbs, G. M. Pjerrou, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters 9, 135 (1962). For the $(KN)_0$ amplitude see J. K. Kim, to be published. For the (πN) amplitudes see C. Lovelace, in Proceedings of the International Conference of Elementary Particles, Heidelberg, Germany, 1967, edited by H. Filthuth (North-Holland Publishing Company, Amsterdam, The Netherlands 1968), p. 79. Since Lovelace quotes no errors on these numbers, we give in brackets the difference of his results and those of J. Hamilton, Phys. Letters 20, 687 (1966), as a measure of the uncertainty.

A FIELD-THEORETIC MODEL FOR ELECTRON-NUCLEON DEEP INELASTIC SCATTERING*

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The structure functions for inelastic electron-nucleon scattering are studied in the Bjorken limit in the framework of canonical field theory. The "parton" model of Feynman is derived and the structure functions' asymptotic behavior for large energy transfers is computed.

In this note we report some preliminary results derived from a field-theoretical study of the structure functions W_1 and W_2 summarizing inelastic electron-nucleon scattering, as defined by

$$W_{\mu\nu} = 4\pi^2 \frac{E_p}{M} \int dx\, e^{+iqx} \langle P | J_\mu(x) J_\nu(0) | P \rangle$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2, \nu) + \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right) W_2(q^2, \nu), \qquad (1)$$

where $|P\rangle$ is a one-nucleon state with four-momentum P_μ, $J_\mu(x)$ is the total hadronic electromagnetic current operator, q_μ is the four-momentum of the virtual photon, $q^2 \equiv -Q^2 < 0$ is the square of the virtual photon's mass, and $M\nu \equiv P\cdot q$ is the energy transfer to the photon in the laboratory system. An average over the nucleon spin is understood in the definition of $W_{\mu\nu}$.

Bjorken[1] has shown on general grounds that W_1 and W_2 become experimentally important probes of small-distance nucleon structure in the experimentally accessible limit $Q^2, M\nu \to \infty$ and $w = 2M\nu/Q^2$ fixed. In particular, if they are nonvanishing in this asymptotic limit, W_1 and νW_2 should become nontrivial functions of w. We have studied the structure functions in the Bjorken limit on the basis of canonical quantum field theory, starting