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[6]Any moment sum rule is good enough to prove our result. This is essentially because we concentrate mostly on the high-energy region to derive our result.
[7]See J. Mandula and R. Slansky, Phys. Rev. Letters 20, 1402 (1968); C. Goebel, Phys. Rev. Letters 21, 383 (1968).
[8]This however does not agree with the conclusions obtained from calculations on the basis of the eikonal approximation and centrifugal barriers. See H. Goldberg, Phys. Rev. Letters 21, 778 (1968).
[9]Similar results have been obtained by R. M. Spector (to be published) by demanding consistency of Regge behavior both in s and t channels. However, our point of view is different, since we derive our result in the context of finite-energy sum rules.

INFORMATION ABOUT CURRENT-DIVERGENCE ALGEBRA FROM PION-BARYON SCATTERING*

C. H. Chan and F. T. Meiere
Department of Physics, Purdue University, Lafayette, Indiana 47907
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It is shown that the connection between even scattering lengths and σ terms of current algebra depends strongly upon dynamics. The σ terms for $\pi\Lambda$ and $\pi\Sigma$ scattering are evaluated and shown to be consistent with a model algebra of current divergences in which the SU(3)-symmetry breaking parameter is not small.

For the scattering of pions from any target, current algebra provides a direct connection between the off-mass-shell, crossing-even scattering amplitude, $T(\nu, q^2)$ at $\nu=0$, $q^2=0$, and the σ term (the commutator of an axial divergence with an axial generator). To relate the matrix elements of the σ term to the physical scattering lengths requires a continuation in both energy and external mass. For πN scattering, it has been found that the σ terms and the effect of mass and energy continuation are relatively small.[1] This Letter investigates the σ terms for $\pi\Lambda$ and $\pi\Sigma$ scattering in order to obtain information about SU(3) symmetry breaking. It is shown by explicit dispersion calculation using experimental data that the $\pi\Lambda$ scattering length is not small and that there is no simple connection between the scattering length and the σ term.

To investigate the σ term, we evaluate directly the amplitudes for $\pi\Lambda$ and $\pi\Sigma$ at the current-algebra point rather than the scattering length and compare with the algebra of current divergencies of Gell-Mann, Oakes, and Renner.[2] Within the framework of their model, we eliminate the unknown matrix elements of the scalar densities and compare directly differences of amplitudes with a specific function of the SU(3) symmetry-breaking parameter c. Our results give a lower bound, $|c| \geq 0.7$, and are compatible with the predicted value near $-\sqrt{2}$.

We consider first the $\pi\Lambda$ system since it does not have an unphysical region to complicate the discussion. The $\pi\Lambda$ scattering length has not been measured directly from experiment, but it is possible to draw some indirect conclusions. Consider the forward $\pi\Lambda$ scattering amplitude, $T(\nu)$. Some experimental information on $T(\nu)$ is available from the known Y_1^* resonances. However for our purposes here, the most important experimental information comes from Kim's multichannel analysis of $\bar{K}N$ scattering near threshold.[3] This also provides the low-energy $\pi\Lambda$ and $\pi\Sigma$ amplitudes for $I=0$ and 1 which couple to the $\bar{K}N$ system. This information has been shown to be compatible with the known features of the $\pi\Lambda$ and $\pi\Sigma$ systems[3] and the known $\bar{K}N$ total cross sections.[4] The simplest calculation is to evaluate the experimental parametrization of the $\pi\Lambda$ amplitude at the $\pi\Lambda$ threshold. This gives $a_{\pi\Lambda} = 1.71$ F,[5] which is large compared with other known meson-baryon scattering lengths, e.g.,

$$a_{\pi N}^{(+)} = 0.009, \quad a_{\pi N}^{(-)} = 0.093,$$

$$a_{K^+p} = -0.29.$$

Even if the multichannel parameters are varied arbitrarily, it is found that $a_{\pi\Lambda}$ cannot be reduced below 1 F without altering significantly the known $\bar{K}N$ scattering lengths. One may object that Kim's parameters are determined from data above $\bar{K}N$ threshold so that this procedure involves an extrapolation of at least 180 MeV (center-of-mass energy). To avoid this, we evaluate the following once-subtracted dispersion relation for $T(\nu)$ [6]:

$$\mathrm{Re}T(\nu) = \mathrm{Re}T(\nu_0) + T_{\mathrm{pole}}(\nu) + \frac{2(\nu^2-\nu_0^2)}{\pi} \mathrm{P}\int_\mu^\infty \frac{k'\nu'\sigma_{\pi\Lambda}(\nu')d\nu'}{(\nu'^2-\nu_0^2)(\nu'^2-\nu^2)}, \quad (1)$$

where

$$T_{\mathrm{pole}} = \frac{2\pi g_{\pi\Lambda\Sigma}^2 \nu_\Sigma(\nu^2-\nu_0^2)}{(\nu_\Sigma^2-\nu_0^2)(\nu_\Sigma^2-\nu^2)} \frac{(M_\Sigma-M_\Lambda)^2-\mu^2}{M_\Lambda^2},$$

$\nu=(s-u)/4M_\Lambda$ is the lab energy, $k=(\nu^2-\mu^2)^{1/2}$. At threshold, $T(\mu)=4\pi(1+\mu/M_\Lambda)a_{\pi\Lambda}$. The subtraction point ν_0 was chosen at $\bar{K}N$ threshold since, at this point, Kim's analysis for $\mathrm{Re}T(\nu_0)$ is most accurate. For the integration below ν_C, corresponding to total center-of-mass energy $W = 1600$ MeV, we used Kim's $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ partial waves. Above ν_C we used the known Y_1^* resonances in the narrow-width approximation and a constant background of 22 mb. We used[7] $g_{\pi\Lambda\Sigma}^2=21$. The pole contribution is so small that use of the pure-SU(3) value of 7 changes $a_{\pi\Lambda}$ by only a few percent. (This coupling constant plays a more important role in the current-algebra calculation.) Numerical evaluation of Eq. (1) determines $a_{\pi\Lambda}=1.52$ F. The separate contributions are as follows: subtraction $=-16.82$, pole $=1.20$, low energy $=39.77$, resonances $=-1.18$, and background $=-1.47$. This value is quite close to the scattering length obtained by extrapolation of the multichannel parametrization to the $\pi\Lambda$ threshold. This is perhaps not surprising considering that the S-wave $\pi\Lambda$ amplitude contains no resonances nor poles.

To reconcile this with current-algebra results, we use the conventional definition of the off-mass-shell amplitude applied to $\pi\Lambda$ scattering, denoted by $T_\Lambda(\nu,q^2)$, where q^2 is the "external pion mass." The physical amplitude with $q^2=\mu^2$ will sometimes be denoted $T_\Lambda(\nu)$. For the $\pi\Lambda$ system, the only baryon pole arises from the Σ. However, our whole calculation can be repeated exactly for the crossing-even $\pi\Sigma$ and πN amplitudes. In these cases, the zero-mass limits are taken by keeping the external baryon mass not equal to the internal baryon masses which occur in pole terms. The physical masses are inserted at the end. It is well established that this procedure yields the correct results for both current-algebra[8] and consistency-condition calculations[9] and will simplify our presentation. The current algebra gives[10]

$$T(0,0) = 4if_\pi^2\langle\Lambda|[\partial_\mu A_\mu^3, F_5^3]|\Lambda\rangle, \quad (2)$$

where f_π is the decay constant of the pion,[10] F_5 is an axial generator, and $\partial_\mu A_\mu$ is the divergence of the weak axial current. To calculate $T(0,0)$ using dispersion relation from physical $\pi\Lambda$ scattering data, we have to make a mass continuation of Eq. (2) to $q^2=\mu^2$. It is conventional to deal exactly with the Born term $T_{\mathrm{Born}}(\nu,q^2)$ and use models for $T-T_{\mathrm{Born}}$. The effect of mass continuation for the latter part is expected to be small on general grounds (order μ^2/m_Λ^2) whether the q^2 continuation is done for fixed ν or fixed s or some linear combination. This is supported by model calculations.

Before doing the mass continuation, we present here a model-independent calculation for $T_\Lambda(0,\mu^2)$. $T_\Lambda(\nu,q^2)$ satisfies a once-subtracted dispersion relation which is the straightforward generalization of Eq. (1). For crossing-even scattering amplitudes, the pole term in the dispersion relation, $T_{\mathrm{pole}}(\nu,q^2)$, and the Born term are not the same because of the subtraction but are related by $T_{\mathrm{pole}}(\nu,q^2)=T_{\mathrm{Born}}(\nu,q^2)-T_{\mathrm{Born}}(\nu_0,q^2)$. For simplicity, we use $\tilde{T}(\nu,q^2)\equiv T(\nu,q^2)-T_{\mathrm{pole}}(\nu,q^2)$. It is easy to check that use of the Born term gives the same results, but with more algebra. Figure 1 shows the results for $\mathrm{Re}\tilde{T}_\Lambda(\nu,\mu^2)$ between $\nu=0$ and $\bar{K}N$ threshold, obtained from the on-mass-shell dispersion relation Eq. (1) using the experimental information described earlier. (The multichannel parametrization is also shown. Both methods agree well between $\pi\Lambda$ and $\bar{K}N$ threshold but clearly the parametrization breaks down near the Σ pole.) The important point is that the variation of $T_\Lambda(\nu,\mu^2)$ between $\nu=0$ and threshold is large, ruling out any simple relation between the σ term and the scattering length.

As stated, the effect of mass continuation, namely $\tilde{T}_\Lambda(0,0)-\tilde{T}_\Lambda(0,\mu^2)$, is expected to be small. To check this statement, we made a dispersion calculation using a model originally suggested by Adler[11] to calculate this difference. In the dispersion relation, $\mathrm{Im}T_\Lambda{}^l(s,q^2)=[p(s,q^2)/p(s,\mu^2)]^{2l}\mathrm{Im}T_\Lambda{}^l(s,\mu^2)$ for each partial wave,

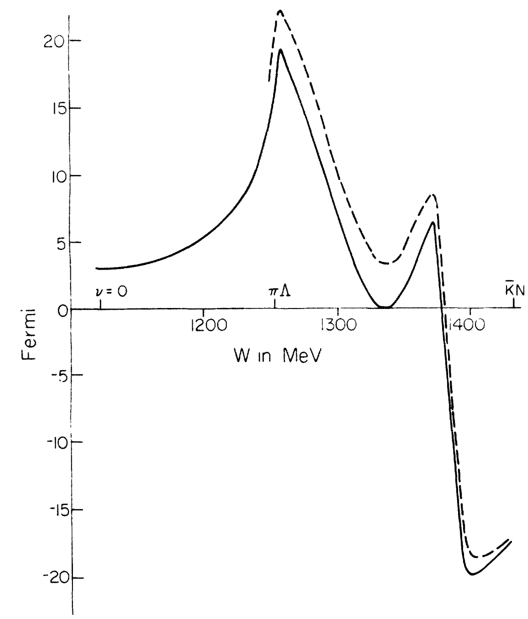

FIG. 1. Variation of the on-mass $\pi\Lambda$ amplitude without the pole term between $\nu=0$ and the $\bar{K}N$ threshold. The solid curve is $\mathrm{Re}\tilde{T}_\Lambda(\nu)$ calculated from dispersion relation, the dashed curve is $\mathrm{Re}\tilde{T}_\Lambda(\nu)$ calculated from extrapolating Kim's parametrization. W is the center-of-mass energy.

where $p(s,q^2)$ is the center-of-mass momentum. Although the model reporduces the proper threshold behavior and is approximately true for Feynman diagrams with direct-channel resonances, it has no firm theoretical basis. The dispersion relation contains one subtraction, $\mathrm{Re}T(\nu_0,q^2)$ for which we also use the $p^{2l}(s,q^2)$ dependence. The subtraction point is considerably above $\pi\Lambda$ threshold. We carefully avoid using any model to directly extrapolate $\mathrm{Re}T_\Lambda(\nu,q^2)$ for energies at or below threshold. The result is that $\tilde{T}_\Lambda(0,\mu^2)-\tilde{T}_\Lambda(0,0)=+0.83$, which is negligible in comparison with that, $\tilde{T}_\Lambda(\mu,\mu^2)-\tilde{T}_\Lambda(0,\mu^2)=17.29$, due to the variation in ν, confirming our previous statement. With $g_{\pi\Lambda\Sigma}{}^2=21$, we finally obtain $T_\Lambda(0,0)=3.78$ F.

Before turning to other applications, we quote the result of a similar calculation for the even $\pi\Sigma$ amplitudes. Since the σ term is real and the $\pi\Sigma$ scattering length is complex, naive extrapolation of the amplitude is less appealing. Such complications have been considered by others[12] for the crossing-odd scattering length. We consider the crossing-even $\pi\Sigma$ amplitude introduced in Ref. 7, $T_\Sigma=2T(I=1)-T(I=0)$. This choice is made to eliminate the unknown $I=2$ phase shifts. Apart from the inclusion of both Λ and Σ poles, the discussion for $\pi\Lambda$ can be repeated exactly. We obtain $\mathrm{Re}a_{\pi\Sigma}=-0.18$ F. The difference $\mathrm{Re}\tilde{T}_\Sigma(\mu,\mu^2)-\tilde{T}_\Sigma(0,\mu^2)=-4.40$ is found to be large while $\tilde{T}_\Sigma(0,\mu^2)-\tilde{T}_\Sigma(0,0)=-1.08$ is small. With $g_{\pi\Lambda\Sigma}{}^2=21$, we obtain $T_\Sigma(0,0)=3.98$ F.

Possibly the most interesting application of the results of this calculation is to the algebra of current divergencies proposed by Gell-Mann, Oakes, and Renner.[2] They introduce a set of scalar and pseudoscalar densities u_α and v_α forming a $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ representation of SU(3) $\otimes$SU(3) which specify the σ term. These operators also appear in the Hamiltonian density, $\mathcal{H}=\mathcal{H}_{\mathrm{inv}}-u_0-cu_8$. An important feature of the model is that c, the SU(3)-symmetry breaking parameter is not small, but near $-\sqrt{2}$. Applied to Eq. (2), this algebra gives

$$T(0,0)=\tfrac{4}{3}(\sqrt{2}+c)f_\pi{}^2\langle B|\sqrt{2}u_0+u_8|B\rangle, \qquad (3)$$

where $B=N$, Λ, or Σ. If we assume that SU(3) holds to the extent that the Λ and Σ matrix elements of $\mathcal{H}_{\mathrm{inv}}$ (and u_0) are equal, $m_\Sigma-m_\Lambda=-c(\langle\Sigma|\times u_8|\Sigma\rangle-\langle\Lambda|u_8|\Lambda\rangle)$ so that

$$\frac{3}{4f_\pi{}^2}\frac{T_\Sigma-T_\Lambda}{m_\Sigma-m_\Lambda}=-1-\frac{\sqrt{2}}{c}. \qquad (4)$$

If the evaluation of T_Σ and T_Λ is taken seriously, they imply $c=-1.0$. Realistically, however, the difference of amplitudes is more sensitive to details in our calculation than are the individual terms or the scattering lengths. Although the error in Eq. (4) is hard to establish, small values of c are definitely ruled out. Small values of c imply a large difference of amplitudes. The largest negative value for the left-hand side of Eq. (4) is obtained by arbitrarily reducing the high energy and pole contributions by 50%. This gives $c\simeq+0.7$. Similar considerations for the largest positive value for Eq. (4) gives $c\simeq-0.8$.

Thus we conclude here that although current algebra makes a definite prediction for $T(0,0)$, in general, the scattering length is not simply related to this prediction and depends strongly on the dynamics of the system. We have evaluated the σ terms for $\pi\Lambda$ and $\pi\Sigma$ scattering and shown that they are consistent with the algebra-of-current divergencies of Gell-Mann, Oakes, and Renner. The necessity of evaluating the σ term directly is emphasized by the fact that use of the πN, $\pi\Lambda$, and $\pi\Sigma$ scattering lengths in Eq. (4) gives a value for c of +0.04 ($\Lambda\Sigma$), −0.08 (ΛN), and +5.9 (ΣN) as opposed to the predicted value near $-\sqrt{2}$ or our value near -1.0.

Similar considerations are applicable to πN and KN even scattering amplitudes. For the KN case, in fact, one may avoid the difficulty of comparing two small numbers since the small factor $\sqrt{2}+c$ does not occur.

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SYSTEMATIC TEST OF THE SOFT-MESON THEOREMS AND OF A THEORY OF SU(3)⊗SU(3) SYMMETRY BREAKING*

Frank von Hippel†
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California

and

Jae Kwan Kim
Department of Physics, Harvard University, Cambridge, Massachusetts
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We make a systematic test of soft-meson theorem predictions for both elastic and inelastic P-B threshold scattering amplitudes. The predictions are obtained by using an extrapolation procedure developed by Fubini and Furlan and by ourselves. Our results are in reasonable agreement with a theory of SU(3)⊗SU(3)-symmetry breaking proposed recently by Gell-Mann, Oakes, and Renner.

In this paper we use the experimental values of the real parts of 13 elastic and inelastic pseudoscalar meson scattering amplitudes,

$$P_\alpha+B_i\to P_\beta+B_f \qquad (1)$$

evaluated at initial threshold, to test soft-meson theorems. The details of our analysis will be presented elsewhere.

The soft-meson theorems. – These are, as is well known, deduced from the hypotheses of partial conservation of axial-vector currents (PCAC) and SU(3)⊗SU(3) charge algebra.[1] In their exact form they depend, not only upon the matrix element of the equal-time commutator

$$\mathfrak{M}_Q\equiv-\langle f|[Q_\beta{}^5,A_\alpha{}^0(0)]|i\rangle, \qquad (2)$$

but also on the customarily neglected matrix element of the symmetrized equal-time commutator

$$\mathfrak{M}_\sigma=+\tfrac{1}{2}i\langle f|[Q_\beta{}^5,\partial_\mu A_\alpha{}^\mu(0)]+[Q_\alpha{}^5,\partial_\nu A_\beta{}^\nu(0)]|i\rangle. \qquad (3)$$

Here $Q_\beta{}^5$ is the axial charge with the SU(3) quantum numbers of P_β, and $A_\alpha{}^\mu$ is the axial current associated with P_α. In the SU(3)⊗SU(3)-symmetry limit the axial-vector currents are divergenceless and $\mathfrak{M}_\sigma=0$. Therefore, if one is to predict the values of $\mathfrak{M}_\sigma$, it is obviously necessary to have a theory of the symmetry breaking. Recently Gell-Mann, Oakes, and Renner have shown[2] that, if one makes the assumption that the symmetry-breaking term in the hadron energy density transforms like a member of the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$, one may use the PCAC hypothesis to calculate all the $\mathfrak{M}_\sigma$ up to a single pa-