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A STUDY OF DIFFRACTION-PRODUCED $K^*\pi$ AND $\Delta\pi$ SYSTEMS IN 12.6-GeV/c K^-p INTERACTIONS*

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Two different peripheral quasi three-body final-state processes are studied in the reaction $K^-p \to K^-p\pi^+\pi^-$. Near-threshold mass enhancements are observed in both the $\bar{K}^{*0}(890)\pi^-$ and $\Delta^{++}(1238)\pi^-$ systems. In both cases the data are suggestive of a production mechanism involving inelastic diffraction scattering. For each case a comparison is made with the predictions of a double-Regge-pole exchange model.

Recent studies of high-energy πp, pp, and Kp interactions[1-8] have shown that an important contribution to the total interaction cross section arises from the diffractionlike production of three-body final states with the invariant mass of one pair of final-state particles enhanced at near-threshold values. We comment on the observation of two of these effects in the reactions

$$K^-p \to \bar{K}^{*0}(890)p\pi^- \to K^-p\pi^+\pi^-, \tag{1a}$$

$$K^-p \to K^-\Delta^{++}(1238)\pi^- \to K^-p\pi^+\pi^-, \tag{1b}$$

and point out the very high degree of similarity in the dynamical details of the two classes of events in Reactions (1). In both cases the salient features of the data are well described by a Reggeized double-peripheral production model.

From an exposure of the Brookhaven National Laboratory 80-in. liquid-hydrogen bubble chamber to a beam of rf-separated K^- mesons at 12.6-GeV/c incident momentum, we have obtained a sample of 1300 four-pronged events fitting the reaction $K^-p \to K^-p\pi^+\pi^-$, the cross section for this reaction being 0.70 ± 0.05 mb. Two-thirds of the events in this reaction fall either in the $K^*(890)$ region of the $K^-\pi^+$ mass spectrum or in a $p\pi^+$ mass peak at the mass of the $\Delta(1238)$. Both the $K\pi\pi$ and $p\pi\pi$ mass plots (Fig. 1) are characterized by a large low-mass enhancement. In the $K\pi\pi$ case this enhancement is the familiar Q effect in the region $1100 \leq M(K\pi\pi) \leq 1400$ MeV, and is made up almost entirely of events with $K^-\pi^+$ forming $\bar{K}^{*0}(890)$ or with $\pi^+\pi^-$ in the ρ^0 band. The relative amount of the two vector mesons in the decay of the Q is difficult to estimate because of interference effects, however the rho contribution is small and this discussion will be confined to events in the K^* band.[9] We estimate the contribution from $K^{*-}(1420) \to \bar{K}^{*0}(890)\pi^-$ to be less than 10 events. The L-meson signal,

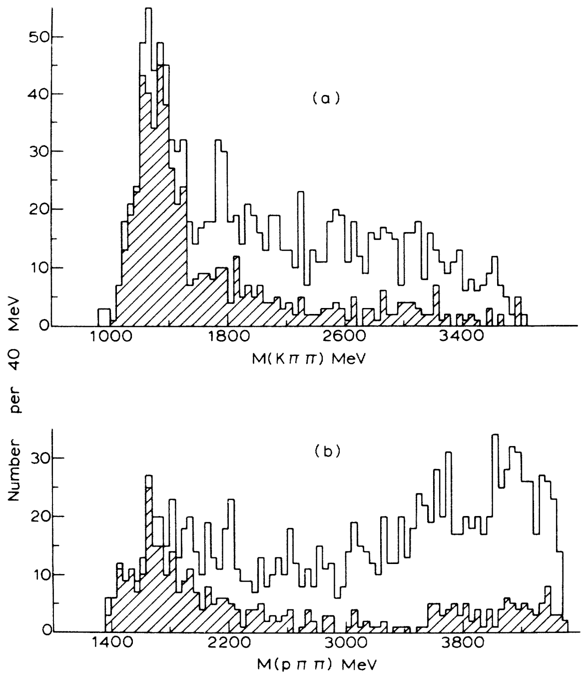

FIG. 1. (a) $K^-\pi^+\pi^-$ invariant-mass distribution. The shaded portion contains only K^* events. (b) $p\pi^+\pi^-$ invariant mass. The shaded portion contains events with $M(p\pi^+)$ in the $\Delta(1238)$ region, with a wide cut applied in this selection: $1120 \leq M(p\pi^+) \leq 1360$ MeV (see text).

which is clearly seen in the total $K\pi\pi$ mass spectrum, is suppressed when K^* events are selected.

We find, as have others,[2,3] that the production of Q is suggestive of diffraction scattering at the proton vertex (Pomeranchukon exchange). We do not see the effect in the reaction $K^-p \to \bar{K}^0 n\pi^+\pi^-$, which would require nonzero isospin exchange. The angle of the outgoing K^- in the decay of the K^*, measured with respect to the incident beam direction in the K^* rest frame, shows a $\cos^2\theta$ distribution indicative of total alignment of the K^* with $m=0$ along the beam direction. We observe no dropoff in the Q-production cross section when comparing our data with that of lower energy $K^\pm p$ experiments.[10] The distribution in the square of the four-momentum transfer (t) to the proton [Fig. 2(a)] for events in the Q peak has a form $\exp(bt)$ with $b = 11 \pm 2$ $(\mathrm{GeV}/c)^{-2}$. This value of the slope for small $K^*\pi$ mass is steeper than that for elastic K^-p and π^-p scattering at this energy.[11] The observation of steeper-than-elastic momentum-transfer distributions is reported in Refs. 5 and 7 and by Foley et al.,[12] (among others), and appears to be a signature of inelastic diffraction scattering processes.

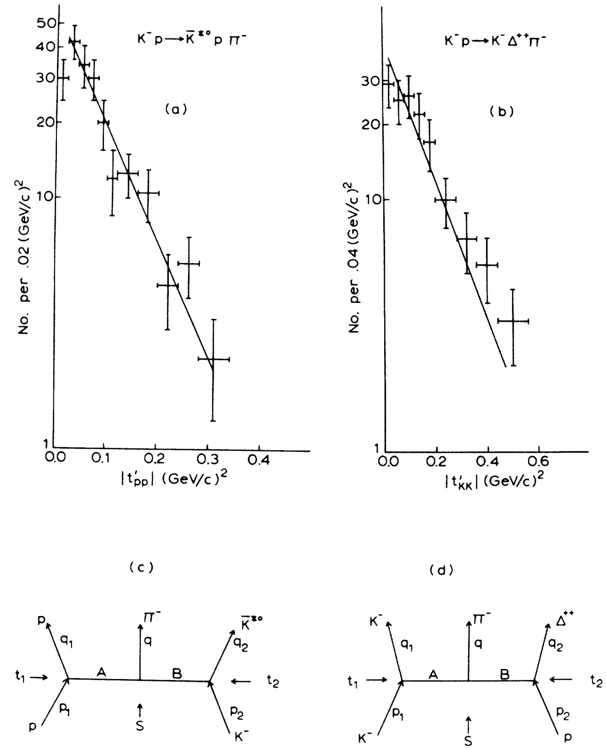

FIG. 2. (a) The distribution in $|t_{pp}'|$ (where $t' = t - t_{\mathrm{min}}$) for K^* events in the Q enhancement [$M(K\pi\pi) \leq 1400$ MeV]. t_{pp} is the square of the four-momentum transfer from the target to the outgoing proton. (b) The distribution in $|t_{kk}'|$ for shaded events in Fig. 1(b) with $p\pi\pi$ mass less than 2200 MeV. The solid curves in (a) and (b) are the result of the calculations described in the text, normalized to the number of events in each plot. (c), (d) The double-Regge-pole exchange diagrams appropriate to Reactions (1a) and (1b). A and B are exchanged trajectories; q_i and p_i are four-momenta.

No Q signal is observed in our data in the reaction $K^-p \to \bar{K}^0\pi^-p$. The absence of a $K\pi$ decay mode of the Q places it in the unnatural-parity sequence. A fit of the $K\pi\pi$ Dalitz plot for events in the Q-mass range in reactions $K^-p \to K^-p\pi^+\pi^-$ and $K^-p \to \bar{K}^0 p\pi^-\pi^0$ by a decay matrix element similar to that suggested by Chien et al.[2] gives acceptable results for $J^P = 1^+$ and 2^-, with 1^+ preferred.[13] We find no statistically convincing evidence for fine structure in the Q region of the $K\pi\pi$ mass plot; however, a comparison of high-energy K^-p data over a range of incident momenta indicates that such structure may be present.[3]

The low-mass $p\pi\pi$ enhancement, as shown in Fig. 1(b), is associated mainly with events having $p\pi^+$ mass in the region of the $\Delta^{++}(1238)$. The enhancement is broader and less symmetric than the Q, with a sharp spike centered near $M(p\pi\pi) = 1680$ MeV. This spike rapidly disappears as we narrow the cut on $p\pi^+$ mass in the selection of $\Delta^{++}(1238)$ events, leaving us to conclude that the 1680 peak consists primarily of uncorrelated $p\pi\pi$ events on top of a predominantly $\Delta\pi$ background.

Our data suggest a production mechanism for the $\Delta\pi$ part of the $p\pi\pi$ bump which is very similar to that of the $K^*\pi$ enhancement—in this case involving diffraction scattering at the kaon vertex. The enhancement is not present in the charge-exchange reaction $K^-p \to \bar{K}^0 p\pi^-\pi^0$. The distribution in square of the four-momentum transfer from the incident to the outgoing K in the enhanced mass region [Fig. 2(b)] is adequately fitted by a straight line on a semilog scale with a slope approximately equal to 5 $(\mathrm{GeV}/c)^{-2}$. This result is reasonable for the expected shape of the diffraction peak in a diagram such as that of Fig. 2(d) involving virtual $K^-\pi^-$ elastic scattering on the left-hand side.[14]

Among the many qualitative similarities between the $K^*\pi$ and $\Delta\pi$ enhancements is the extent to which each defies a clearcut understanding in terms of resonant production versus "kinematic effect,"[2-8] although the distinction between these interpretations may be removed by the recent suggestion of the duality of a Regge-pole exchange description of high-energy scattering in the t channel and resonance contributions in the direct (s) channel.[15] We have compared our data for both the $K^*\pi$ and $\Delta\pi$ enhancements with the same Reggeized double peripheral model which has given good agreement with the main features of the $\Delta\pi$ and A_1 enhancements in other experimental data.[1,4,6] The model is described in detail by Berger.[16] For Reaction (1a) we consider only diagram 2(c), and for Reaction (1b) we use diagram 2(d). In each case the exchanged trajectory A is taken to be that of the Pomeranchukon, and B is taken to be the pion trajectory. The squared matrix element is

$$|M|^2 = N_0 \frac{(\pi\alpha_\pi')^2}{2(1-\cos\pi\alpha_\pi)} [S_0^{-1}(S_2\cdots)]^{2\alpha_\pi} \times (S_1\cdots)^2 \exp(\beta t_1),$$

where $S_i = (q = q_i)^2$, $t_i = (q_i - p_i)^2$, $\alpha_\pi = (t_2 - m_\pi^2)\alpha_\pi'$ and the notation is otherwise the same as in Ref. 16. In both cases, the scale constant S_0 was fixed at 0.8 GeV^2, and α_π', the slope of the pion trajectory, was taken as 1.2 GeV^{-2}; these values were determined from previous fits.[1,6,16] As explained in Ref. 16, the damping factor β may be obtained directly from two-body forward elastic scattering data as the slope of the diffraction peak. Thus, for Reaction (1a), we used $\beta = 8.0$ $(\mathrm{GeV}/c)^{-2}$, derived from elastic πp data, and for the $\Delta\pi$ events we employed $\beta = 5.0$ $(\mathrm{GeV}/c)^{-2}$ appropriate to elastic πK scattering.[14] The normalization constants are,[14,16] for $(K^*\pi)$,

$$N_0 = g_{K^*K\pi}^2 (M_{K^*}^2 - 4M_\pi^2)\sigma_{\mathrm{tot},\pi p}^2,$$

and for $(\Delta\pi)$,

$$N_0 = g_\Delta^2 \sigma_{\mathrm{tot},\pi K}^2,$$

with $(g_{K^*K\pi}^2/4\pi) = 1.6$ and $g_\Delta^2 = 42.0$ GeV^2. Thus the matrix element in both cases is completely determined; there are no free parameters.

In keeping with the requirements of double-peripheralism for each reaction, we restricted $|t_1|$ and $|t_2|$ to values less than 1.0 $(\mathrm{GeV}/c)^2$. Because our study was limited to the diagrams shown in Figs. 2(c) and 2(d) with A the Pomeranchukon, the comparison was made both with and without a cut to restrict $M(\pi p)$, in the $K^*\pi$ case, and $M(\pi K)$, in the $\Delta\pi$ case, to large values where the Pomeranchukon should dominate. The agreement was equally good in both cases, and the essential features of both the model and the data were not greatly affected by this selection. Curves are shown here without this selection in order to take best advantage of the limited statistics.

Some results of the calculation are compared with the data in Figs. 2 and 3. All of the momentum-transfer distributions (not all are shown) are well described by the calculated curves, including the extra-steep diffraction slope associated with the Q region. (A detailed study of the diffraction slope as a function of $K^*\pi$ and $\Delta\pi$ mass over a more extended mass range was not possible due to the small number of events outside the enhanced mass regions. The predictions of the Regge model in this regard are discussed in Ref. 16: b is nearly twice elastic at $K\pi\pi$ threshold and decreases systematically with increasing $K\pi\pi$ mass. The data are in qualitative agreement with these predictions: The slope becomes significantly smaller with increasing mass.) Of particular interest are the distributions in the Treiman-Yang angles [Figs. 3(b) and 3(e)] which are very distorted in comparison with the isotropic prediction of elementary pion exchange, and in good agreement with the Regge-

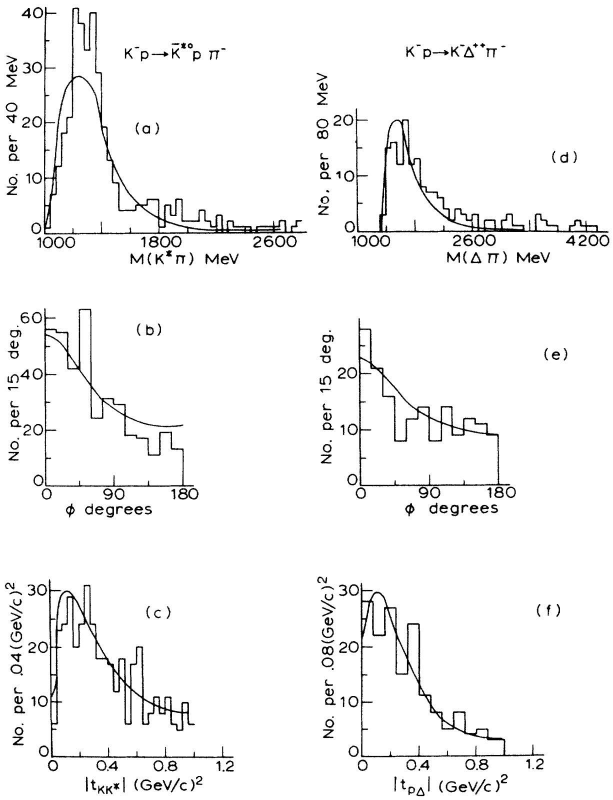

FIG. 3. (a) $K^*\pi$ mass distribution. (b) The Treiman-Yang angular distribution for Reaction (1a). (c) Square of the four-momentum transfer from the beam to the K^* in Reaction (1a). (d) $\Delta\pi$ mass distribution. (e) Treiman-Yang distribution for Reaction (1b). (f) Square of the four-momentum transfer from the target to the Δ in Reaction (1b). The smooth curves are the model calculations described in the text normalized to the data. For (a) and (d) the normalization is to the enhanced region only. The cut on Δ^{++} is $1160 \leq M(p\pi^+) \leq 1320$ MeV. Event selection is discussed in the text. The Treiman-Yang angles are

$$\cos\varphi = \frac{(\vec{p}_1\times\vec{q}_1)\cdot(\vec{p}_2\times\vec{q}_2)}{|\vec{p}_1\times\vec{q}_1||\vec{p}_2\times\vec{q}_2|}$$

evaluated in the frame in which $\vec{q}_1+\vec{q}=0$.

model results. For both reactions, the shape of the near-threshold mass enhancement is in good agreement with the model, although the predicted Q bump is somewhat too broad. The partial-wave analysis of the Regge-model amplitude concludes that 80% of the predicted cross section in the Q region is in an S-wave ($L=0$) $J^P=1^+$ state, consistent with the data. The model yields a total cross section in the Q region [specifically, here, $M(K^*\pi)\leq 1500$ MeV and $M(p\pi^-)\leq 2000$ MeV] of 89 μb which is 80% of that observed; for Reaction (1b), the model yields 59 μb in the region $M(\pi\Delta)\leq 2200$ MeV, 87% of that observed.

The reasonably good agreement with the shapes of the experimental distributions and the absolute normalization shows that the double-Regge model describes the $\Delta\pi$ enhancement seen here as well as that seen in the 28.5-GeV/c Brookhaven data[4] in a perfectly consistent manner. Increased statistics on this process would provide further tests of Pomeranchukon factorization which was used to derive the damping factors and normalizations. For the $K^*\pi$ events, the agreement in overall normalization may be improved by adding the contribution from a diagram with K^* and Pomeranchukon exchanges which contributes 20 to 40 μb in the Q region, depending on the value taken for $\sigma_{\mathrm{tot},K^*p}$. This calculation will be discussed in an expanded version of this work. We have shown that the Regge model describes the observed Q adequately, but stress that this does not vitiate a resonant interpretation of that effect; rather, we present the comparison in an attempt to further examine the duality concept of Ref. 15 in a quantitative way.

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[9]Assuming the Q decays only to $K^*(890)\pi$ or $K\rho$, a fit to the projections of the decay Dalitz plot, with no interference, requires 15 to 20% rho. For economy of writing we adopt the convention $K^* \equiv \bar{K}^{*0}(890)$, $\Delta \equiv \Delta^{++}(1238)$. The K^* band is defined as $800 \leq M(K^-\pi^+) \leq 1000$ MeV.
[10]The cross section for Q production in Reaction (1), obtained by taking all events with $K^-\pi^+\pi^-$ mass less than 1500 MeV after removing $\Delta^{++}(1238)$ background, is 0.19 ± 0.02 mb. The cross section for production of the $p\pi\pi$ enhancement, obtained by taking all events in the shaded portion of Fig. 1(b) with $p\pi\pi$ mass less than 2200 MeV, is 0.10 ± 0.02 mb.
[11]The apparent dip at low momentum transfer in Fig. 2(a) can be accounted for in terms of detection losses due to very slow protons which do not produce a visible track in the bubble chamber. For $|t_{pp}| \leq 0.06$ (GeV/c)^2 the detection efficiency varies with momentum transfer, and the second and third points in Fig. 2(a) have been corrected for this effect. The quoted error in the slope includes the uncertainty introduced by this correction.
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[14]Elastic πK data are not directly observable. However, by (1) assuming that high energy πp, Kp, and pp elastic scattering are dominated by Pomeranchukon exchange, and (2) employing factorization, one may determine that the slope of the πK elastic-diffraction peak is given by $\beta_{\pi K} = \beta_{Kp} + \beta_{\pi p} - \beta_{pp} \approx 5$ (GeV/c)$^{-2}$ and the total cross section is $\sigma_{\pi K} = \sigma_{\pi p}\sigma_{Kp}/\sigma_{pp} \approx 12.7$ mb.
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FINITE-ENERGY SUM RULE AND NONLINEARLY RISING REGGE TRAJECTORIES*

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Demanding the consistency of finite-energy sum rules at high energies, we predict that Regge trajectory $\alpha(s)$ increases at large energies as $(s \ln s)^{1/2}$ and the elastic widths of resonances go as $s^{\alpha_t(0)-\frac{1}{2}}/\ln s$ for large s.

Present experimental evidence tends to suggest that Regge trajectories do rise indefinitely; however, a subject of much controversy is how fast they rise as energy becomes high enough. Assuming polynomial boundedness for the scattering amplitude and making other standard assumptions about the analycity property of trajectory parameters, Khuri[1] has shown that as s increases, $\mathrm{Re}\,\alpha(s) \sim \sqrt{s}$ at best. His arguments have been recently refined by Childers.[1] Making a partial-wave projection of the Regge amplitude, Kugler[2] has made it quite plausible that such behavior really does exist. In spite of all these arguments, it is customary to assume a linearly rising trajectory rather than a nonlinear one and indeed all crossing-symmetric Regge-behaved amplitudes constructed recently by Veneziano[3] and others depend crucially on this assumption. It is, therefore, useful and indeed necessary to reexamine this problem more systematically. The present Letter attempts to give arguments, based on finite-energy sum rules[4] and the resonance saturation assumption, in support of nonlinearly rising trajectories. As a consequence of our analysis, we also predict the behavior of the residue function at large energies. The theoretical implication of such a result, especially in connection with behavior of resonance widths at high energies, has been discussed.

For simplicity we assume scattering of spin-0 particles like $\pi\pi \to \pi\pi$, and we choose an amplitude for which the crossed channel does not have the Pomeranchukon.[5] Also, for simplicity, we choose to work in a single-trajectory model. At the end, we will make some comments about the case in which there is more than one trajectory present. To derive our result, we first write down the zeroth-moment finite-energy sum rule[6]:

$$\int^N ds' \,\mathrm{Im}A(s',t) \simeq \sum_i \beta_i(t) N^{\alpha_i(t)+1}. \qquad (1)$$

It is obvious that, as N is increased, the finite-energy sum rules become more and more exact. So we go to the limit when N is very large. Then the finite-energy sum rules become consistency conditions on the Regge parameters in the s and t channels and some kind of a bootstrap relation