GRAPHICAL FORM OF DUALITY*

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We present a simple visual form of duality. It can be derived from SU(3) couplings for trajectories and the absence of resonances in most exotic channels, and has many physical implications.

The complementary description of reactions by Regge poles or resonances has come to be known as "duality." For inelastic processes most t-channel trajectories behave as if "built" of direct-channel resonances.[1] The identification of Pomeranchukon (P) exchange with background completes this picture.[2] If the t-channel exchange of a given SU(3) representation is to give no "exotic" s-channel contributions,[3] families of trajectories must obey severe constraints, which seem true in nature.[4] This suggests an underlying redundancy in the usual Regge descriptions.

In this note we show this to be so; adding duality to the usual Regge model suggests the following simple rule for seeing many of these constraints: (1) Represent mesons by $q\bar{q}$ and baryons by qqq. (2) Write all "connected" graphs as in Fig. 1. (3) A given graph will then exhibit duality among the channels in which it can be written in "planar" form, i.e., without quark lines crossing one another.

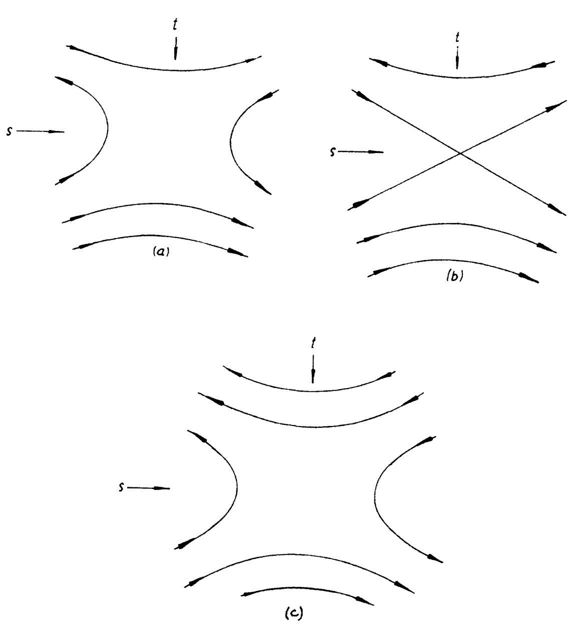

FIG. 1. Connected graphs for four-point functions. (a) Graph with an imaginary part at high s. (b) Graph with no imaginary part at high s. (c) Graph for baryon-antibaryon scattering with an imaginary part at high s.

Graphs for $AB \to CD$ will be "planar" in two channels. For example, Fig. 1(a) is "planar" in the s and t channels. It represents baryons in s and mesons in t. The imaginary parts corresponding to s-channel baryon intermediate states will thus "build" an imaginary part for high s if duality is valid. On the other hand Fig. 1(b) is "planar" in u and t. There are no s-channel resonances to build an imaginary part; so we expect such graphs to be purely real at high s.[2] We now verify these properties in the SU(3) Regge model.

We define a "non-P" amplitude[5] $\{ABCD\}$ for $AB \to \overline{D}\overline{C}$ with normalization such that

$$\sigma_T(AB)-\sigma_T(AB)|_{\text{Pom}} = \operatorname{Im}\{AB\bar{B}\bar{A}\}|_{t=0}(E_L/p_L\nu), \quad (1)$$

where $\nu \equiv \frac{1}{2}(s-u)$. For 0^- meson (M)–$\frac{1}{2}^+$ baryon (B) scattering, $\{ABCD\} \sim A'(\nu, t)$ and for $t=0$ BB or $B\bar{B}$ scattering it is related to the sum of the two helicity-nonflip t-channel amplitudes.

According to the usual Regge description[6]

$$\{ABCD\} \xrightarrow[\binom{\nu\to\infty}{\text{fixed } t}]{} \sum_E g_{DEA}(t) g_{C\tilde{E}B}(t) \times \frac{\pm 1-\exp[-i\pi\alpha_E(t)]}{\sin\pi\alpha_E(t)}\left(\frac{\nu}{\nu_0}\right)^{\alpha_E(t)}, \quad (2)$$

where $E = T\ (2^+)$ or $V\ (1^-)$, the tilde denotes charge conjugation, and the sign is $(-,+)$ for $E = (T, V)$. Assuming SU(3) and "ideal" nonet mixing, and using 3×3 matrices and angular brackets for their trace, we have[6]

$$g_{M_1TM_2} = \sqrt{2}\,\gamma_{MT}\langle M_1\{T, M_2\}_+\rangle, \quad (3)$$

$$g_{M_1VM_2} = \sqrt{2}\,\gamma_{MV}\langle M_1[V, M_2]_-\rangle, \quad (4)$$

$$g_{\bar{B}_1EB_2} = \sqrt{2}\,\gamma_{BE}\{[1-F_E]\langle\bar{B}_1\{E, B_2\}_+\rangle + F_E\langle\bar{B}_1[E, B_2]_-\rangle - (1-2F_E)\langle\bar{B}_1B_2\rangle\langle E\rangle\}. \quad (5)$$

(For $g_{C\tilde{E}B}$ we use $\tilde{T}=T^{\mathrm{T}}$, $V=-\tilde{V}^{\mathrm{T}}$, where the superscript T denotes transposition.)

The couplings (3)-(5) agree with experiment.[7] The absence of $\langle M_1M_2\rangle\langle T\rangle$ in (3) is supported by $f'\not\to\pi\pi$. The absence of $\langle M_1M_2\rangle\langle V\rangle$ in (4) comes from C invariance. The first term in (5) is "D" coupling, the second is "F," and the third decouples f' and φ from nucleons (as seems to hold).

These couplings imply that all graphs will be connected. Equations (3) and (4) clearly give connected vertices. In (5) we write B as

$$B_{ij}=2^{-1/2}\epsilon_{klj}([kl]i), \tag{6}$$

where repeated indices denote summation, and $([kl]i)$ represents an octet-baryon wave function antisymmetrized in its first two quarks. Similar expressions for $\tilde{B}$ and mesons are

$$\tilde{B}_{ij}=2^{-1/2}\epsilon_{kli}([\bar{k}\bar{l}]\bar{j}) \tag{7}$$

and

$$M_{ij}=(i\bar{j}); \tag{8}$$

so (5) becomes

$$g_{\overline{B}EB}=\sqrt{2}\,\gamma_{BE}(j\bar{k})\{([\bar{m}\bar{n}]\bar{j})([mn]k)+(2F_E-1)\{([\bar{m}\bar{j}]\bar{p})([mk]p)+([\bar{j}\bar{n}]\bar{p})([kn]p)\}\}. \tag{9}$$

Hence (5) entails connected vertices only; terms $(i\bar{i})([\bar{m}\bar{n}]\bar{j})([mn]j)$ cancel.

In the absence of $f'\to f$ and $\varphi\to\omega$ in the t-channel, four-point functions are then also "connected" for high s and fixed t.[8]

Now we impose duality by assuming that when $AB\to\overline{DC}$ lacks s-channel resonances, $\mathrm{Im}\{ABCD\}=0$ for high ν. In the context of the above model this requires[4] $\gamma_{MT}=\gamma_{MV}\equiv\gamma_M$, $\gamma_{BT}=\gamma_{BV}\equiv\gamma_B$, $F_T=F_V\equiv F$, $\alpha_{P'}=\alpha_{A_2}=\alpha_\omega=\alpha_\rho\equiv\alpha_0$, $\alpha_{K^*}=\alpha_{K^{**}}\equiv\alpha_1$, and $\alpha_{f'}=\alpha_\varphi\equiv\alpha_2$. With these constraints the $t=0$ data are still fitted fairly well when $F\sim1.5$ and $\alpha_0\sim\frac{1}{2}$,[9] and one can write $\{ABCD\}$ very compactly as

$$\{M_1M_2M_3M_4\}\xrightarrow[\substack{\nu\to\infty\\ \text{fixed }t}]{}4\gamma_M^{\,2}(k\bar{m})_2(m\bar{j})_3\{(j\bar{l})_4(l\bar{k})_1\{-\cot[\pi\alpha_{j\bar{k}}(t)]+i\}$$
$$+(j\bar{l})_1(l\bar{k})_4\{-\csc\pi\alpha_{j\bar{k}}(t)\}\}(\nu/\nu_0)^{\alpha_{j\bar{k}}(t)}, \tag{10}$$

$$\{M_1B_2\tilde{B}_3M_4\}\xrightarrow[\substack{\nu\to\infty\\ \text{fixed }t}]{}4\gamma_M\gamma_B\{([\bar{m}\bar{n}]\bar{j})_3([mn]k)_2+[2F-1]\{([\bar{m}\bar{j}]\bar{p})_3([mk]p)_2+([\bar{j}\bar{n}]\bar{p})_3([kn]p)_2\}\}$$
$$\times\{(j\bar{l})_4(l\bar{k})_1[-\cot\{\pi\alpha_{j\bar{k}}(t)\}+i]+(j\bar{l})_1(l\bar{k})_4[-\csc\pi\alpha_{j\bar{k}}(t)]\}(\nu/\nu_0)^{\alpha_{j\bar{k}}(t)}, \tag{11}$$

where $\alpha_{j\bar{k}}$ is α_0, α_1, or α_2 when none, one, or two of its subscripts are 3. Similar expressions hold for baryon-baryon and baryon-antibaryon scattering.

The coefficient of $-\cot[\pi\alpha]+i$ in (11) is shown in Fig. 1(a), while that of $-\csc\pi\alpha$ is shown in Fig. 1(b). The first is almost purely imaginary when $\alpha\sim\frac{1}{2}$, while the second is purely real. As we set out to prove, the lack of baryon intermediate states in (b) is reflected in the lack of an imaginary part for high s and fixed t.

Such graphs are therefore an excellent way to visualize duality, and have many interesting consequences.

(a) Absence of imaginary parts.—Equation (11) shows that, for example, $K^-p\to\pi\Sigma$ and $K^-p\to\pi\Lambda$ should be purely real at high s,[10] as only graphs of the form (b) can be involved. At lower energies the average contributions of s-channel resonances to $I=0$ and $I=1$ amplitudes must therefore vanish separately, implying more relations among coupling constants than one obtains from superconvergence alone.

(b) Exotic baryon-antibaryon systems.—An equation similar to (10) or (11) predicts that the imaginary part at high s in $B\overline{B}$ must arise from graphs of the form 1(c) if the conventional $(q\overline{q})$ trajectories dominate. The s-channel $B\overline{B}$ states are then made of $qq\overline{q}\overline{q}$, and so may be exotic (unless $F=\frac{1}{2}$, a very unreasonable value[4,11]). They may be resonances or annihilation states.[4,11] The latter may not preclude the former, as sums over intermediate states may very well produce Argand circles[12] in elastic partial waves.[13] The observation of exotic baryon-antibaryon resonances would certainly clarify this point, as they may well have been missed up to now.[14]

Figure 1(c) rotated by 90° predicts that saturation of $B\bar{B}$ finite-energy sum rules with conventional s-channel resonances will give exotic t-channel exchanges. Some evidence for this has been quoted in $\bar{p}p \to \bar{Y}_1{}^{*+}Y_1{}^{*-}$ at 3.25 GeV/c,[15] but the effect is not visible at 7.0 GeV/c[16] and is, in any case, much smaller than that associated with the conventional exchanges. The exchanges with conventional isospin and hypercharge built by $q\bar{q}$ s-channel resonances will then also have little effect at high s: They will also be made of $qq\bar{q}\bar{q}$ and may correspond to cuts, for example. Hence saturation of $B\bar{B}$ finite-energy sum rules by the conventional resonances is unlikely to "build" the important trajectories at high energy. The $qq\bar{q}\bar{q}$ states, be they resonances or annihilation states, are what one expects to matter most.[17]

(c) Branching ratios.—Equation (10) indicates that $\{K^-K^0\bar{K}^0K^+\}$ is pure f' and φ in the t channel, whereas $\{\pi^-\pi^0\pi^-\pi^0\}$ is pure P'.[18] The imaginary part of the former must then fall off faster with s than that of the latter, which can only happen if $\Gamma(M_J \to K\bar{K})/\Gamma(M_J \to \pi\pi)$ decreases as $s \to \infty$ for $I = 1$ mesons of spin J. Similar arguments apply to $N^* \to (K^+ \text{ or } K^0) + \text{hyperons}$.

(d) SU(3) for the Veneziano model.—We may generalize (10) to a crossing-symmetric form:

$$\{M_1M_2M_3M_4\} = \sum_{P(\alpha\beta\gamma)} 4\gamma_M{}^2 (j\bar{k})_1 (k\bar{l})_\alpha (l\bar{m})_\beta (m\bar{j})_\gamma f(\alpha_{j\bar{l}}(s_{1\alpha}), \alpha_{k\bar{m}}(s_{\alpha\beta})), \tag{12}$$

where the sum is over permutations of $\alpha, \beta, \gamma = 2, 3, 4$, and $s_{\alpha\beta} \equiv (p_\alpha + p_\beta)^2$, etc. $f(x, y) = f(y, x)$ has poles at x or y = integers and Regge behavior as $x \to \pm\infty$ for fixed y, but is otherwise arbitrary.

Equation (12) allows the introduction of SU(3) breaking in intercepts alone. For high s and fixed t the couplings still obey SU(3) and factorize as expected.[6,7]

A sample form for which $f(\alpha(s), \alpha(t))$ clearly "interpolates" between s- and t-channel configurations is the beta-function model,[19]

$$f(\alpha(s), \alpha(t)) = [1 - \alpha(s) - \alpha(t)] \int_0^1 dx\, x^{-\alpha(s)} (1-x)^{-\alpha(t)} \tag{13}$$

in which x looks like an internal coordinate. With (12) and (13) one can then write the complete meson-meson scattering amplitude in closed form.[18]

(e) Depression of $I = 0$ $^3P_0$ masses.—Figure 1(a), which contributes to $\mathrm{Im}\{ABCD\}$ at high ν, can be interpreted as a quark-antiquark annihilation into (and creation from) the vacuum, i.e., as an especially strong isosinglet $q\bar{q}$ force[17] in the $^3P_0$ state (which has $J^{PC} = 0^{++}$, the quantum numbers of the vacuum). The depressed masses of the $\sigma(750)$ and $S(1068)$ relative to those of an "ideal" nonet including the $\delta(965)$ and $\kappa(1080)$ might be indicative of such a force. We therefore suspect the S (1068) to be a genuine resonance rather than an enhanced scattering length, and await its confirmation. If the above force is the only spin-dependent SU(3)-breaking one for triplet states we expect the $^3P_1$ nonet to be "ideal" (as are the $^3P_2$ and $^3S_1$). Hence an $I = 0$ 1^{++} meson (whose decay modes would include 4π, $\pi\pi\eta$, and $K\bar{K}\pi$) should lie very close to the A_1 in mass.

If the graphs we draw mean something not just about SU(3) indices but actually about quarks, one may be able to learn more by coupling their spin indices in four-point functions in the same way we have coupled their SU(3) indices in (12). However, it is not clear how to do this in a relativistically invariant way.[20]

Our approach exhibits duality for production processes as well,[21] but the SU(3) couplings are harder to test. However, rule (3) above may provide useful estimates of production amplitude phases for use in multiperipheral bootstraps.[13] For example, if used in the unitarity relation, it predicts that some inelastic two-body amplitudes can have the same phase correlations of $\langle f | T^+ | n \rangle$ and $\langle n | T | i \rangle$ that one usually associates only with elastic processes, and therefore explains (qualitatively, at present) the possibility of P exchange in such reactions as $\pi N \to A_1 N$ and $NN \to N^*N$.

The graphical technique described here may be much more general than the SU(3) model used to derive it. It is certainly simpler.

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[2]H. Harari, Phys. Rev. Letters 20, 1395 (1968); F. Gilman, H. Harari, and Y. Zarmi, ibid. 21, 323 (1968).

[3]Except for baryon-antibaryon cases (to be discussed). "Exotic" will mean not having Y or I characteristic of $q\bar{q}$ or qqq.

[4]See, for example, J. Rosner, Phys. Rev. Letters 21, 950 (1968); M. Kugler, Phys. Rev. (to be published); H. Lipkin, to be published; C. Schmid, CERN Report No. CERN Th 960, 1968 (to be published); and Ref. 2.

[5]All particles are taken as incoming.

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[7]V. Barger, M. Olsson, and D. Reeder, Nucl. Phys. B5, 411 (1968); D. Reeder and K. V. L. Sarma, Phys. Rev. 172, 1566 (1968).

[8]The importance of "connectedness" was first noted for vertices by G. Zweig, CERN Report No. CERN Th 402, 1964 (to be published). We thus suggest "Zweig graphs" may be generalized to "tree graphs."

[9]One fails to fit the slow variation of $\sigma_T(pp)$ and $\sigma_T(pn)$ below 10 GeV/c and the apparent breaking of ρ-A_2 exchange degeneracy in $\pi^-p\to\pi^0n$ and $\pi^-p\to\eta n$, which are distinctly secondary effects. The low value of $F_V(\sim1.2)$ obtained by Reeder and Sarma (Ref. 7) has large errors (D. Reeder, private communication) and is sensitive to $t\neq0$ model-dependent assumptions.

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[11]D. P. Roy and M. Suzuki, CERN Report No. CERN Th 976, 1968 (to be published).

[12]C. Schmid, Phys. Rev. Letters 20, 689 (1968).

[13]Such behavior is expected in a multiperipheral bootstrap, for instance. See W. R. Frazer, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 419.

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[17]Cf. H. Lipkin, Phys. Rev. Letters 16, 1015 (1966), and Ref. 4.

[18]Cf. K. Kawarabayashi, S. Kitakado, and H. Yabuki, Phys. Letters 28B, 432 (1969).

[19]G. Veneziano, Nuovo Cimento 57A, 190 (1968); C. Lovelace, Phys. Letters 28B, 264 (1968).

[20]See, however, S. Mandelstam, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (unpublished).

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ERRATA

METHOD OF MEASURING THE BETA-DECAY COUPLING CONSTANT OF THE RHO MESON. Byron P. Roe [Phys. Rev. Letters 21, 1666 (1968)].

The equation for f_ρ^2 should read $4\times10^{-2}m_\rho^2M_p^2$. The calculated cross sections should be lowered accordingly. It would thus appear that there is little chance of observing diffraction production of ρ mesons by neutrinos at Brookhaven National Laboratory or CERN. However, possibly at Serpukhov and certainly at the National Accelerator Laboratory the data should be sufficient to examine this process. I wish to thank L. Stodolsky for calling my attention to this error.

DISCREPANCY BETWEEN THE VECTOR-DOMINANCE MODEL AND PION PRODUCTION BY POLARIZED PHOTONS. R. Diebold and J. A. Poirier [Phys. Rev. Letters 22, 255 (1969)].

The left-hand sides of Eqs. (8) should each contain an additional multiplicative factor of $\frac{1}{2}$. This in no way affects any of the figures, conclusions, or other equations.

MEASUREMENT OF PLASMA END LOSSES IN A Q MACHINE. R. W. Motley and D. L. Jassby [Phys. Rev. Letters 22, 333 (1969)].

In line 7 of the second column on page 333, $P_{\rm Cs}=0.74$ should be changed to $P_{\rm Cs}=0.074$. This latter value is that calculated from Eq. (3), and was the value used in the theory.

FIELD-THEORETICAL NUCLEON-NUCLEON POTENTIAL. M. H. Partovi and E. L. Lomon [Phys. Rev. Letters 22, 438 (1969)].

Replace the letter R, appearing in Eqs. (1)-(3) and on the line immediately preceding Eq. (5), by the letter k.