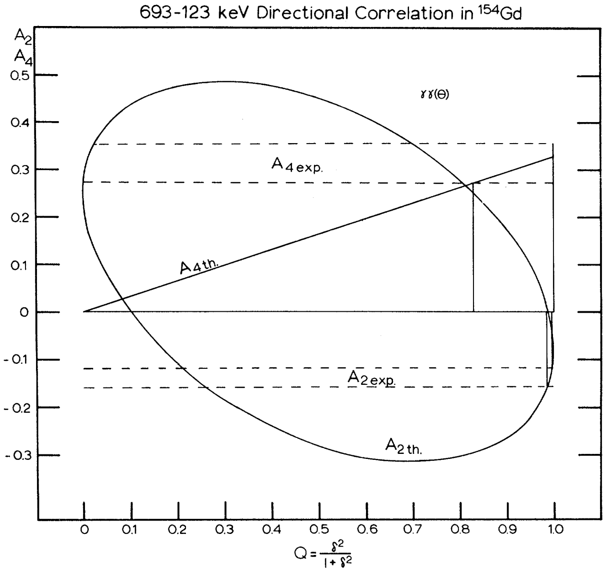

FIG. 1. Theoretical correlation coefficients A_2 and A_4 for a $2 \rightarrow 2 \rightarrow 0$ cascade. The dashed lines are the limits of the experimental values.

How one puts both the ^{154}Gd and ^{178}Hf results into a consistent scheme is an open question. It is possible, of course, that both experiments are right and that they reflect a difference in the softness of nuclear deformation in the two nuclei as ^{154}Gd lies at the beginning of this deformed region but ^{178}Hf more towards the center. In an effort to understand the possible sources of these discrepancies in Z_0, a mixing of the beta and gamma bands was tried[9] in ^{152}Gd and ^{154}Gd. While this improved the agreement in the gamma band, it did not help the consistency in the beta band. Further experiments are very important to clarify our understanding of these vibrational states where a new description may be needed.

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COULOMB DISTORTION EFFECTS IN LARGE-ANGLE $M1$ ELECTROEXCITATION*

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A reanalysis of inelastic electron-scattering measurements from giant $M1$ states in ^{28}Si and ^{12}C and in ^{26}Mg is made using Coulomb-distortion corrections in place of the plane-wave Born-approximation analysis. It is shown that the corrections even for light nuclei can reduce the nuclear form factors for $M1$ transitions outside the experimental error quoted in the plane-wave Born approximation analysis of electron scattering. This reduction in $B(M1,q)$ is magnified at the photon point.

Many experiments have been performed by groups at Stanford,[1] Darmstadt,[2] and the Naval Research Laboratory[3] using $100° \leq \theta \leq 180°$ electron scattering to pick out prominent magnetic-dipole transitions in light nuclei from ^{6}Li to ^{28}Si. Complementary measurements at Illinois[4] of radiative widths of some of these giant $M1$ states have been made by nuclear resonance fluorescence. The measured nuclear form factors, $B(M1,q)$, have been important theoretically[5] in understanding the role of spin-orbit coupling and particle-hole configurations in light nuclei as the $1p$, $2s$, and $1d$ shells are being filled and in determining the transition radius[6] where the magnetic-dipole transitions occur in nuclear matter. For spin-, isospin-flip transitions, $00 \rightarrow 11$, in self-conjugate nuclei ($A = 4N$), Kurath[5] has shown that the spin-orbit coupling term in the interaction Hamiltonian is dominant and that fragmentation of the $M1$ strength into several final states is unlikely. When expressed through Kurath's sum rule,

$$\sum_\nu (E_\nu - E_0) B(M1; 0 \rightarrow \nu)$$

$$\simeq -a(\mu_n - \mu_p + \tfrac{1}{2})^2 \langle 00 | \sum_k \vec{l}_k \cdot \vec{s}_k | 00 \rangle,$$

the ground-state configurations for $A = 4N$ may be determined from a single strong $M1$ transition with an accurate measurement of $B(M1)$ at the photon point, $q = k$. Careful analysis of nuclear resonance fluorescence measurements from giant $M1$ states indicates that in ^{12}C, ^{24}Mg, and ^{28}Si, Doppler broadening of the nuclear level, $\Delta \sim \Gamma$, prevents unambiguous determination of the ground-state radiation width Γ_0 which is the quantity of interest in the sum rule $[\Gamma_0 \propto B(M1, q = k)]$. One relies, therefore, on inelastic electron scattering from the same state as that which is excited by elastic photon scattering, to determine Γ_0 from determination of nuclear form factors versus momentum transfer near the photon point. This is the beauty of inelastic electron scattering as an electromagnetic probe of nuclear structure where the momentum-transfer four-vector, $q_\mu q^\mu = -\vec{q}^2 + k^2$, is not a null vector.

To date, the inelastic electron scattering results have been analyzed using a plane-wave Born-approximation calculation (PWBA) for one-photon exchange.[7] The most common analysis of the data uses a ratio of areas under peaks to determine the ratio of inelastic to elastic cross sections, both of which are calculated in PWBA. It is recognized that this reduction of the nuclear form factor is approximate and that distortion of the electrons by the nuclear Coulomb field in elastic scattering and by the nuclear transition currents for inelastic scattering must be considered. Partial-wave analysis of the Dirac equation for an extended charge distribution has been used for elastic electron scattering, $E_0 > 100$ MeV, in deducing the shapes and the parameters of the charge distribution in heavy nuclei[8] and recently in lower-energy elastic electron scattering, $E_0 \sim 50$ MeV, in determination of very accurate rms radii in light nuclei.[9] Inelastic electron scattering inducing electric transitions in medium and heavy nuclei, Ni to Bi, has been analyzed using the Duke distorted-wave Born-approximation calculation (DWBA).[10,11] Furthermore, Schucan[12] has investigated the separability of the nuclear form factor from the electron physics in scattering inducing nuclear transitions through an examination of a higher-order Born-approximation calculation. He finds that the transition nuclear form factor is separable and that the ratio $(d\sigma/d\Omega)_{\text{DWBA}}/(d\sigma/d\Omega)_{\text{PWBA}}$ is a meaningful Coulomb-distortion correction if it is independent of the nuclear model employed for the transition charge and current densities.

The purpose of this note is to show the effect of Coulomb distortion of the electron waves in magnetic dipole transitions induced by large-angle electron scattering from threshold energies to 60 MeV in light nuclei, $A < 30$. The Coulomb distortion is greater for $M1$ than $E2$ excitations because of the larger s-wave contribution in spin-flip transitions,[13] and the correction to the cross section using the DWBA reduces the nuclear form factor $B(M1, q)$ outside the experimental error usually quoted in reducing the measurements using the PWBA. This change in the nuclear form factors, in turn, has not negligible effects on both the determination of Γ_0 and the transition radius R_{tr} and will affect the nuclear-model calculations which attempt to fit $B(M1, q)$ vs q^2. We have used the Duke DWBA calculation of inelastic electron scattering. It has been generalized for magnetic-dipole transitions by Tuan, Wright, and Onley,[14] using an incompressible, irrotational liquid-drop model of the nucleus with $j_{L,L}(r) = d\rho_0/dr$, where ρ_0 is the ground-state charge distribution.

The results are shown in Fig. 1 for the ratio of the DWBA to PWBA cross section, versus atomic number for $E_0 \simeq 40$ MeV, $k \simeq 10$ MeV, and $\theta > 90°$. The insensitivity with angle $\theta > 90°$ at 40 MeV is, however, in general both energy- and atomic-number-dependent. For example, the distortion ratio in inelastic electron scattering from $^{12}_{6}$C, 15.1-MeV $M1$ state decreases from 1.15 at 30 MeV to 1.07 at 70 MeV approximately independent of angle, $\theta > 90°$. However, for $^{28}_{14}$Si, 11.42-MeV $M1$ state the distortion is 1.31 at 35 MeV for $90° \leq \theta \leq 180°$ but decreases from 1.14 at 90° to 1.07 at 150° for $E_0 = 60$ MeV. At 40 MeV one notes that the silicon results are angle independent. Figure 2 displays our reanalysis of the nuclear form factor for the electron scattering measurements of Liesem[2] for all 11.42-MeV giant $M1$ transition in $^{28}_{14}$Si excited by 33- to 56-MeV electrons with $104° < \theta < 165°$. The photon point, $q = k$, in Fig. 2 is from the nuclear resonance-fluorescence work of Kuehne, Axel, and Sutton,[4] with the assumption that $\Gamma_0/\Gamma = 1$. The Coulomb distortion decreases linearly, $d\sigma(\text{DWBA})/d\sigma(\text{PWBA}) = 1.33$ at 33 MeV to 1.12 at 55 MeV, for

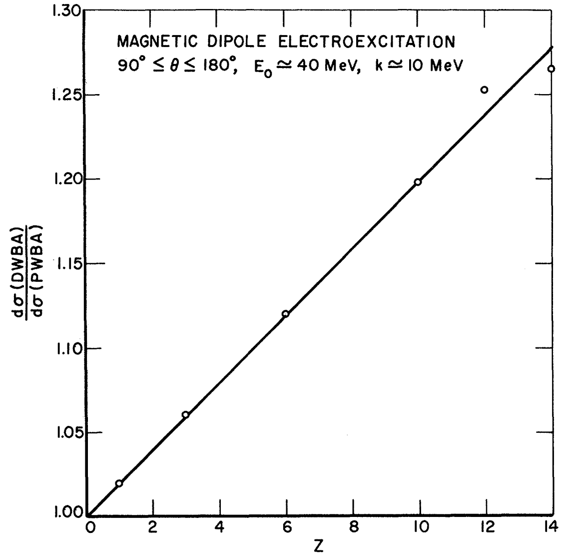

FIG. 1. The ratio of the Duke DWBA inelastic electron-scattering cross section to the PWBA result is linear with atomic number, $Z < 14$, for 10-MeV $M1$ transitions induced by 40-MeV electrons. The cross sections are computed for an extended charge distribution and are model independent. The distortion ratio for a particular Z at $\theta > 90°$ decreases with increasing electron energy.

this $0^+ \to 1^+$, 11.42-MeV transition in ${}^{28}_{14}\mathrm{Si}$. Since the original analysis used a ratio of peak areas proportional to the ratio $(d\sigma/d\Omega)_{\mathrm{inel}}/(d\sigma/d\Omega)_{\mathrm{el}}$, where the elastic cross section was also computed in the PWBA, we have used a partial-wave-analysis computer code[15] to determine $(d\sigma/d\Omega)_{\mathrm{el}}$. The deduced nuclear form factor depends on a ratio of an inelastic to elastic correction and the net Coulomb correction decreases from 1.2 at 33 MeV to 1.1 at 56 MeV dependent on backward angle. The improvement in the agreement between the electron and photon scattering data is evident. As seen in Fig. 2, the change in the ${}^{28}\mathrm{Si}$ nuclear form factors is significant near $q \to k$. One observes that $B(M1, q=0)$ has decreased by 26% so that $\Gamma_0 = 25.7 \pm 3.6$ eV vs 32.4 ± 4.5 eV from the PWBA analysis of the data. Further the transition radius is smaller, $R_{\mathrm{tr}} = 3.0 \pm 0.3$ fm vs 3.2 ± 0.3 fm.

It is clear that better data are needed near 30 MeV if electron scattering is to be used to determine Γ_0 to better than 3%. The same remarks on accuracy of the radiative width apply to the other giant $M1$ transitions in self-conjugate nuclei. The large decrease in Γ_0 from Coulomb effects during the 11.42-MeV $M1$ transitions is to be contrasted to a much smaller effect, a 5% decrease, for the 1.78-MeV $E2$ transition in ${}^{28}\mathrm{Si}$.[2] The radiative width from inelastic electron scattering, $\Gamma_0 = 25.7 \pm 3.6$ eV, is now in agreement with the nuclear resonance fluorescence value of 23 ± 4 eV and confirms the assumption that $\Gamma_0/\Gamma \simeq 1$. The comments of Kuehne, Axel, and Sutton on the ground-state configuration of ${}^{28}\mathrm{Si}$ from the value of $\langle 00|\sum_k \vec{1}_k \cdot \vec{s}_k|00\rangle$ still apply although it is worth pointing out that one can exhaust the entire sum rule for j-j coupling for a completed $d_{5/2}$ ground-state configuration, $\langle 00|\sum_k \vec{1}_k \cdot \vec{s}_k|00\rangle \simeq 8.1$, by inclusion of the weakly excited 10.91- and 12.33-MeV states as $M1$ transitions. High-resolution, large-angle, inelastic electron-scattering studies can determine this point and place constraints on allowable deformation in the ${}^{28}\mathrm{Si}$ ground state.

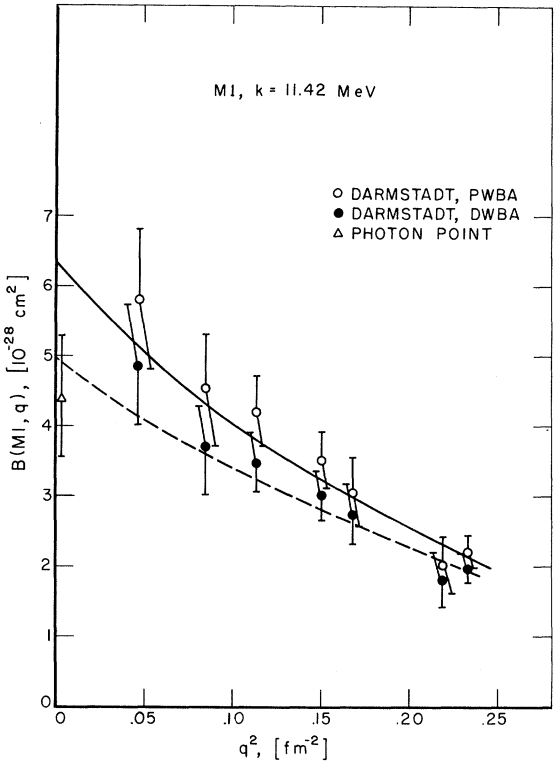

FIG. 2. The solid curve is from Liesem's measurements (Ref. 2) of inelastic electron scattering from ${}^{28}\mathrm{Si}$. Three measurements each near $q^2 = 0.16$ and 0.23 fm^{-2} constrain his best-fit curve. To be able to display both PWBA and DWBA points, only single data points at $q^2 \simeq 0.16$ and 0.23 fm^{-2} are shown. The DWBA data are our Coulomb-distortion corrections to the measurements of both inelastic and elastic cross sections. The photon point is from a different experiment (Ref. 4). The electron data are extrapolated to $q = k$ with pairs of data and $[B(M1,q)/B(M1,0)]^{1/2} = 1 - \frac{1}{5}(qR)^2 + (3/280)(qR')^4$, $R_{\mathrm{tr}} = R \simeq R'$.

When the same analysis is made of the electron-scattering measurements of the 15.1-MeV giant $M1$ in ^{12}C,[16-18,1] one gets $\Gamma_0 = 32.6 \pm 2.7$ eV in contrast to photon-scattering $\Gamma_0 = 37 \pm 5$ eV using $\Gamma_0/\Gamma = 0.96$. The inconsistencies among laboratories in the $^{12}\mathrm{C}(e,e')$ data at low momentum transfer for the 15.1-MeV excitation indicate the need for caution in using our Coulomb-corrected radiative width. In this context one notes that $\Gamma_0 = 32.6$ eV agrees well with the prediction of $\Gamma_0 \simeq 30.6$ to 33.6 eV using experimentally determined $1p$-shell wave functions.[19] Further, the statement of universality of electric charge through the conserved vector current theory of β decay uses the 15.1-MeV Γ_0 in predicting the shape-dependent factors in ^{12}B and ^{12}N decay to ^{12}C.[4,20] Using $\Gamma_0 = 32.6$ eV, we obtain $A = (0.85 \pm 0.17)\,\%$ from the conserved vector current prediction versus an experimental value of 1.07 ± 25 %.[20]

Finally we report on the analysis of the nuclear form factors in the recent 180° electron scattering from the 10.63-MeV $M1$ state in ^{26}Mg at electron energies of 39.0 and 56.0 MeV.[3] Since these measurements are not relative to the elastic peak, the Coulomb-distortion corrections apply only once to the inelastic electron cross section, and one obtains $\Gamma_0 = 6.80 \pm 1.5$ eV, $R_{\mathrm{tr}} = 3.25 \pm 0.13$ fm vs $9.10^{+2}_{-1.7}$ and 3.47 ± 0.14,[3] respectively, from the PWBA analysis.

It should be noted that the transition radius R_{tr} decreases since the Coulomb effects are larger on $B(M1,0)$ than $B(M1,q)$. This decrease in R_{tr} makes even greater the difference already noted between the radii for electric and magnetic transitions.[6,10] The $E0$, $E1$, $E2$, and $E3$ transition radii determined from inelastic electron scattering for light nuclei give $R_0 = R/A^{1/3} \simeq 1.6$ fm, while the magnetic-dipole-transition distribution is at a different location near the surface of the nucleus, $R_0 \simeq 1.1$ fm. The separate problem of magnetic-dipole electrodisintegration of the deuteron at threshold energies led us into the consideration of Coulomb-distortion effects. It is significant that the Coulomb-distortion effects are of the same order of magnitude as meson-exchange current effects at these electron energies. This will be discussed elsewhere. It is clear that in the analysis of electron scattering (even) for low-Z nuclei and particularly for $M1$ transitions, the nuclear physics can only be extracted when the electron dynamics is properly taken care of, which means abandoning the Born-approximation and using partial-wave analysis and/or the distorted-wave Born-approximation calculations.

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