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HIGH-ENERGY ELASTIC SCATTERING IN QUANTUM ELECTRODYNAMICS

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We have examined all two-body, elastic-scattering amplitudes in quantum electrodynamics. Aside from the photon-pole contribution, the amplitude for any elastic process is imaginary and proportional to s in the limit $s\to\infty$ with t held finite. The coefficients of s, to the lowest nonvanishing orders of e, can be expressed simply in terms of "impact factors." The results contradict both the Regge-pole model and the droplet model in their most straightforward interpretations.

High-energy scattering processes have been of central interest in both theoretical and experimental physics in the past decade. Among the existing theories, the Regge-pole model is beyond dispute the most explored. This model owes its existence to potential theory,[1] with its generalization to relativistic cases relying mainly on "intuition." Its recent difficulties, as for example summarized by Chan,[2] must be taken as a sign of possible dangers, and it is time to question whether too much faith has been exercised. The droplet model[3] is concerned mainly with diffraction scattering; it is also based primarily on intuition gained from studying potential scattering.

Instead of trusting the relevance of potential scattering in relativistic processes, we turned to the only relativistic theory which has produced both theoretical and experimental triumphs in the past, i.e., quantum electrodynamics. We have made a systematic study, at high energies, of all two-body elastic-scattering amplitudes in quantum electrodynamics.

Consider first the electron Compton-scattering amplitude. The second-order diagrams are illustrated in the fourth row of the first column in

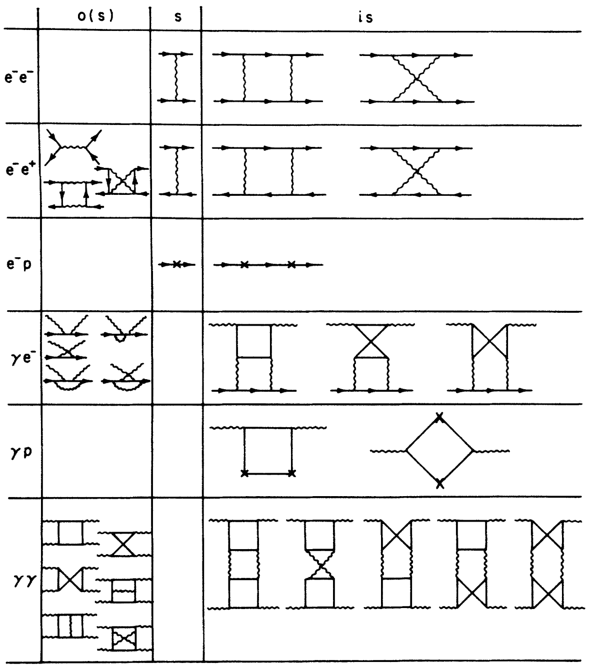

FIG. 1. Feynman-Dyson diagrams for six of the two-body, elastic-scattering processes in quantum electrodynamics. (In the first column some of the diagrams which do not contribute to s are given; in the second column the lowest order diagrams for each of the processes which contribute to the real coefficient of s are illustrated; in the third column the lowest order diagrams for each of the processes which contribute to the imaginary coefficient of s are illustrated. The s channel is from left to right and the t channel is from bottom to top.)

Fig. 1. The corresponding amplitude approaches a constant in the limit $s\rightarrow\infty$ with t held finite, where s is the square of the center-of-mass system energy and $-t$ the square of the momentum transfer. The second-order amplitude therefore contributes zero amount to the differential cross section $d\sigma/dt$ for the electron Compton-scattering process in this limit. Similarly, the fourth-order diagrams illustrated in the same place of Fig. 1 do not contribute to

$$\lim_{s\rightarrow\infty} d\sigma/dt.$$

The lowest order diagrams which contribute to

$$\lim_{s\rightarrow\infty} d\sigma/dt$$

are of sixth order and are illustrated in the fourth row of the third column in Fig. 1. At this point we note that if the external electron is replaced by a proton or a nucleus, the process becomes Delbrück scattering as illustrated in the fifth row of Fig. 1.

We have studied all the Feynman-Dyson diagrams shown in Fig. 1. Most of them fail to contribute to

$$\lim_{s\rightarrow\infty} d\sigma/dt,$$

such as the second-order and fourth-order Compton diagrams just mentioned. Examples of these negligible diagrams are shown in the first column of Fig. 1. In the second column of Fig. 1 three diagrams involving a photon exchange are shown. Their amplitudes contribute to the real part of the matrix element and are easily understood to be the photon-pole terms. The most interesting ones are the remaining 15 diagrams, which are shown in the third column of Fig. 1 and contribute to the absorptive cross section at high energies, i.e., their contributions to the matrix elements are imaginary and proportional to s at extremely high energies.

The last statement requires clarification. The matrix element obtained from the last diagram in the first row, for example, must be real. In fact, in the Feynman gauge, this matrix element is proportional to $s\ln s$ at high energies. This contribution, however, is completely cancelled by the other diagram in the first row of the third column. The sum of their contributions is of the form is at high energies. It is imperative to keep together diagrams related by gauge transform.

We find, after 16 months and more than 2000 pages of calculation, that the results from these 15 diagrams can be summarized as follows. (Details of this calculation are to be published elsewhere.) The matrix element for the forward elastic scattering $a+b\rightarrow a+b$ is, at high energies,

$$\mathfrak{M}_0^{(ab)}\sim i(2r_2r_3)(2\pi)^{-2}\int d^2\vec{q}_\perp[(\vec{q}_\perp+\vec{r}_1)^2+\lambda^2]^{-1}[(\vec{q}_\perp-\vec{r}_1)^2+\lambda^2]^{-1}\mathcal{I}^a(\vec{r}_1,\vec{q}_\perp)\mathcal{I}^b(\vec{r}_1,\vec{q}_\perp). \qquad (1)$$

In (1), $2\vec{r}_1$ is the momentum transfer, r_2 and r_3 are the averages of the incoming and outgoing four-momenta of particles a and b, respectively, λ is a photon mass introduced to avoid infrared divergences in some cases, and $\mathcal{I}^a$ and $\mathcal{I}^b$ are the "impact factors" for particles a and b. Here a and b can each be interpreted as an electron, a positron, a photon, or a nucleus in the sense of a static Coulomb field. The four impact factors are

$2\pi\alpha\delta_{12}/m$ for both electron and positron, $2\pi Z^2\alpha/M$ for a nucleus,

and

$$8\alpha^2\int_0^1 d\beta\, d\beta'\delta(1-\beta-\beta')\int_0^1 dx\left\{\frac{8\beta^3\beta' x(1-x)r_{1i}r_{ij}-\beta^2\vec{r}_1^{\,2}\delta_{ij}[1-8\beta\beta'(x-\frac{1}{2})^2]}{4\vec{r}_1^{\,2}\beta^2x(1-x)+m^2}-\frac{8\beta\beta' x(1-x)Q_iQ_j-\vec{Q}^2\delta_{ij}[1-8\beta\beta'(x-\frac{1}{2})^2]}{4\vec{Q}^2x(1-x)+m^2}\right\} \tag{2}$$

for a photon. In (2), m is the electron mass, M is the nuclear mass, Ze is nuclear charge, α is the fine-structure constant, i and j are the polarization indices for the incoming and outgoing photons, and Q is given by $\vec{Q}=\frac{1}{2}(\vec{q}_\perp+\vec{r}_1)-\beta\vec{r}_1$. Note that the impact factor of a particle is equal to that of its antiparticle.

As an application of (1), let us consider the Delbrück scattering process. To obtain the Delbrück scattering amplitude from (1), we simply substitute for g^a and g^b in (1) the photon and the nuclear impact factors given by (2). In the limits $|t|\gg m^2$ and $|t|\ll m^2$, the resulting expression takes simple asymptotic form. Our asymptotic form in the limit $|t|\ll m^2$ disagrees with that of Bethe and Rohrlich.[4] We believe that this is due to the failure of the method of impact-parameter approximation used by these authors. In particular, we find that, contrary to the results of Bethe and Rohrlich,[4] the scattered photon is highly polarized, over 50% for momentum transfers exceeding 2.3 MeV.

In the Regge-pole model, the elastic-scattering amplitude is dominated by the Pomeranchukon[5] which is a moving Regge pole with factorized residues.[6] Our result (1) demonstrates that the leading term in the high-energy elastic-scattering amplitude is not factorized, the factorized quantities being the impact factors which are functions of the variable of integration $\vec{q}_\perp$. We believe that, in the language of Regge poles, (1) can be interpreted as a branch cut with factorized discontinuity functions. However, it is physically more transparent to describe the amplitudes in terms of impact factors.

Applying the droplet model to the various processes considered here, we find that the high-energy amplitudes in this model can also be cast in the form (1). However, the impact factor $g^\gamma(\vec{r}_1,\vec{q}_\perp)$ in the droplet model is in the form $D(\vec{r}_1)-D(\vec{q}_\perp)$. The form factor $g^\gamma(r_1,q_\perp)$ as given by (2) cannot be equated to this form, even approximately, except in the nonrelativistic limit $m^2\gg\vec{r}_1^{\,2}$, $m^2\gg\vec{q}_\perp^{\,2}$.

The disagreement with both the Regge-pole model and the droplet model in their most straightforward interpretations is probably to be expected. Regge poles are obtained from the ladder diagrams in the t channel, while the ladder diagrams in the s channel are included in the droplet model. For the processes of Delbrück scattering, electron Compton scattering, and photon-photon scattering, the dominating diagrams are not all ladder diagrams. In fact, most of them are even nonplanar. Furthermore, vacuum polarization figures prominently in many of the diagrams. All these effects are foreign to potential scattering but have now been proven to be essential to high-energy relativistic scattering. It is precisely the diagrams with these new features which lead to disagreements with the existing theories.

The present consideration does not depend in any way on the massless nature of the photon. In fact, the photon is given a mass λ in Eq. (1). It is therefore difficult to imagine that either the Regge-pole theory or the droplet model in their simplest form can hold for hadron interactions but not for quantum electrodynamics. In other words, it is totally inconceivable that hadron physics is closer to potential theory than is quantum electrodynamics.

In retrospect we observe that all of the diagrams in the third column of Fig. 1 involve two photon exchanges. One may raise the question whether our results still apply for diagrams of higher order, where more than two photons can be exchanged. A case of practical interests is the Delbrück scattering process, where $Z\alpha$ (Ze is the charge of the nucleus) may not be considered small. In this case, one has to include all of the diagrams where an arbitrary number of photons is exchanged between the nucleus and the created pair. We have summed up, for high energies, the contribution from all such diagrams and the Delbrück amplitude $\mathfrak{M}^{(D)}$ is obtained to be

$$\mathfrak{M}^{(D)} \sim i(2r_2r_3)(2\pi)^{-2}\int d^2\vec{q}_\perp[(\vec{q}_\perp+\vec{r}_1)^2]^{-1+i\alpha Z}[(\vec{q}_\perp-r_1)^2]^{-1-i\alpha Z}\mathfrak{g}^\gamma(\vec{r}_1,\vec{q}_\perp)\mathfrak{g}^N, \tag{3}$$

where $\mathfrak{g}^\gamma(\vec{r}_1,\vec{q}_\perp)$ and $\mathfrak{g}^N$ are precisely the photon and the nuclear impact factors given by (2). A comparison of (1) and (3) shows that the only effect of multiphoton exchange is the modification of the two propagators. Thus the impact factor appears to express an intrinsic property of a particle.

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PHOTOPRODUCTION OF VECTOR MESONS AND OTHER RESONANCES AT 4.3 BeV

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The Stanford Linear Accelerator Center 40-in. hydrogen bubble chamber was exposed to 4.3-BeV positron-annihilation radiation. Results are presented on reactions containing no neutrals and one neutral particle in the final state. For $\gamma p \rightarrow \rho^0 p$ and $\omega^0 p$ we find cross sections of 19.2 ± 1.2 and 2.8 ± 0.5 μb, respectively. Assuming vector dominance, we determine the V^0p cross sections. Evidence for other resonant states is presented, in particular $\Delta^{++}V^0\pi^-$, where we find $\Delta^{++}\rho^0\pi^-/\Delta^{++}\omega^0\pi^-$ about 1.5.

Previous studies of the photoproduction of multibody final states in bubble chambers were done by using a bremsstrahlung beam.[1,2] In these studies no cross sections could be given for reactions having a neutral particle in the final state, except for some special cases such as ω^0 production. Recently, in order to overcome this difficulty, an e^+-annihilation photon beam was constructed at the Stanford Linear Accelerator Center (SLAC) by the SLAC Group.[3] Some results of the SLAC Group at photon energies of 5.2 and 7.5 BeV were already published.[3] In the present experiment we report on cross sections and resonance production at 4.3 BeV. The SLAC 40-in. hydrogen bubble chamber was exposed to a 4.3-BeV photon beam, obtained by the annihilation of an 8.5-BeV/c (±0.5 %) e^+ beam in a liquid H_2 target. The beam details are given in Ref. 3. A total of 400 000 pictures were taken and the present results are based upon the analysis of about half the pictures. 5200 nuclear interactions containing three or more prongs (or one prong plus a V^0) were found in the above sample. We estimate that about one-half of all our observed nuclear events are due to the monochromatic photons resulting from the e^+ annihilation. Since the energy of the monochromatic photons was known rather well (to within ±2.5 %), the kinematical analysis was similar to the usual one used for charged particle beams.