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EXPERIMENTAL TEST OF QUANTUM ELECTRODYNAMICS BY MUON BREMSSTRAHLUNG*

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We have measured muon bremsstrahlung of 9- to 13-GeV/c muons by using a carbon-plate spark chamber as a target. Our results in the region of invariant four-momenta of the recoil muon-photon system 200-650 MeV/c are consistent with the predictions of quantum electrodynamics.

We have measured muon bremsstrahlung in an experiment designed to observe the processes by which a 9- to 13-GeV muon loses most of its energy in a carbon target. In the previous Letter,[1] results of a total inelastic cross section were reported, along with a description of the apparatus and of the muon beam. In this paper, we will discuss the analysis of the bremsstrahlung events and compare the results with the predictions of quantum electrodynamics (QED).

A total of 70 000 pictures were taken (61% with positive beam polarity and 39% with negative polarity). With all of the data analyzed, we have measured 729 events which satisfied the following criteria: (1) No charged secondary prongs longer than three sparks were in the target spark chamber. (2) There was only one shower in the lead-plate spark chambers and it had a clearly defined first spark. (3) The event vertex was in the region of the carbon plates, to within the errors of the reconstruction. (4) The extrapolations of the track of the scattered muon, as measured in the aluminum-foil section of the target chamber and in the aluminum chamber following the large-aperture analyzing magnet, agreed within limits in both the side and top views (the limits were momentum dependent to allow for multiple scattering). (5) The track measured in the aluminum chambers after the magnet, on extrapolation, intersected the tagging counters, $F(AB)(CD)$, which participated in the trigger. Allowance was made for the multiple scattering in the 1-m iron absorber which became severe at low recoil momenta due to the 1.3-GeV energy loss there. (6) The scattered muon momentum was between 1.5 and 6.0 GeV/c. (7) The tracks of the incident muon in the beam chambers, upstream of the target chamber, could be reconstructed to give a consistent trajectory and a momentum in the region 8.60 to 12.9 GeV/c. (8) The transverse momentum of the scattering muon-photon system q_t was less than 150 MeV/c.

To compute the theoretical cross section, we took the expression for the square of the QED matrix element from Drell and Walecka[2] and inserted the harmonic-well form for the elastic carbon form factor; this formula includes all effects of nuclear recoil and the muon mass. We chose the following as the four free variables: the muon polar angle, the photon polar angle (both relative to the incident muon direction), the scattered muon energy, and the invariant momentum, $Q=|P_\mu'+K|$, of the scattered muon and photon system. Q/c is the invariant mass of the propagating virtual muon in the timelike bremsstrahlung diagram [see Fig. 1(a)]. To facilitate the comparison with pair-production experiments, a symmetric lepton pair with an invariant pair

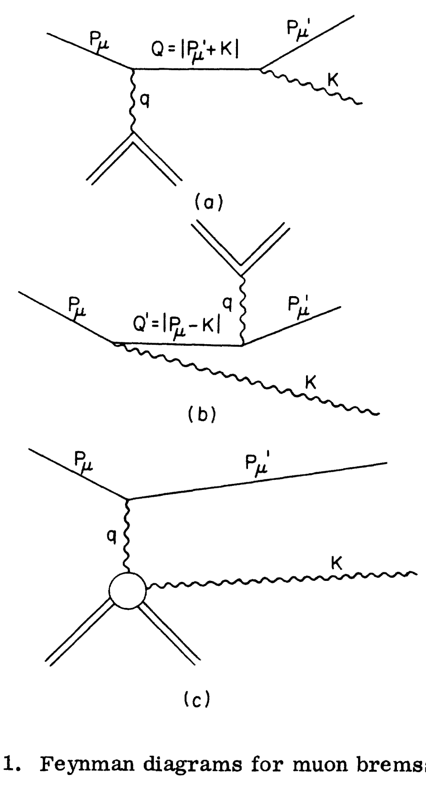

FIG. 1. Feynman diagrams for muon bremsstrahlung.

mass of 900 MeV/c^2 has a Q/c of $900/\sqrt{2}\approx 650$ MeV/c^2. In the experimental configuration, we triggered on all values of Q from 200 to 750 MeV/c. To compare our results with the theory in given regions of this variable, it was necessary to integrate the theoretical cross section over the phase space available for detection. The limits on the angular integrations were set by imposing a cut on the transverse momentum of the scattered muon-photon system; in effect, we limited the integrations to regions where the cross section was large since the cross section varies approximately as $1/q_t^2$ for q_t larger than a few MeV/c. These angular limits were dependent on the incident-beam energy which was randomly picked from the experimental beam-momentum distribution. The geometric detection efficiency as a function of the final momenta and polar angles of the scattered muon was included in the calculation. This efficiency was obtained by a Monte Carlo technique.[1] The overall accuracy of this theoretical cross section is estimated to be 3%, including possible errors in the detection efficiency.

We chose 150 MeV/c as the maximum allowable transverse momentum. The inelastic bremsstrahlung contribution to the cross section then is less than 2%, which we estimated by calculating the inelastic form factor[3] and assuming the elastic bremsstrahlung momentum-transfer dependence. Radiative corrections[4] and second Born terms are also of this order, and we have neglected them. A negligible amount of elastic bremsstrahlung has transverse momentum larger than 150 MeV/c. The difference between the positive- and negative-beam momentum distributions result in a calculated negative-muon bremsstrahlung cross section 4% larger than the positive.

The interference between the Bethe-Heitler [Figs. 1(a) and 1(b)] and Compton terms [Fig. 1(c)] vanishes when equal amounts of positive- and negative-muon bremsstrahlung are added. We have combined all our data, which have a 6:4 ratio of positive to negative events, and we feel that whatever interference effects remain are small enough to neglect.

To compare the expected total number of events with the experimental number, we applied several corrections to the yield predicted by QED. We multiplied the theoretical number by factors to account both for extra bremsstrahlung events from additional material in the beam (1.18 ± 0.05) and for the loss of events due to the following:

scanning efficiency (0.93 ± 0.03), measurement and reconstruction efficiency (0.74 ± 0.05), conversion of photons before the third plate of the first spark chamber after the analyzing magnet (0.70 ± 0.05), poor extrapolation of the rear track to the tagging counters (0.97 ± 0.02), and the presence of a knock-on electron in the target chamber (0.97 ± 0.02). The predicted number of pictures is proportional to the useful flux and our calculation of this number is dependent upon the reliability of the measurement and reconstruction of the beam chambers; the estimated uncertainty is 5 %. We estimate the overall accuracy of this calculation of the predicted number of events to be 11 %. The ratio of measured events to predicted events is 0.91 ± 0.12 for both beam polarities, where the error includes statistics.

In Fig. 2(a), we plot the expected and observed number of events as a function of Q; Fig. 2(b) shows the comparison versus the scattered muon energy, and Fig. 2(c), the comparison versus the angle between the scattered muon and the photon.

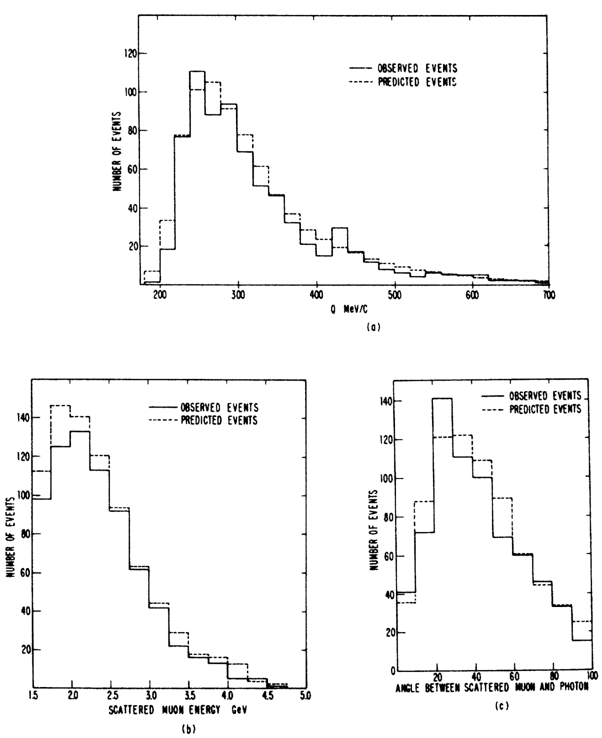

FIG. 2. (a) Observed (solid line) and predicted number (dashed line) of events for the values of Q seen in this experiment. The predicted numbers have an uncertainty of 11%. (b) Same comparison in 0.25-GeV intervals of final muon energy. (c) Same comparison in 10-mrad intervals of the angle between the photon and the scattered muon.

In Fig. 3, we present the ratio R of experiment to theory from our results, with data from adjacent Q bins combined to improve the statistical precision; also plotted are the results of the Cornell electron-bremsstrahlung experiment.[5]

A deviation of the ratio R from unity could be explained by the following:

(1) A breakdown in QED, which, to be consistent with general considerations, might be of the form[6]

$$R = A(1.0 \pm Q^4/\Lambda^4).$$

We find a fit to this from our results:

$$R = 0.93[(1.0 \pm 0.045) - (0.69 \pm 1.37)10^{-12}(\mathrm{MeV}/c)^{-4}Q^4].$$

This corresponds to a Λ of 1.1 GeV/c; within 95% confidence limits, $\Lambda > 0.73$ GeV/c. Only statistical errors have been included in making this fit.

(2) A process which distinguishes muons from electrons. Because Q is timelike, bremsstrahlung is sensitive to the existence of a heavy muon, with mass $Q/c = M^+ = |P_\mu' + K|/c$, which decays into a muon and a photon. We do not have any statistically significant evidence for a heavy muon with mass less than 600 MeV/c^2. Our experiment is not as sensitive as the muon $g-2$ experiment[7] to a heavy lepton with a magnetic moment coupling, as proposed by Low,[8] because ours is at small momentum transfers and because our detection efficiency is good only for low-energy recoil muons.

In conclusion, the results of this experiment agree with the predictions of QED. Also, we find no evidence for a heavy muon.

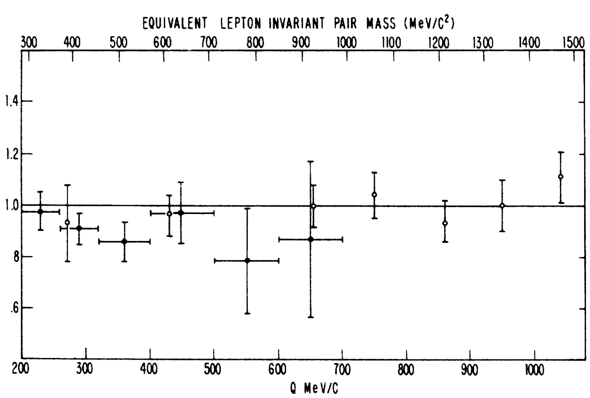

FIG. 3. The ratio R of our experiment (solid dots) to theory as a function of Q, together with the electron bremsstrahlung results of Ref. 5 (open dots). The 11% normalization uncertainty is not shown.

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HIGH-ENERGY ELASTIC SCATTERING IN QUANTUM ELECTRODYNAMICS

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We have examined all two-body, elastic-scattering amplitudes in quantum electrodynamics. Aside from the photon-pole contribution, the amplitude for any elastic process is imaginary and proportional to s in the limit $s \to \infty$ with t held finite. The coefficients of s, to the lowest nonvanishing orders of e, can be expressed simply in terms of "impact factors." The results contradict both the Regge-pole model and the droplet model in their most straightforward interpretations.

High-energy scattering processes have been of central interest in both theoretical and experimental physics in the past decade. Among the existing theories, the Regge-pole model is beyond dispute the most explored. This model owes its existence to potential theory,[1] with its generalization to relativistic cases relying mainly on "intuition." Its recent difficulties, as for example summarized by Chan,[2] must be taken as a sign of possible dangers, and it is time to question whether too much faith has been exercised. The droplet model[3] is concerned mainly with diffraction scattering; it is also based primarily on intuition gained from studying potential scattering.

Instead of trusting the relevance of potential scattering in relativistic processes, we turned to the only relativistic theory which has produced both theoretical and experimental triumphs in the past, i.e., quantum electrodynamics. We have made a systematic study, at high energies, of all two-body elastic-scattering amplitudes in quantum electrodynamics.

Consider first the electron Compton-scattering amplitude. The second-order diagrams are illustrated in the fourth row of the first column in