HIGH-ENERGY MUON INELASTIC SCATTERING*

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Muon inelastic scattering has been measured using muons with momenta between 8.6 and 12.9 GeV/c striking a carbon target. The virtual-photoproduction cross section is presented as a function of the four-momentum transfer from the muon.

An experiment to measure muon inelastic scattering and muon bremsstrahlung has recently been completed at the Brookhaven National Laboratory alternating-gradient synchrotron. The data for both processes were taken simultaneously with the same beam and apparatus because both occur with roughly equal rates when a muon scatters in carbon with appreciable energy loss. The inelastic scattering is described here and the bremsstrahlung is discussed in the following Letter.[1] Inelastic muon scattering is also being measured at the Stanford Linear Accelerator Center and preliminary results have been reported.[2]

A plan view of the experimental apparatus is shown in Fig. 1. The muon beam was produced by the decay of a high-intensity pion beam.[3] The muons entered the experimental area through a 24-ft-long carbon-filled collimator which served to attenuate the remaining pions. There were 2×10^4 μ^+ per pulse or 10^4 μ^- per pulse with momenta between 8.6 and 12.9 GeV/c and 10^{-4} pion per muon. The beam was defined by the beam counters $S1$, $S2$, and $S3$ in anticoincidence with counter AS, which had a round hole. The angular divergence of the beam was ±20 mrad. Muons with momenta below 8.6 GeV/c were suppressed by requiring a sufficiently high pulse from a momentum-measuring hodoscope system,[4] HA, HB, and HC. The momentum of the triggering muon was ultimately measured to ±2 % using four aluminum-plate spark chambers which flanked magnet $D6$. The carbon target consisted of the first 21 plates of a spark chamber, each $\frac{1}{2}$ in. thick; the second half of this chamber had aluminum-foil plates. The muons then passed through a large-aperture magnet and five spark chambers. The first two of these chambers each had five aluminum plates. This arrangement permitted detection and momentum analysis of recoil muons produced at angles up to 12° with final momenta in the region 1.5-6.0 GeV/c. The final three chambers each had eight plates of lead and a ninth plate of aluminum. The scattered muon was detected in the F bank which had six scintillation counters. After it passed through a 40-in. iron pion filter, the muon was detected by a crossed array of counter banks, A, B, C, and D. A trigger required at least one count in F in coincidence with at least one count from A or B and at least one count in C or D. Three counters centered on the unscattered beam line served to veto unscattered muons. The 54 rear counters were connected to gate circuits which flashed lights that were photographed by one of the cameras, thus recording which of the rear counters actually participated in the coincidence. Each event consisted of four photographs: one of the beam-momentum chambers, one of the target chamber, and two of the rear chambers. The experimental detection efficiency for various final energies and physical scattering angles was calculated by a Monte Carlo program in which trajectories were traced through the apparatus.

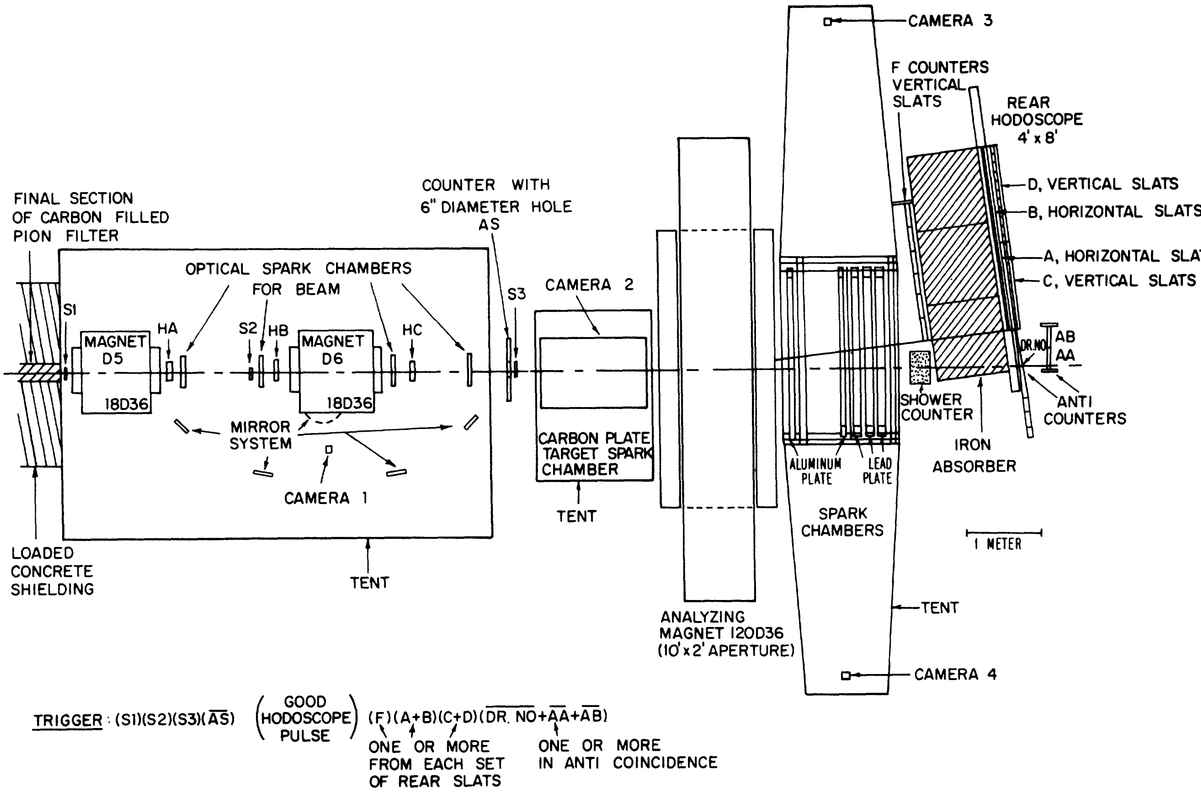

FIG. 1. Experimental arrangement.

We report here on the data from two μ^+ runs: one with the beam as described and one with 101.2 g/cm^2 (2 ft) of the carbon in the collimator removed. The pion contamination in the beam was much greater than we had anticipated. Even though the iron pion filter rejected pions with an efficiency of ~95%, about 10% of the events with the full collimator were pion induced. Since the pion attenuation length in carbon is 95.1 ± 5.8 g/cm^2,[5] the pion flux in the beam with the filter partially removed was increased by a factor of 2.9 ± 0.2. By comparing the two sets of data we are able to deduce the cross section due to muon-induced events alone.

The pictures of the beam-momentum chambers were measured by the SPASM[6] automatic scanning system. Only events which were successfully reconstructed and had incident momenta between 8.6 and 12.9 GeV/c were measured in the other chambers. An event is defined as an interaction in the carbon target which produced two or more prongs; obvious knock-ons, in which the muon was undeflected and the second prong had very low energy, were not included. The incident track and the interaction vertex in the target chamber, and the triggering particle in the rear chambers, were measured manually. We made no attempt to determine from the target-chamber pictures which of the particles emerging from the vertex was the muon. The direction of the scattered muon was reconstructed using the vertex position and the measurement in the rear chambers.

Radiative corrections were calculated using the elastic cross section, inelastic scattering with the production of the first, second, and third nucleon resonances in the zero-width approximation, and an approximate fit to the inelastic continuum beyond the resonances.[7] The correction was 2 to 8% depending on q^2, the square of the four-momentum transfer from the muon, and was assumed to be accurate to within a factor of 2. The corrections for scanning and measuring efficiences were 3 and 6%, respectively.

The cross section for inelastic muon scattering assuming one-photon exchange and detection of only the scattered muon is[8]

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma_T(q^2, k)\sigma_T(q^2, k) + \Gamma_0(q^2, k)\sigma_0(q^2, k),$$

where E and E' are the initial and final laboratory lepton energies, M is the mass of the proton, and $k = E - E' - |q^2|/2M$ is the effective virtual-photon energy.

The first term is the contribution of transversely polarized virtual photons and the second term is that of scalar photons. The Γ factors may be interpreted as the number of virtual photons per MeV per steradian (of the scattered muon). In the limit $q^2 \to 0$, σ_0 vanishes while σ_T becomes equal to the total cross section for real photons of energy k. The Γ factors may be expressed as

$$\Gamma_T(q^2, k) = \frac{\alpha}{4\pi^2}\frac{k}{q^2}\frac{E'}{E}\left[2 + \frac{4E^2E'^2\sin^2\theta}{q^2(\vec{p}-\vec{p}')^2}\right],$$

$$\Gamma_0(q^2, k) = \frac{\alpha}{4\pi^2}\frac{k}{q^2}\frac{E'}{E}\left[\frac{4E^2E'^2\sin^2\theta}{q^2(\vec{p}-\vec{p}')^2} + \frac{4m^2}{q^2}\right],$$

where θ is the muon scattering angle and m is the muon mass. We present our data as

$$\sigma_T(q^2, k) + \sigma_0'(q^2, k) = \frac{d^2\sigma/d\Omega dE'}{\Gamma_T(q^2, k)}.$$

Figure 2 shows $(\sigma_T + \sigma_0')$ as a function of q^2 with the pion contamination removed as described above; the inset shows the distribution of events plotted as a function of the virtual-photon energy k. The errors shown are due to statistics and to uncertainties in the radiative corrections, the pion attenuation length in carbon, and in the scanning and measuring efficiencies. Also shown are two preliminary points from inelastic electron scattering from hydrogen[9] multiplied by 12 (the number of nucleons in carbon). The electron data indicate that in the deep inelastic region, $W_2(q^2, k)$ is independent of q^2 for $q^2 > 0.8$ (GeV/c)^2, where

$$W_2(q^2, k) = \frac{k(\sigma_T + \sigma_0')}{4\alpha\pi^2(0.04\times 10^{-26})}\frac{q^2}{(q^2 + q_0^2)}.$$

The solid line in Fig. 2 shows the fit

$$\sigma_T + \sigma_0' = (1.62 \pm 0.19 \text{ mb}) \times \frac{1}{1 + q^2(2.5 \pm 0.3)},$$

where q^2 is expressed in (GeV/c)^2. This fit has a χ^2 of 2.9 for five degrees of freedom or a 70% confidence level. Its form was chosen to express the constancy of W_2 at high q^2 when $q_0^2 > q^2$ and to give a simple form at $q^2 = 0$. Another fit of the form

$$\sigma_T + \sigma_0' = \frac{A}{(1 + q^2/m_\rho^2)^2} + \frac{Bq^2}{(1 + q^2/m_\rho^2)^2},$$

where m_ρ is the rho-meson mass, gave a similar χ^2 with $A = 1.60 \pm 0.33$ mb.

To the extent that the extrapolation to $q^2 = 0$ is correct, we obtain for the total photoproduction cross section

$$\sigma_{\gamma C} = 1.62 \pm 0.19 \text{ mb for } 5.5 < k < 9.5 \text{ GeV}.$$

The vector-dominance model and the optical theorem may be used to obtain[10]

$$\sigma_{\gamma A} = \left(\frac{16\pi}{1+\beta^2}\right)^{1/2} \times \sum_{V=\rho,\omega,\varphi} \frac{e}{f_V}\left[\frac{d\sigma}{dt}(\gamma A \to VA)\Big|_{t=0}\right]^{1/2},$$

where β is the ratio of the real part of the forward Compton-scattering amplitude to the imaginary part. An integration of the dispersion relation for the real part[11] yields $\beta \simeq 0.12$ at 7.5 GeV,[12] which is negligible. Using the measured cross sections for forward photoproduction of ρ[13] and φ[14] mesons from carbon, and an estimate of the contribution of ω mesons, we find that vector dominance predicts 1.0 ± 0.2 mb for the total photoproduction cross section from carbon.

One would expect the photoproduction cross section to scale as A because the gamma ray does not get absorbed strongly in a nucleus. However a vector-dominance–model calculation indicates that the photoproduction cross section from carbon is about a factor of 10 times the hydrogen

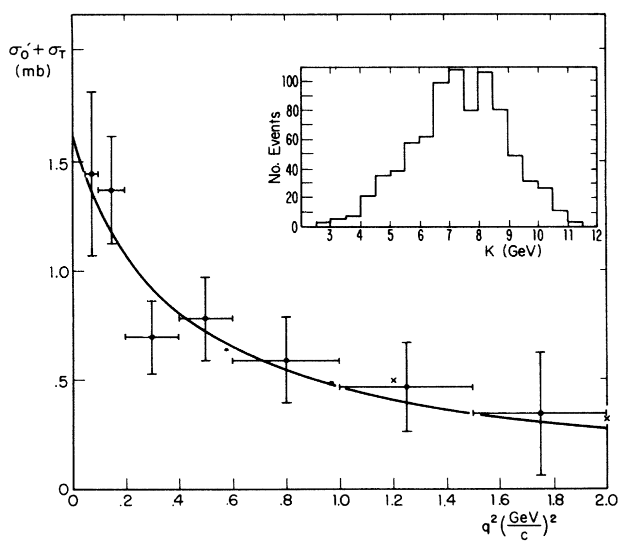

FIG. 2. The total virtual-photon cross section on carbon as a function of the square of the four-momentum transfer from the muon. The inset shows the distribution of the virtual-photon energies. Dots with error bars, this experiment; crosses, the experiment of Bloom et al. multiplied by 12 (number of nucleons in carbon).

Table I. Results from other sources compared with those from the present work.

Reference	Technique used	Photon energy (GeV)	$\sigma_{\gamma p}$ obtained (μb)
This work	Inelastic μ scattering; scale from carbon $\sim A$	$5.5 < k < 9.5$	135 ± 16
This work	Inelastic μ scattering; scale from carbon, factor of 10	$5.5 < k < 9.5$	162 ± 19
ABBHHM[a] Collaboration	Bubble chamber, photoproduction	$3.5 < k < 5.4$	116 ± 17
Ballam et al.	Bubble chamber, photoproduction	$7 < k < 8$	126 ± 17
Davier et al.	Vector-dominance model	$k = 9$	130 ± 30
Knies	Quasielastic optical theorem	$3 < k < 5$	99 ± 12
Buccella and Colocci	Regge-pole theory	$k > 5$	130

[a]Aachen-Berlin-Bonn-Hamburg-Heidelberg-München.

cross section,[15,16] not a factor of 12, at 7.5 GeV. We compare our results with these two different scalings with other experimental[17,18] and theoretical[19-21] results for the total photoproduction cross section from hydrogen in Table I. Our data are consistent with all previous determinations of the hydrogen cross section if we scale as A, while we do not agree well if we scale with a factor of 10.

We also verify the lack of a strong q^2 dependence of the cross section at large k observed in inelastic electron scattering.[9] Because even vector dominance predicts an $\sim A$ behavior at medium and large q^2,[15] we see no difference between muon and electron inelastic scattering. However electron data at lower momentum transfers and more accurate muon data are needed before any definite conclusions can be drawn.

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EXPERIMENTAL TEST OF QUANTUM ELECTRODYNAMICS BY MUON BREMSSTRAHLUNG*

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We have measured muon bremsstrahlung of 9- to 13-GeV/c muons by using a carbon-plate spark chamber as a target. Our results in the region of invariant four-momenta of the recoil muon-photon system 200-650 MeV/c are consistent with the predictions of quantum electrodynamics.

We have measured muon bremsstrahlung in an experiment designed to observe the processes by which a 9- to 13-GeV muon loses most of its energy in a carbon target. In the previous Letter,[1] results of a total inelastic cross section were reported, along with a description of the apparatus and of the muon beam. In this paper, we will discuss the analysis of the bremsstrahlung events and compare the results with the predictions of quantum electrodynamics (QED).

A total of 70 000 pictures were taken (61% with positive beam polarity and 39% with negative polarity). With all of the data analyzed, we have measured 729 events which satisfied the following criteria: (1) No charged secondary prongs longer than three sparks were in the target spark chamber. (2) There was only one shower in the lead-plate spark chambers and it had a clearly defined first spark. (3) The event vertex was in the region of the carbon plates, to within the errors of the reconstruction. (4) The extrapolations of the track of the scattered muon, as measured in the aluminum-foil section of the target chamber and in the aluminum chamber following the large-aperture analyzing magnet, agreed within limits in both the side and top views (the limits were momentum dependent to allow for multiple scattering). (5) The track measured in the aluminum chambers after the magnet, on extrapolation, intersected the tagging counters, $F(AB)(CD)$, which participated in the trigger. Allowance was made for the multiple scattering in the 1-m iron