THE η-x^0 MIXING ANGLE AND THE κ MESON*

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The SU(3) singlet-octet mixing angle between η and x^0 is expressed in terms of the masses of the members of the pseudoscalar meson nonet and a scalar strange meson κ. Using the recent experimental value $m_\kappa = 1100$ MeV, one obtains $\theta = 23°$, or $\theta = 14°$, depending on whether the angle is defined for mesons being at rest or moving with infinite momentum.

It is generally accepted that in the nonets of mesons, the $I=0$ members are mixtures of pure octet and singlet states. The mixing angle can be calculated from the Gell-Mann–Okubo mass formula. For the η-x^0 mixing angle in the nonet of pseudoscalar mesons one obtains[1] $\theta = \pm(10.4 \pm 0.2)°$ using a quadratic mass formula and $\theta = \pm(24 \pm 1)°$ using a linear one. From the commutators of charges with their time derivatives one can obtain either a linear or quadratic mass formula depending on the choice of the Lorentz frame for the evaluation of the commutators.[2] The rest system gives a linear formula while the infinite-momentum system gives a quadratic formula. Other considerations seem to support a quadratic mass formula.[3] However, till now there is no experimental decision as to which value of the mixing angle is correct.[1]

In this note we calculate the weak decay constants f_η and f_{x^0} of the η and x^0 mesons, respectively, in the framework of the assumptions stated below. They are obtained in terms of the masses of the members of the pseudoscalar-meson nonet and a scalar $K\pi$ resonance[4] κ. Since we use charge commutators we have no freedom in the choice of the Lorentz frame. The mixing angle θ, however, cannot be defined invariantly. It is given by $\tan^2\theta = (f_{x^0}/f_\eta)^2(E_{x^0}/E_\eta)^2$, where E_{x^0} and E_η are the energies of the particles. Using[4] $m_\kappa = 1100$ MeV, one gets $\theta = 23°$ for mesons at rest, and $\theta = 14°$ for particles moving with infinite momentum.[5] These values correspond well to a linear or quadratic mass formula, respectively, although they are derived from quite a different method which takes into account the symmetry breaking by the explicit use of a κ mass. The formula for the mixing angle is only weakly dependent on the κ mass. It is interesting to note that it exhibits a lower limit for the κ mass, $m_\kappa \sim 625$ MeV ($\theta = 0$), below which $\tan^2\theta$ is negative, and an upper limit for the angle, $|\theta| = 31°$ (defined in the rest frame) for $m_\kappa \sim \infty$.

For the derivation of the relevant formulas we use (1) the commutation relations of certain vector and axial-vector charges,[6] (2) the hypothesis of partially conserved axial-vector current, and partially conserved vector current for the strangeness-changing vector current, and (3) pole dominance for the form factors corresponding to the divergences of the currents, and consistently we neglect continuum states in saturating the commutators.

The (real) weak decay constants f_K, f_η, f_{x^0}, and f_κ are defined by[7]

$$\langle 0|F_5^{K^\mp}|K^\pm\rangle = i f_K m_K (2\pi)^{\frac{3}{2}}\delta^3(\vec{p}), \quad (1a)$$

$$\langle 0|F^{K^\mp}|\kappa^\pm\rangle = \pm f_\kappa m_\kappa (2\pi)^{\frac{3}{2}}\delta^3(\vec{p}), \quad (1b)$$

$$\langle 0|F_5^{M}|\eta(x^0)\rangle = i f_{\eta(x^0)} m_{\eta(x^0)} (2\pi)^{\frac{3}{2}}\delta^3(\vec{p}), \quad (1c,d)$$

where $F^\alpha = \int d^3x\, V_0^\alpha(\vec{x},0)$, $F_5^\alpha = \int d^3x\, A_0^\alpha(\vec{x},0)$ are, respectively, the vector and axial-vector charges transforming according to the SU(3) octet quantum number α. The mass of particle a is denoted by m_a and $\vec{p}$ is its momentum. The three commutators obtained from the charges used in Eqs. (1) are sandwiched between the vacuum and the states of the corresponding mesons K, η, x^0, and κ, leading to four equations:

$$\langle 0|[F_5^{M}, F^{K^+}]|K^-\rangle = \tfrac{1}{2}\sqrt{3}\, i f_K m_K (2\pi)^{\frac{3}{2}}\delta^3(\vec{p}), \quad (2a)$$

$$\langle 0|[F_5^{K^+}, F_5^{M}]|\kappa^-\rangle = \tfrac{1}{2}\sqrt{3}\, f_\kappa m_\kappa (2\pi)^{\frac{3}{2}}\delta^3(\vec{p}), \quad (2b)$$

$$\langle 0|[F^{K^+}, F_5^{K^-}]|\eta(x^0)\rangle = \tfrac{1}{2}\sqrt{3}\, i f_{\eta(x^0)} m_{\eta(x^0)} \times (2\pi)^{\frac{3}{2}}\delta^3(\vec{p}). \quad (2c,d)$$

On the right-hand side of these equations the commutation relations of the charges[6] together with Eqs. (1) have been used. In Eqs. (2) a set of intermediate states is introduced. We call attention to the fact that only states carrying zero total angular momentum contribute. The sum over intermediate states is approximated by taking into account only one-particle meson states and the "semidisconnected" two-particle states, where one of the particles is annihilated by the right-hand side meson, e.g.,

$$\langle \kappa^+ K_p^- | F^{K^+} | K_{p'} \rangle = \langle \kappa^+ | F^{K^+} | 0 \rangle 2p_0 \delta^3(\vec{p}-\vec{p}'). \tag{3}$$

Equations (2) then lead to four relations between matrix elements, of which the following is an example. From Eq. (2a) one obtains

$$(2\pi)^3 \{ if_\eta [\langle \eta | V_0^{K^+} | K^- \rangle + \langle 0 | V_0^{K^+} | \eta K^- \rangle] + (\eta \to x^0) + f_\kappa [\langle \kappa^- | A_0^M | K^- \rangle + \langle 0 | A_0^M | \kappa^+ K^- \rangle] \} = \sqrt{3}\, if_K m_K, \tag{4}$$

where $A_0 = A(x=0)$, and $V_0 = V(x=0)$. There is no freedom in the choice of the Lorentz system, since the use of charges in Eq. (2) forces all three-momenta of the particles in Eq. (4) to be zero.

We use the hypothesis of partially conserved axial-vector current (partially conserved vector current) in the form

$$\partial A^K = f_K m_K^2 \varphi^K, \tag{5a}$$

$$\partial V^{K^\pm} = \pm i f_\kappa m_\kappa^2 \varphi^{\kappa^\pm}, \tag{5b}$$

$$\partial A^M = f_\eta m_\eta^2 \varphi^\eta + f_{x^0} m_{x^0}^2 \varphi^{x^0}, \tag{5c}$$

where the φ's are the fields of the corresponding mesons. Note that the κ field is defined to have positive charge-conjugation properties. For the divergence of the matrix elements appearing in Eq. (4) and in the corresponding equations which are obtained from Eqs. (2b)-(2d) we make the unsubtracted-pole Ansatz:

$$(2\pi)^3 \langle \kappa^- | \partial A^{K^-} | \eta(x^0) \rangle = f_K m_K^2 g_{\kappa K \eta(x^0)} / (m_K^2 - Q^2), \tag{6a, b}$$

$$(2\pi)^3 \langle K^+ | \partial V^{K^+} | \eta(x^0) \rangle = i f_\kappa m_\kappa^2 g_{\kappa K \eta(x^0)} / (m_\kappa^2 - Q^2), \tag{6c, d}$$

$$(2\pi)^3 \langle \kappa^- | \partial A^M | K^- \rangle = f_\eta m_\eta^2 g_{\kappa K \eta} / (m_\eta^2 - Q^2) + f_{x^0} m_{x^0}^2 g_{\kappa K x^0} / (m_{x^0}^2 - Q^2). \tag{6e}$$

Here Q^2 is the square of the momentum transfer and we have used the definition

$$g_{abc} = (2\pi)^3 (m_b^2 - Q^2) \langle a | \varphi^b | c \rangle |_{Q^2 = (c-a)^2 = m_b^2}, \tag{7}$$

where g_{abc} is symmetric in the indices. The quantity g_{abc} is the strong-coupling constant of the three particles and corresponding to the pole Ansatz (6) g_{abc} is assumed to be independent of Q^2. Using Eqs. (6) in Eq. (4) and the corresponding equations obtained from Eqs. (2b)-(2d) by introducing intermediate states, one is led to the following set of equations $[(ab) = m_a^2 - m_b^2]$:

$$f_\kappa f_K f_\eta g_{\kappa K \eta} (\kappa\eta)/(\kappa K)(\eta K) + (\eta \to x^0) = \tfrac{1}{2}\sqrt{3} f_K^2, \tag{8a}$$

$$f_\kappa f_K f_\eta g_{\kappa K \eta} (\eta K)/(K\kappa)(\eta\kappa) + (\eta \to x^0) = \tfrac{1}{2}\sqrt{3} f_\kappa^2, \tag{8b}$$

$$f_\kappa f_K f_\eta g_{\kappa K \eta} (K\kappa)/(\kappa\eta)(K\eta) = \tfrac{1}{2}\sqrt{3} f_\eta^2, \tag{8c}$$

$$f_\kappa f_K f_{x^0} g_{\kappa K x^0} (K\kappa)/(\kappa x^0)(K x^0) = \tfrac{1}{2}\sqrt{3} f_{x^0}^2. \tag{8d}$$

We want to solve this set for the unknown quantities f_η, f_{x^0}, $g_{\kappa K\eta}$, and $g_{\kappa K x^0}$, which are then given in terms of the masses of the mesons and f_K, f_κ, where f_κ is yet unknown.

In order to get information about f_κ, we proceed in exactly the same manner as before for the system of mesons K, π, and κ and their corresponding charges. The formulas look analogous to those given above; however, one has to drop all x^0-dependent terms and to replace η by π^0. Furthermore in Eqs. (2) and (8) the factor $\frac{1}{2}\sqrt{3}$ has to be replaced by $\frac{1}{2}$ because of the different structure constants in the commutation relations being used now.

From the analogs of Eqs. (8a)-(8c) one gets

$$(f_\kappa/f_\pi)^2 = (K\pi)^2/(\kappa K)^2; \quad (f_K/f_\pi)^2 = (\kappa\pi)^2/(\kappa K)^2; \tag{9a}$$

$$(2f_\kappa g_{\kappa K\pi})^2 = (K\pi)^2. \tag{9b}$$

These relations were obtained previously by Dahmen, Rothe, and the author[8] in an investigation of the K_{l3} form factors.

Using Eqs. (9a) in Eqs. (8) we obtain

$$(f_\eta/f_\pi)^2 = [(\kappa x^0)^2(K\pi)^2 - (Kx^0)^2(\kappa\pi)^2]/N, \tag{10a}$$

$$(f_{x^0}/f_\pi)^2 = [(\kappa\pi)^2(K\eta)^2 - (K\pi)^2(\kappa\eta)^2]/N, \tag{10b}$$

where we have used the abbreviation $N = (\kappa x^0)^2 \times (K\eta)^2 - (Kx^0)^2(\kappa\eta)^2$,

and

$$|f_\eta g_{\kappa K\eta}/(K\kappa)| = \tfrac{1}{2}\sqrt{3}\,(f_\eta/f_\pi)^2 |(\kappa\eta)(K\eta)/(\kappa\pi)(K\pi)|, \tag{11a}$$

$$|f_{x^0} g_{\kappa K x^0}/(K\kappa)| = \tfrac{1}{2}\sqrt{3}\,(f_{x^0}/f_\pi)^2 |(\kappa x^0)(Kx^0)/(\kappa\pi)(K\pi)|. \tag{11b}$$

Now we introduce an η-x^0 mixing angle θ by decomposing the physical states $|\eta\rangle$ and $|x^0\rangle$ into its SU(3) octet and singlet parts: $|\eta\rangle = \cos\theta|8_0\rangle - \sin\theta|1_0\rangle$ and $|x^0\rangle = \sin\theta|8_0\rangle + \cos\theta|1_0\rangle$. Corresponding to Eqs. (1c) and (1d) we have in a general Lorentz frame $\langle 0|A_0^M|x^0\rangle = if_{x^0}E_{x^0}(2\pi)^{-\frac{3}{2}}$ and $\langle 0|A_0^M|\eta\rangle = if_\eta E_\eta(2\pi)^{-\frac{3}{2}}$. Recalling that A_0^M transforms as a pure octet, one finds

$$\tan^2\theta = (E_{x^0}/E_\eta)^2(f_{x^0}/f_\eta)^2. \tag{12}$$

With the help of Eqs. (10) and the known masses of the mesons π, K, η, and x^0, one obtains from Eq. (12) for the η and x^0 meson being at rest $|\theta| = 23°$ with[4] $m_\kappa = 1100$ MeV, varying from $|\theta| = 21.5°$ to $|\theta| = 24°$ with m_κ between 1000 and 1200 MeV, respectively. The value of $|\theta|$ decreases, if m_κ decreases, and becomes zero for $m_\kappa \sim 625$ MeV. For m_κ below this value $\tan^2\theta$ becomes negative. For $m_\kappa \sim \infty$ one has $|\theta| = 31°$.

Defining the mixing angle for particles moving with infinite momentum one has $(E_{x^0}/E_\eta)^2 = 1$ in Eq. (12) and finds $|\theta| = 14°$ for $m_\kappa = 1100$ MeV, which again only weakly depends on the value of the κ mass.

We do not feel that our approximations are very far from being correct since for the $K\pi$ system Eqs. (9) give the mass and the width of the κ as a function of f_K/f_π which seems to be supported by experiment.[8] Furthermore, the agreement of our mixing angle $\theta = 23°$ with that of a linear mass formula (and the fair agreement of $\theta = 14°$ with a quadratic one) seems again to give theoretical support to the value $m_\kappa \sim 1100$ MeV used.

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[5]From this explicit formula one might argue that it is more reasonable to define the mixing angle for particles at rest, if used in decay processes or mass formulas. This would support the value $|\theta| = 23°$ and a linear mass formula.

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[7]We have defined $J^{\pi^\pm} = (J^1 \pm iJ^2)/2\sqrt{2}$, $J^{K^\pm} = (J^4 \pm iJ^5)/2\sqrt{2}$, $J^{\pi^0} = J^3/2$, and $J^M = J^8/2$, where the J^i transform like the matrices λ^i of Ref. 6.

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