POMERANCHUKON EXCHANGE PLUS DIRECT-CHANNEL RESONANCES AS A MODEL TO DESCRIBE THE FORWARD DIFFRACTION PEAK IN PION-NUCLEON SCATTERING*

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We show that the t-channel Pomeranchukon with slope $\alpha' = 0.7^{+0.16}_{-0.10}$ plus s-channel resonances provide a good model for the description of near forward pion-nucleon elastic scattering in the range 1.00 to 20 BeV/c.

A few years ago Regge-pole theory became a victim of considerable attack due to its failure to describe the forward diffraction peak of pion-nucleon scattering (i.e., predicting shrinkage when the data showed otherwise). The conclusion then was that a simple model based on an exchange of a Pomeranchukon with ordinary slope alone was not adequate. Subsequent work by Rarita et al.[1] with a purely Regge exchange model using several exchanges succeeded in fitting the data for $p_{\text{lab}} > 6$ BeV/c only if $\alpha' \leq 0.3$.

In this Letter we demonstrate that the s-channel resonances make a considerable contribution in the forward direction, and that their inclusion with that of the Pomeranchukon indeed enables one to describe the forward diffraction peak for small momentum transfers. For the description of wider angles (i.e., secondary maximum and the backward hemisphere) models have been proposed based on pure resonances alone[2] and resonances plus a background amplitude.[3] The point in favor of this model is the feature of duality; that due to a simple analyticity relation for the scattering amplitude (obtained by assuming an asymptotic power-law dependence) the s-channel resonance amplitude averaged over energy has been shown[4] to have the same properties as the high-energy exchange amplitude. Further, it has been shown by Harari[5] that in this model the difficulty of double counting, which is inherent in the interference model of Barger and Olsson,[6] is avoided. We, therefore, consider our model, in which direct-channel resonances are used, as an alternative to fitting with P', ρ, and other lower trajectories as in an exchange-model approach. The resonance model works over a wider energy range.

The data existing at present show shrinkage in the forward direction only for pp and K^+p scattering and not for $\pi^\pm p$ and K^-p. A strong motivation for this work came with the obvious observation that there are no known direct-channel resonances for K^+p and pp, while other systems such as $\pi^\pm p$ and K^-p have an abundance of direct-channel resonances.

We have done the calculation for the pion-nucleon elastic scattering system with a set of resonances, all of which lie on three straight-line trajectories, namely Δ_δ, N_α, and N_γ, used in an earlier work by the author to fit $\pi^- p$ elastic scattering in the very backward direction.[7] As much as one desires a complete list of resonances with exact parameters, for the purpose of this Letter the existing resonances were fairly sufficient to give us an overall magnitude and sign in making up the resonance amplitude. The elastic differential cross section in the forward direction is very sensitive not to the exact and detailed organization of the parameters but only to the density of the resonances and their distribution with respect to l and J. The resonance contribution to be added to that of the Pomeranchukon is made up of a summation of Legendre polynomials which all start positive in sign, decrease rapidly toward zero, and change sign as one moves away from the very forward direction. In the range of s where direct-channel resonances are known to exist the resonance contribution tends to fall off more rapidly with $-t$ than the Pomeranchukon contribution. The combination of the two results in a diffraction peak which fits the data and does not appear to expand at low energies in the 1- to 4-BeV/c beam momentum range.

Resonances are added and the final amplitude is calculated according to the formula

$$A_{\text{nonflip}}^{\text{res}} = \frac{1}{k} \sum_j \frac{x_j (J_j + \frac{1}{2}) P_{l_j}}{\epsilon_j - i}, \tag{1}$$

$$A_{\text{flip}}^{\text{res}} = \frac{1}{k} \sum_j \frac{(-1)^{(J_j - l_j - \frac{1}{2})} x_j P_{l_j}'}{\epsilon_j - i}, \tag{2}$$

$$\epsilon_j = (M_j^2 - s)/M_j \Gamma_j,$$

where j is the index of summation over the resonances, J_j the spin of the resonance, Γ_j the

width, M_j the mass of the resonance, x_j the elasticity of the resonance, P_{lj} the corresponding Legendre polynomial, and $P_{lj}' = dP_{lj}/d\theta$.

The Pomeranchukon pole contribution was calculated using the following formula:

$$A_{NF}^{P}(s,t) = \alpha\xi C e^{-(\alpha' \ln s/s_0)t}, \quad (3)$$

$$\xi = -(1+e^{-i\pi\alpha})/\sin\pi\alpha,$$

$$\alpha(t) = \alpha(0) - \alpha' t.$$

C, s_0, and α' are all constants with $\alpha(0) = 1$.

For best fits to all $\pi^+ p$- and $\pi^- p$-system data the Pomeranchukon parameters were found to be

$$C = 5.90, \quad \alpha' = 0.7^{+0.16}_{-0.10}, \text{ and } s_0 = 0.27 \simeq 2 m_\pi m_p.$$

The fits of this model were sensitive to the value of α'. We tried different values of α' and s_0. It was discovered that one can fit the region 6-20 BeV/c with a value of $\alpha' \sim 0.35$ or less, but the low-energy fits, $P_{\text{lab}} < 6$ BeV/c, shrank away from the data by better than 4-5 standard deviations. Higher values of α' up to 0.94 were also tried, and the fits started bowing due to the arrival of an excessive real part, thus losing their exponential shape both at low and high energies.

Differential cross sections were calculated using the formulas

$$d\sigma/dt = |A_{NF}|^2 + |A_F|^2, \quad (4)$$

$$A_{NF} = A_{NF}^{\text{res}} + A_{NF}^{P}(s,t), \quad (5)$$

$$A_F = A_F^{\text{res}}, \quad (6)$$

where NF stands for spin-nonflip amplitude and F stands for spin flip.

In Figs. 1 and 2 we show our $\pi^- p$ and $\pi^+ p$ fits to the experimental data[8-14] in the range 0.685 to 18.90 BeV/c of the incoming pion momentum.[15] Only a few of the cross sections have been included in the figures.[16] In the low and intermediate regions of energy, 1-5 BeV/c, the fits of the model (solid lines) are very reasonable. At these energies the fits could be considerably improved by altering the elasticities of the resonances or by adding more resonances. For comparison we have also plotted the Pomeranchukon pole contribution alone (dashed lines) to emphasize the size of the resonance contribution. In this region where resonance contribution is sizable (i.e., $P_{\text{lab}} \lesssim 5$ BeV/c) we wish to point out in Figs. 1 and 2 the delicate interference between

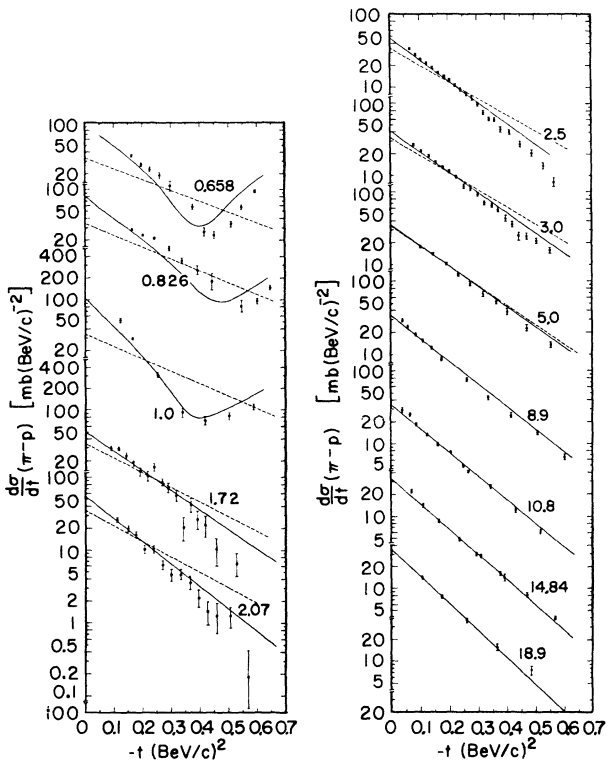

FIG. 1. $\pi^- p$ elastic differential cross-section experimental data at 0.658 (Ref. 12), 0.826 (Ref. 12), 1.0 (Ref. 11), 1.72 (Ref. 14), 2.07 (Ref. 14), 2.5 (Ref. 10), 3.0 (Ref. 10), 5.0 (Ref. 10), 8.9 (Ref. 8), 10.8 (Ref. 8), 14.84 (Ref. 8), and 18.9 (Ref. 8) BeV/c compared with the fits of the model (solid line). The dashed lines are those due to Pomeranchukon amplitude alone.

the resonance and the Pomeranchukon amplitudes. Both in $\pi^+ p$ and $\pi^- p$ systems starting at very low energies there is constructive interference for $|t| \lesssim 0.2$ and destructive interference for $|t| \gtrsim 0.2$. This crossing point appears to remain constant in t within the range of energies covered in this work. This can be easily understood since for small values of t, the Legendre polynomials corresponding to different momenta all cross zero at approximately the same value of t. We believe that this characteristic will persist for even higher energies up to where the resonance contribution becomes negligible. If the parameters of the Pomeranchukon are fixed at a point in energy where the resonance contribution is zero and which is notably much higher than available, and the Pomeranchukon exchange alone extended to the lower energies, it will be seen that the calculated forward point will lie below the data (σ_T is decreasing for increasing energies), and the crossing point will occur at a

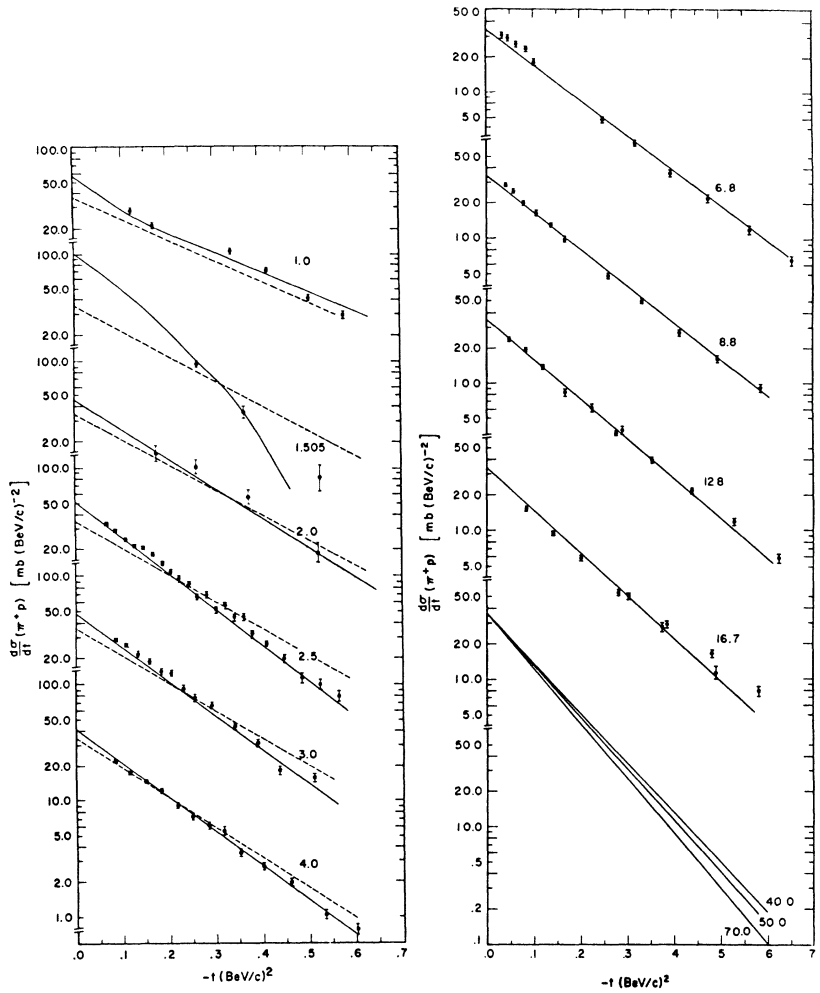

FIG. 2. π^+p elastic differential cross-section experimental data at 1.0 (Ref. 11), 1.505 (Ref. 11), 2.0 (Ref. 13), 2.5 (Ref. 9), 3.0 (Ref. 9), 4.0 (Ref. 9), 6.8 (Ref. 8), 8.8 (Ref. 8), 12.8 (Ref. 8), and 16.7 (Ref. 8) BeV/c compared with the fits of the model (solid line). The dashed lines are those due to Pomeranchukon amplitude alone.

$|t|$ value such that the resonances will add constructively to the left of the crossing point and destructively to the right. One can get a fair idea on the resonance contribution that is needed to fit the data by examining the differences between the extrapolated data and the Pomeranchukon contribution at $t=0$.

For the high-energy region $P_{\text{lab}}>6$ BeV/c the fits to both the π^+p and the π^-p systems deviate slightly from the data exhibiting a need for some resonance amplitude. The present set of resonances used in this analysis was minimal and extends only up to 6 BeV/c. Upon closer examination of the Figs. 1 and 2, the model shows better agreement with π^+p than the π^-p system. One very obvious conclusion is that the π^+p data shows slightly more shrinkage of the forward diffraction peak than the π^-p system for increasing energies. The other obvious conclusion, which is connected with the first, is that the π^-p system is more populated with resonances (as is evident from the existing tables at low energies due to the mixture of two isospins), than that of the π^+p system and would keep this behavior at even higher energies. In short there is a need for more resonances to interfere with the Pomeranchukon amplitude for the π^-p than that of the π^+p system.

As is seen in Figs. 1 and 2 the model fits the low-energy data around $P_{\text{lab}}\approx 1.0$ BeV/c reasonably well. In this region where phase-shift analysis exists, as a further check, it would be very helpful to compare the background of this model with that resulting from phase-shift analysis.

The present analysis has been extended only to $t\sim -0.5$ since more structure sets in for higher

$-t$ than can be accounted for with the present set of resonances. The parameters of the Pomeranchukon amplitude and the resulting fits established in this work are not to be considered definitive and may be subject to modification as a better set of resonances and experimental data become available.

We compared our polarization calculations with that of the data and we discovered rapid fluctuations both with variations in $-t$ and also with s. Nothing conclusive can be said about our polarization calculation since it is so sensitive to the parameters of the resonances used. Today's existing data on polarization show structure up to ~15 BeV/c, and structure may still exist even at higher energies. We feel that resonances will play a considerable role even at these energies in giving rise to polarization.

Finally we summarize our argument in stating that resonance contribution in the forward direction is sizable and most probably sufficient to remedy the shrinkage that is inherent in the Pomeranchukon amplitude alone. We would like to emphasize that "shrinkage or nonshrinkage" is closely tied to the amount of resonant amplitude contribution and cannot be discussed except in a region where resonances are no longer present, i.e., resonance contribution is negligible. The present model explains the absence of shrinkage only in the resonance region. Polarization, especially, and charge-exchange scattering measurements at higher energies could shed considerable light on the validity of the model. Predictions for energies greater than 30 GeV are given in Fig. 2.

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[15]In Fig. 2 we include the predictions of the present model at 40, 50, and 70 BeV/c.

[16]Fits of the present model to the remaining data could be requested directly from the author.