tensity ratio of electrons to nuclei is ~10^{-2} from 1 to 300 BeV and they both have the same spectral index of ~2.6. But in this energy range the electron spectrum has already steepened by one-half power because of radiative losses. Therefore the injection ratio $Q_e(E)/Q_p(E)$ increases progressively for higher energy and already approaches unity at the present observational limit if one accepts the existence of the infrared field. This conclusion holds independent of other details as long as the electrons and nuclei of cosmic rays are produced in the same source and diffuse in the same manner. One would then have to look for mechanisms which accelerate electrons more efficiently than nuclei.[13]

In conclusion, one cannot claim that the current data on cosmic-ray electrons and high-energy radiation have "proved" the existence of a cosmic (or at least galactic) infrared flux, but they are certainly compatible with the presence of such a radiation field. The origin of the radiation, which we have made no attempt to discuss in this Letter, is still an open question at present. The author wishes to thank Professor G. W. Clark for helpful discussions.

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[1]K. Shivanandan, J. R. Houck, and M. O. Harwit, Phys. Rev. Letters 21, 1460 (1968).

[2]G. W. Clark, G. P. Garmire, and W. L. Kraushaar, Astrophys. J. 153, L203 (1968).

[3]V. L. Ginzburg and S. I. Syrovatskii, The Origin of Cosmic Rays (The Macmillan Company, New York, 1965).

[4]C. S. Shen and G. Berkey, Astrophys. J. 151, 895 (1967).

[5]K. C. Anand, R. R. Daniel, and S. A. Stephens, Phys. Rev. Letters 20, 764 (1968).

[6]The source Q and the isotropic diffusion equation used here are, of course, gross approximations for propagation of cosmic rays in the galaxy. As pointed out by J. R. Jokipii and P. Meyer [Phys. Rev. Letters 20, 752 (1968)], D should increase toward the halo where the magnetic field is weaker. In the disk one also expects diffusion along the spiral arms to be faster than across the lines of force. The distribution of sources, while likely to be confined in the galactic plane, may not lie as evenly as suggested by a Gaussian. Nevertheless, refinements along the above lines (C. S. Shen and G. Berkey, to be published) do not alter the essential features presented here.

[7]C. S. Shen, Phys. Rev. Letters 19, 399 (1967). There is a typographical error in Eq. (7) of this paper. The exponential factor should be $\exp[-\{\xi+4/9(1+2\xi)\}]$.

[8]A detailed discussion was given by Jokipii and Meyer, Ref. 6.

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THEORIES WITH TOO MUCH SYMMETRY

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It is shown that a class of theories, of which Sugawara's theory of currents and certain Yang-Mills type theories are examples, possess a particular symmetry which is not, even approximately, present in the observed strong interactions. The symmetry would imply that all mesons and all massive fermions have partners with opposite parity and opposite charge conjugation.

There is a class of possible strong-interaction theories, of which Sugawara's theory of currents[1,2] is perhaps the most interesting, in which the energy-momentum tensor $\theta^{\mu\nu}$ can be written as

$$\theta^{\mu\nu}=\theta_{(+)}{}^{\mu\nu}+\theta_{(-)}{}^{\mu\nu}, \tag{1}$$

where $\theta_{(+)}{}^{\mu\nu}$ and $\theta_{(-)}{}^{\mu\nu}$ are two (independently) conserved tensors which commute with each other at equal times and are interchanged by parity, i.e., $P\theta_{(\pm)}{}^{\mu\nu}P^{-1}=\theta_{(\mp)}{}^{\mu\nu}$. The existence of these two independent, conserved tensors leads in the usual way to a symmetry group $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$, where $\mathcal{P}_{(+)}$ and $\mathcal{P}_{(-)}$ are two commuting Poincaré groups. This is to be contrasted to the usual case where there is only one conserved tensor and (apart from internal symmetries) the theory is invariant only under the usual Poincaré group $\mathcal{P}$.

It will be shown below that the possible existence of a $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ symmetry is in conflict with the observed properties of hadrons and therefore that theories which possess this extra symmetry must be rejected on experimental grounds. Such a symmetry would imply that <u>all mesons and all massive fermions have partners degenerate in mass but with opposite parity and charge-conjugation properties.</u>

In Sugawara's theory this extra symmetry comes about because his $\theta^{\mu\nu}$ can be written in the form of Eq. (1) with

$$\theta_{(\pm)}{}^{\mu\nu}=\frac{1}{c}[J_{(\pm)}{}^{a\mu}J_{(\pm)}{}^{a\nu}+J_{(\pm)}{}^{a\nu}J_{(\pm)}{}^{a\mu}-g^{\mu\nu}J_{\lambda(\pm)}{}^{a}J_{(\pm)}{}^{a\lambda}], \tag{2}$$

where

$$J_{\pm}{}^{a\mu}=\tfrac{1}{2}[V^{a\mu}\pm A^{a\mu}] \tag{3}$$

and $V^{a\mu}$ and $A^{a\mu}$ are the usual octets of vector and axial-vector currents. The vanishing of the commutator $[\theta_{(+)}{}^{\mu\nu}, \theta_{(-)}{}^{\lambda\sigma}]$ at equal times is a consequence of Sugawara's currents satisfying $[J_{(+)}{}^{a\mu}, J_{(-)}{}^{b\nu}]=0$ for equal times.

Another, otherwise attractive,[3] theory in which our phenomenon occurs is a Yang-Mills theory of eight vector and eight axial-vector mesons interacting with quarks. In order that this theory have the SU(3)⊗SU(3) symmetry which appears to be (approximately) present in the strong interactions, we must give all the mesons a common bare mass and provide no bare mass for the quarks. Then, writing the vector fields as $\psi_{(+)}{}^{a\nu}+\psi_{(-)}{}^{a\nu}$, the axial fields as $\psi_{(+)}{}^{a\nu}-\psi_{(-)}{}^{a\nu}$, and introducing quark fields $q_{(\pm)}=\frac{1}{2}(1\pm\gamma_5)q$, the Lagrangian for this theory can be written as $\mathcal{L}=\mathcal{L}_{(+)}+\mathcal{L}_{(-)}$, where $\mathcal{L}_{(+)}$ depends only on the fields $\psi_{(+)}$ and $q_{(+)}$, and $\mathcal{L}_{(-)}$ depends only on $\psi_{(-)}$ and $q_{(-)}$. Whenever a Lagrangian splits into two independent pieces, as is the case here, one immediately obtains the two independent tensors of Eq. (1).[4] One more example of a theory with $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariance is the SU(3)⊗SU(3)-invariant theory of zero-bare-mass quarks with an interaction[5] $G[(\bar{q}\gamma^\mu q)^2+(\bar{q}\gamma_5\gamma^\mu q)^2]$.

One can easily break the $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ symmetry of the above Lagrangian models. For example, one could give the quarks a bare mass or split the masses of the Yang-Mills mesons. However, either of these ways of breaking $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ also breaks SU(3)⊗SU(3). The same is true of Sugawara's theory: In the broken symmetry version of his model,[2,6] SU(3)⊗SU(3) and $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ are not conserved by the same mechanism. The point here is that in the class of theories and types of symmetry breaking under consideration, the SU(3)⊗SU(3) symmetric limit is also $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ symmetric so that both symmetries should be broken by roughly equal amounts. Now SU(3)⊗SU(3) appears to be a rather good symmetry of hadrons, being broken to about the same extent that SU(3) is.[7] However, as will be shown below, there is no trace of $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ in the strong interactions. We will conclude, then, that symmetry breaking does not resolve the difficulties inherent in these overly symmetrical theories.

At this point it is well to observe that SU(3)⊗SU(3) does <u>not</u>, in general, imply a $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ symmetry. For example, an SU(3) version of the σ model which contains scalar and pseudoscalar mesons interacting with quarks (to which Yang-Mills mesons can be added, if one wishes) can be made SU(3)⊗SU(3) symmetric without obtaining $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ at the same time. Also, a theory of zero-bare-mass quarks with $G(\bar{q}\gamma^\mu q)^2$ for an interaction is SU(3)⊗SU(3) but not $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariant. However, it does not seem possible to have SU(3)⊗SU(3) without $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ within a theory of currents like Sugawara's. Adding scalar and pseudoscalar fields to this model in the manner of Ref. 2 breaks $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ only to the extent that SU(3)⊗SU(3) is broken.[8]

Having discussed some examples of $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$-symmetric theories, we will now show, as promised above, that this symmetry is not consistent with the observed strong interactions.

The experimental predictions of a symmetry depend critically on whether or not the vacuum is invariant. Examples of the usual case where the vacuum is invariant are the ordinary Poincaré group or SU(3) in the limit that it is exact. An example of a noninvariant vacuum is provided by the hadrons in the limit that SU(3)⊗SU(3) is exact.[7] In this case one does not obtain SU(3)⊗SU(3) multiplets but rather an octet of massless 0^- Goldstone bosons corresponding to the π, K, and η. Offhand, one might think that $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ could be realized by a degenerate vacuum as is SU(3)⊗SU(3). It will be shown below however that, in a local Lorentz-invariant theory, this is impossible. Thus we need only consider the case where the vacuum is $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariant.

It is not hard to see that in a $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$-invariant theory whose vacuum is $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariant, all mesons and massive fermions must come in degenerate pairs, the members of which have opposite parity and charge-conjugation properties. This circumstance, which is clearly present in the above Lagrangian models, is arrived at as follows. Consider acting with $H_5=\int d^3x[\theta_{(+)}{}^{00}(\vec{x})-\theta_{(-)}{}^{00}(\vec{x})]$ on a state containing, say, a proton at rest. Since H_5 is a conserved rotational scalar with negative parity and charge conjugation there must be a new state $|p'\rangle=H_5|p\rangle$ with the same mass and spin as a proton but of negative parity and charge conjugation relative to the proton. {H_5 cannot annhiliate $|p\rangle$: The commutation relation $i[\vec{K}_5,H_5]=\vec{P}$, where $\vec{P}$ is the momentum operator and $\vec{K}_5=\int\vec{x}[\theta_{(+)}{}^{00}(\vec{x})-\theta_{(-)}{}^{00}(\vec{x})]$, cannot be satisfied if $H_5|p\rangle=0$.} Furthermore, since H_5 is an SU(3) singlet, $|p'\rangle$ must have the same SU(3) quantum numbers as $|p\rangle$. Now if the vacuum is $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariant, $|p'\rangle$ must be a single-particle state with the result that $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ would require an opposite parity and charge-conjugation partner for the proton. Clearly the same argument works for any massive state. Massless states require some special consideration. Here we simply state the result that massless mesons must be doubled, but massless fermions need not be doubled.

Evidently, the lack of opposite parity partners for, say, the nucleon or pion rules out the existence of a slightly [to about the same extent as SU(3)] broken $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ symmetry in the strong interactions. Actually, the situation is even worse than this. One can convince himself that if in a $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ symmetric theory one forms linear combinations of particles and their partners like $|p\pm\rangle=|p\rangle\pm|p'\rangle$, then the "plus" states never interact with the "minus" states.[9] Thus one would be living in the midst of two worlds which do not interact with each other. There is certainly no evidence for such a situation.

Yet another objection to $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ is that, as will be shown below [see Eq. (6)], if a theory becomes $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariant in the limit that it respects SU(3) ⊗ SU(3), then SU(3) invariance of the vacuum implies SU(3) ⊗ SU(3) invariance for the vacuum.[10] In reality it appears, as was pointed out above, that the vacuum is SU(3) but not SU(3) ⊗ SU(3) invariant in the symmetry limit.

It remains to show that the vacuum must be invariant. The proof is quite simple: It was noted above that $[\theta_{(+)}{}^{\mu\nu}(\vec{x},t),\theta_{(-)}{}^{\lambda\sigma}(\vec{y},t)]$ vanishes. Multiplying this equal-time commutator from the left by $U(\tau)$ and the right by $U^{-1}(\tau)$, where $U(\tau)=\exp[-i\tau\int d^3x\,\theta_{(-)}{}^{00}(\vec{x},t)]$, yields

$$[\theta_{(+)}{}^{\mu\nu}(\vec{x},t),\theta_{(-)}{}^{\lambda\sigma}(\vec{y},t+\tau)]=0, \tag{4}$$

so that $\theta_{(+)}{}^{\mu\nu}$ and $\theta_{(-)}{}^{\lambda\sigma}$ commute at different times. Obviously $\langle 0|[\theta_{(+)}{}^{\mu\nu}(x),\theta_{(-)}{}^{\lambda\sigma}(0)]|0\rangle=0$, and by using Fourier transforms with positive or negative frequencies one obtains that $\langle 0|\theta_{(+)}{}^{\mu\nu}(x)\times\theta_{(-)}{}^{\lambda\sigma}(0)|0\rangle=\langle 0|\theta_{(-)}{}^{\lambda\sigma}(0)\theta_{(+)}{}^{\mu\nu}(x)|0\rangle=0$. This immediately leads to the result that

$$\langle 0|\theta^{\mu\nu}(x)\theta^{\lambda\sigma}(0)|0\rangle=\langle 0|\theta_5{}^{\mu\nu}(x)\theta_5{}^{\lambda\sigma}(0)|0\rangle, \tag{5}$$

where $\theta_5{}^{\mu\nu}=\theta_{(+)}{}^{\mu\nu}-\theta_{(-)}{}^{\mu\nu}$. From Eq. (5) it is easy to see that if the usual Poincaré generators P^μ and $M^{\mu\nu}$ obtained from $\theta^{\mu\nu}$ annihilate the vacuum, so do the corresponding generators $P_5{}^\mu$ and $M_5{}^{\mu\nu}$ obtained from $\theta_5{}^{\mu\nu}$. Consequently, the vacuum is $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariant.

There is a loophole in the above proof that should be patched up. One might argue that while the density $\theta_{(-)}{}^{00}$ is all right, the integral $\int\theta_{(-)}{}^{00}d^3x$ used to obtain Eq. (4) is not a well-defined operator. However, in a local theory, acting with $U_f(\tau)=\exp[-i\tau\int f(\vec{x})\theta_{(-)}{}^{00}(\vec{x},t)d^3x]$, where $f(\vec{x})$ is a suitable test function which is identically equal to unity in some finite region around the point $\vec{y}$, will also lead to Eq. (4).

Reasoning similar to that used in arriving at Eq. (5) also leads to[11]

$$\langle 0|V^{a\mu}(x)V^{b\nu}(0)|0\rangle=\langle 0|A^{a\mu}(x)A^{b\nu}(0)|0\rangle \tag{6}$$

in our $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$-invariant theories. From this it follows that if a vector charge annihilates the vacuum, then so does the corresponding axial charge.

Having stated our results, we now turn to a brief discussion of possible ways out of the impasse.

First, we should emphasize that verification of the $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ structure of the specific theories discussed above involves manipulations with products of operator densities at the same point. Essentially identical manipulations are used to obtain the usual Poincaré group from a conserved tensor $\theta^{\mu\nu}$, but it could be that one or more of these theories are singular in such a way that the usual manipulations with $\theta^{\mu\nu}$ are valid but manipulations with $\theta_5{}^{\mu\nu}=\theta_{(+)}{}^{\mu\nu}-\theta_{(-)}{}^{\mu\nu}$ are not. In our discussion we have assumed that the commutator structure is that given by straightforward calculation.[12] Also, we have assumed that both $\theta^{\mu\nu}$ and $\theta_5{}^{\mu\nu}$ connect physical states to physical states. (It might happen that $\theta_5{}^{\mu\nu}$ also connects physical states to unphysical states where, for example, the metric need not be positive.)

Secondly, there is the possibility that the vacuum is not Lorentz invariant and therefore need not be $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariant. (Of course, there should be no observable consequences.) It is obviously rather difficult to see how such a theory would operate. The only real work in this direction seems to be Bjorken's formal arguments that such a spontaneous breakdown of Lorentz invariance can lead to a massless spin-1 Goldstone boson (the photon in his case). Bjorken's calculation is based entirely on badly divergent diagrams in perturbation theory or, equivalently, on singular products of operators at a point so that it is not clear that this possibility really exists or, if it does, whether it could apply in the present context.[13] As a partial check we have tried the assumptions that the vacuum depends on a fixed four-vector η^μ and that the theory contains spin-1 and/or spin-0 Goldstone bosons.[14] Under these circumstances we were not able, using Mandelstam's diagramatic methods,[15] to satisfy the equal-time commutator

$$i[\theta_5^{00}(\vec{x}),\theta_5^{00}(\vec{x}')]=[\theta^{0k}(\vec{x})+\theta^{0k}(\vec{x}')]\partial_k\delta^3(\vec{x}-\vec{x}')$$

sandwiched between proton states (unless the partner $|p'\rangle$ also exists).

To conclude, we seem to have shown that a class of highly symmetrical theories of strong interactions are actually too symmetrical to agree with experiment. In the case of Sugawara's theory this is, we feel, unfortunate since this particular theory is rather beautiful and is probably the only internally consistent theory which can be constructed from usual vector and axial-vector currents alone.

Note added in proof.—After completing this work we were informed that H. J. Lipkin also pointed out similar troubles in theories invariant under two commuting Poincaré groups.[16]

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[1]H. Sugawara, Phys. Rev. 170, 1659 (1968).

[2]K. Bardakci, Y. Frishman, and M. Halpern, Phys. Rev. 170, 1353 (1968).

[3]This theory might be considered attractive because it is the "minimal" Lagrangian theory which is SU(3) $\otimes$ SU(3) symmetric, yields the "algebra of fields," and contains fermions.

[4]The $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ symmetry of this theory is not unrelated to that of Sugawara's model. In Ref. 2 it is shown that Sugawara's theory can be obtained as the limit of a Yang-Mills theory.

[5]The $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariance of this theory was first pointed out by R. Marshak and S. Okubo, in Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1966 (W. H. Freeman & Company, San Francisco, Calif., 1966).

[6]H. Sugawara, Phys. Rev. Letters 21, 772 (1968).

[7]See, for example, S. Weinberg, Phys. Rev. 166, 1568 (1968); M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); R. Dashen, to be published.

[8]In this theory $\theta^{\mu\nu}$ is the sum of three terms $\theta_{(+)}^{\mu\nu}+\theta_{(-)}^{\mu\nu}+\theta^{\mu\nu}(\pi,\sigma)$, where $\theta^{\mu\nu}(\pi,\sigma)$ commutes, at equal times, with all quantities used in constructing $\theta_{(\pm)}^{\mu\nu}$. Clearly, a theory of this type is still $\mathcal{P}_{(+)}\otimes\mathcal{P}_{(-)}$ invariant and our conclusions are unchanged.

[9]This is obviously the case in a model where the Lagrangian is the sum of two independent terms.

[10]This is not quite trivial. The presence of parity and charge-conjugation doubling means that massless pseudoscalar mesons must be accompanied by massless scalar mesons which the vector charges could make out of the vacuum. Equation (6) shows that the scalar mesons do not decouple from the vector currents unless the pseudoscalars decouple from the axial currents.

[11]Equation (6) was also derived in Sugawara's theory by W. Bierter and K. Bitar, to be published, using multiple commutators with the Hamiltonian, that is, expanding both sides of Eq. (6) as a power series in the relative time.

[12]A situation like this occurs in spinor electrodynamics where direct application of the equations of motion does not lead to the correct Ward identity for the axial current, the correction being proportional to the fine structure constant. See S. L. Adler, to be published.

[13]That Bjorken's model [J. D. Bjorken, Ann. Phys. (N.Y.) 24, 174 (1963)] does not work in the usual way can be seen as follows. In a four-fermion-interaction theory he begins with a current J^μ satisfying $\langle 0|J^\mu(0)\times|0\rangle=\eta^\mu\neq 0$. The commutator $i\langle 0|[M^{\mu\nu},J^\lambda(0)]|0\rangle=g^{\mu\nu}\eta^\nu-g^{\nu\lambda}\eta^\mu$ can be satisfied by means of a massless scalar particle χ with the couplings $\langle\chi(q)|J^\lambda(0)|0\rangle\propto q^\lambda$ and $\langle\chi(q)|\theta^{\mu\nu}(0)|0\rangle\propto[q^\mu\eta^\nu+\eta^\mu q^\nu-g^{\mu\nu}\eta\cdot q]$. However, it is easy to see that the above commutator cannot be satisfied with a massless spin-1 particle χ' since $\langle\chi'(q,\epsilon)|\theta^{\mu\nu}(0)|0\rangle$ has the wrong kinematic structure. In the case of noninvariant vacuum and only a scalar massless particle, we believe that the noninvariance is trivial and can be easily removed or else that there would be observable violations of Lorentz invariance. Note that although a coupling of the current to a massless vector particle is possible, $\langle 0|J_\lambda(0)|\epsilon(Q)\rangle\propto[\epsilon_\lambda(Q)-Q_\lambda\eta\cdot\epsilon/\eta\cdot Q]$, the propagator will not yield covariant results without a Coulomb interaction in the Hamiltonian, which is not present originally.

[14]According to P. Higgs, Phys. Rev. 145, B1156 (1966), in theories which contain long-range forces

there are other possible ways of losing invariance of the vacuum. In the theories discussed here there are no obvious long-range effects, but they might arise from singular products of operators.

[15]S. Mandelstam, Phys. Rev. 168, 1884 (1968).
[16]H. J. Lipkin, in Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1968 (to be published).