DUALITY DIAGRAMS*

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The duality between the s- and t-channel descriptions of strong-interaction scattering amplitudes and the assumption that in every channel the scattering proceeds via nonexotic resonances are exhibited by simple diagrams. The diagrams lead to simple derivations of some known consequences of these assumptions as well as to many new predictions which can be tested experimentally.

For many years it has been implicitly or explicitly assumed that, in strong-interaction scattering processes, one may add the contributions of s-channel resonances to those of the t-channel exchanges. This belief was partly motivated by the fact that such an addition is legitimate in a perturbation-type Feynman diagram approach. Recently, it has become clearer and clearer that, in general, a certain amount of double counting is involved in this procedure.[1] We now know that it is possible to construct a complete description of a strong scattering amplitude[2] either in terms of its s-channel poles or as a sum of t-channel poles. Furthermore, if the poles in all channels are associated only with nonexotic resonances,[3] this duality requirement[1] leads to self-consistency conditions on the scattering amplitudes.[4] The same idea can be generalized to many-particle final states[5] and in all cases the simple addition of the various Feynman diagrams (even when all internal lines are treated as Reggeons) leads to double (or multiple) counting.

In this paper we propose a simple type of diagrams which exhibit the duality principle in a striking way. We assume that all the incoming and outgoing particles as well as the poles in all channels are not exotic[3] and may therefore be mathematically represented by three-quark or quark-antiquark combinations. The diagrams enable us to reproduce easily a large number of known predictions of the duality picture, to understand the reason for its apparent failure in $B\bar{B}$ scattering,[6] and to derive a large number of new predictions.

The rules for drawing a legal diagram are extremely simple: (i) There are three types of lines, corresponding to the $\mathcal{P}$, $\mathfrak{N}$, and λ quarks. Lines do not change their identity. (ii) Every external baryon is represented by three lines running in the same direction. (iii) Every external meson is represented by two lines running in opposite directions. (iv) The two ends of a single line cannot belong to the same external particle. (v) In any $B = 1$ channel (s, t, or u) it is possible

to "cut" the diagram into two by "cutting" only three quark lines (and not $4q + \bar{q}$, etc.). Similarly, in any mesonic channel we should be able to split the diagram by cutting only two lines (and not $2q + 2\bar{q}$, etc.).

The physical assumptions involved are the following: (a) All baryons are in the $\underline{1}$, $\underline{8}$, or $\underline{10}$ SU(3) multiplets and can be mathematically described as three-quark structures. (b) All mesons are quark-antiquark structures in the $\underline{1}$ or $\underline{8}$ multiplets. (c) The entire scattering amplitude (except for the contribution of the Pomeranchukon) is given in any channel by a sum of single-particle states.[2,4]

The diagrams describing meson-meson scattering, and backward meson-baryon scattering are illustrated in Figs. 1(a)-1(c). The duality property is clearly demonstrated since the diagrams can be viewed either as a sum of single-particle states in one channel or as a sum of such terms in the other channel. It is clear from the dia-
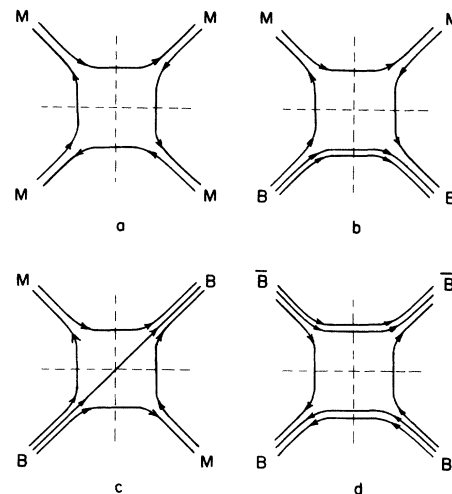

FIG. 1. Diagrams for (a) meson-meson scattering, (b) forward meson-baryon scattering, (c) backward meson-baryon scattering, and (d) an "illegal" diagram for baryon-antibaryon scattering. The s- and t-channel intermediate states are marked by dashed lines.

grams that in MM and MB scattering there is at least one self-consistent set of amplitudes which satisfy duality and resonance dominance in the s and t channels. On the other hand, Fig. 1(d) demonstrates that in $B\bar{B}$ scattering there is no way of describing the amplitude by a sum of nonexotic mesons in both the s and the t channels. At least one of these channels must have $2q + 2\bar{q}$ intermediate states, contrary to our basic rules. The inconsistency between the duality requirement and the resonance-dominance assumption in $B\bar{B}$ scattering is not a new result,[6] but here we have deduced it in a trivial way from our simple diagrams.

Two-body scattering reactions are essentially described here by a combination of single $q\bar{q}$ annihilations in every channel.[7] The only term in the scattering amplitude which does not involve such annihilations is presumably the diffraction contribution (Pomeranchukon exchange).

Similar diagrams can be drawn for processes such as $MB \rightarrow MMB$ [Fig. 2(a)]. The entire amplitude can be, in principle, described in terms of any of the five ordinary diagrams in Figs. 2(b)-2(f). Figure 2(a) includes all of these possibilities, and indicates that every one of them may be a complete description.[5]

If a certain scattering amplitude with two-particle final state is completely "explained" by a sum of direct-channel resonances, we approximate its imaginary part at a given energy by the contributions of the resonances in the neighborhood of that energy. The real part of the amplitude at the same energy will not be described in terms of nearby resonances. In general it will include effects of resonances which are far away in energy. The reason for this distinction is, of course, the fact that the imaginary part of a resonant amplitude is large around the resonance energy, while the main influence of a resonance on the real part of the amplitude is spread over a wider energy range and actually vanishes at the resonance energy. If a process such as elastic K^+p scattering does not exhibit any s-channel resonances, only the imaginary part of its forward amplitude will vanish (except for the Pomeranchuk term). The real part of the K^+p amplitude will not vanish and will have contributions from its distant $s < 0$ resonances—the ones which appear in K^-p scattering! Only if both the s and the u channels of such a process do not show any resonances will the real part of the amplitude also vanish in the resonance-dominance approximation. The immediate moral of these remarks

is that if a certain process cannot be described by our diagrams, only the imaginary part of its amplitude is predicted to vanish. The real part may be fed by the u-channel process, and only when the latter also corresponds to an illegal diagram, both the real and the imaginary part of the amplitude will vanish (again, except for the Pomeranchuk term).

The set of predictions presented here is based on searching for processes which cannot be described by a legal diagram. We can list here only a very small fraction of these predictions, and it is a trivial matter to obtain many others simply by trying to construct legal diagrams for various processes:

(1) The following processes cannot be represented by legal diagrams and are among those predicted to have purely real amplitudes at small t values: $K^+n \rightarrow K^0p, K\Delta, K^*N, K^*\Delta$, and K^-p $\rightarrow \pi^-\Sigma^+, \pi^0\Sigma^0, \pi^0\Lambda, \rho^0\Lambda, \omega\Lambda$. The general rule is that all processes of the form $K^-B \rightarrow \pi^-B'$, π^+B $\rightarrow \bar{K}^0B'$, $K^+B \rightarrow K^0B'$, and $\bar{K}B \rightarrow M^0B'$ are predicted to have vanishing imaginary parts, where B, B' are any nonexotic baryons and M^0 is any $Q = Y$ $= 0$ meson which does not contain a $\lambda\bar{\lambda}$ component

a

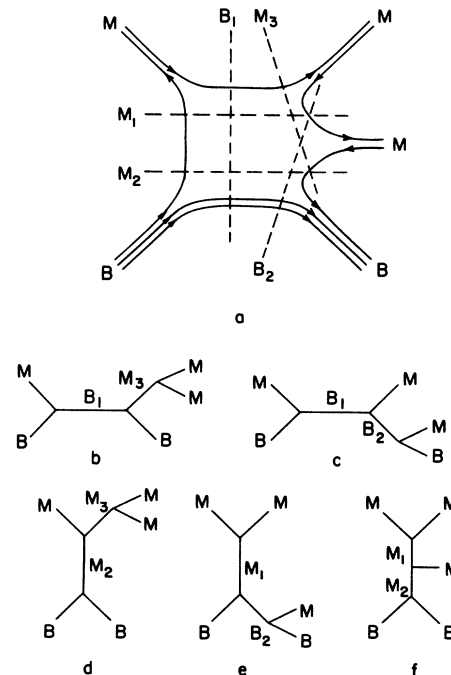b   c

d   e   f

FIG. 2. (a) Diagram for $M + B \rightarrow M + M + B$ and (b)-(f) its various alternative descriptions. Every one of the five descriptions (b)-(f) may, in principle, be a complete picture of the amplitude. They should be summed over all possible intermediate states $(B_1, B_2, M_1, M_2, M_3)$, which are marked by dashed lines in (a).

$(\pi, \rho, \omega, f^0, A_2, A_1,$ etc.). Figure 1(a) shows that in any legal diagram for forward meson-baryon scattering the initial and final meson must include the same type of quark. Hence, transitions like $K^- \to \pi^-$ lead to illegal diagrams and to the above predictions. One way of testing these predictions is to measure the polarizations of the final particles. A purely real amplitude corresponds to zero polarization.

(2) A similar rule forbids the transitions $\pi^+ \to (\mathfrak{N}\bar{\mathfrak{N}})$, $\pi^- \to (\mathcal{P}\bar{\mathcal{P}})$, $K^+ \to (\mathfrak{N}\bar{\mathfrak{N}})$, and $K^0 \to (\mathcal{P}\bar{\mathcal{P}})$. The $\mathcal{P}\bar{\mathcal{P}}$ and the $\mathfrak{N}\bar{\mathfrak{N}}$ states are definite linear combinations of $I = 1$ and $I = 0$ mesons. Assuming that these are ρ-ω of f^0-A_2 combinations, we predict the following at small t values:

$$\text{Im}(\pi^-p \to \rho^0 n) = -\text{Im}(\pi^-p \to \omega n); \quad \text{Im}(\pi^+n \to \rho^0 p) = +\text{Im}(\pi^+n \to \omega p);$$

$$\text{Im}(\pi^+p \to \omega\Delta^{++}) = +\text{Im}(\pi^+p \to \rho^0\Delta^{++}); \quad \text{Im}(\pi^-p \to f^0 n) = -\text{Im}(\pi^-p \to A_2^0 n);$$

$$\text{Im}(\rho^0 p \to K^+\Lambda) = +\text{Im}(\omega p \to K^+\Lambda); \quad \text{Im}(\rho^0 p \to \pi^-\Delta^{++}) = -\text{Im}(\omega p \to \pi^-\Delta^{++}), \text{ etc.}$$

The last two equations are relevant to photoproduction of $K^+\Lambda$ or $\pi^-\Delta^{++}$, if vector dominance is assumed.

(3) The vanishing of $\text{Im}(K^-p \to \rho^0\Lambda)$, $\text{Im}(K^-p \to \omega\Lambda)$, and the combination $[\text{Im}(\rho^0 p \to K^+\Lambda) - \text{Im}(\omega p \to K^+\Lambda)]$ together with the dispersion relations for these processes lead to $\text{Re}(K^-p \to \rho^0\Lambda) = \text{Re}(K^-p \to \omega\Lambda)$ and therefore to $\sigma(K^-p \to \rho^0\Lambda) = \sigma(K^-p \to \omega\Lambda)$ at small t. Similarly, $\sigma(K^-p \to f^0\Lambda) = \sigma(K^-p \to A_2^0\Lambda)$, etc. These predictions were previously derived from the quark model.[8]

(4) The transitions $\pi^\pm \to \varphi$ are not allowed by the diagrams. Hence $\sigma(\pi N \to \varphi N) = 0$, $\sigma(\pi N \to \varphi\Delta) = 0$, etc.

(5) There is no legal diagram for the process $\pi^-p \to K^-K^+n$ if we insist that the outgoing neutron is "tied" to the incoming proton. On the other hand, $\pi^-p \to K^0\bar{K}^0 n$ is perfectly legal. One way of understanding this peculiar prediction is to consider the $K\bar{K}$ pair as coming from intermediate $Q = Y = 0$ mesons. The initial π^- is allowed by our diagrams to produce only coherent mixtures of $I = 0$ and $I = 1$ mesons. These mixtures are forbidden to decay into K^-K^0 while the $K^0\bar{K}^0$ mode is allowed.

All of these predictions can be alternatively obtained (and some of them were indeed derived previously) by considering various combinations of assumptions such as exchange degeneracy, additivity of single-quark amplitudes, universality, SU(3) invariance, etc. We do not attempt here a complete classification of our predictions in terms of the alternative sets of assumptions needed for deriving them. We merely remark that we have obtained the predictions here just from the impossibility of drawing certain diagrams without even knowing how one calculates anything with such diagrams, and certainly without assuming SU(3)-invariant vertices, universality, or additivity of quark amplitudes. Exchange

degeneracy was also not assumed, but it can be derived in most cases from the basic rules of our diagrams and the impossibility of drawing diagrams for $\pi^+\pi^+, K^+p$ scattering, etc.

We conclude with a few remarks:

Due to the difficulty in separating the real and imaginary parts of most scattering amplitudes, it is extremely hard to test many of our predictions. The ones that can be easily tested $[\text{Im}(K^+n \to K^0p) = 0$, $\sigma(\pi N \to \varphi N) = 0$, and $\sigma(K^-p \to \Lambda\rho) = \sigma(K^-p \to \Lambda\omega)]$ agree with experiment,[9] but they were all derived previously by other assumptions and cannot be considered as crucial tests of the basic diagram rules. We have compared some of the other predictions with indirect pieces of evidence (such as phenomenological fits, polarization data, and rough order-of-magnitude estimates) and found neither clear contradictions nor great successes. The polarization tests are especially inadequate since a negligible imaginary part (such as 5% of the cross section) can lead to a large polarization (such as 20%). It is obvious that many more tests are needed before we can start taking our assumptions and methods seriously.

The assumptions involved in drawing are stronger than the requirement that the exotic SU(3) amplitudes vanish in all channels.[4] Our extra assumption can be most easily stated in the SU(3)-invariance limit (although it is not necessary to take this limit here). What we assume in addition is that $4q + \bar{q}$ or $2q + 2\bar{q}$ intermediate states are illegal even if they happen to belong to a singlet or an octet. Another way of stating the same assumption is to note that our annihilated $q\bar{q}$ pairs in all channels must be in SU(3) singlets and not in octets. This assumption is certainly reasonable if real quarks exist, but even if the quarks are only mathematical entities representing some algebraic structure, it is conceivable

that they obey our requirements.

We do not know how to include spin effects in performing quantitative calculations with the diagrams. We may use "quark-counting" assumptions or assume SU(3) invariance for the various vertices, but such requirements do not tell us much about the dynamics of the intermediate baryon and meson resonances. It should be interesting to find out whether the diagrams or some extension of them can be utilized for a more explicit understanding of the hadronic spectrum.

After completing this paper we were informed of a related work done by J. L. Rosner. We would like to thank him for helpful discussions. We also gratefully acknowledge discussions with M. Kugler, H. J. Lipkin, and A. Schwimmer.

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[6]J. L. Rosner, Phys. Rev. Letters 21, 950 (1968). See also H. J. Lipkin, Ref. 4.

[7]A similar assumption was proposed by H. J. Lipkin [Phys. Rev. Letters 16, 1015 (1966)] on pure phenomenological grounds, without appealing to the duality concept or the resonance-dominance hypothesis. See also J. J. J. Kokkedee and L. Van Hove, CERN Report No. CERN 66/248/5-TH642, 1966 (unpublished).

[8]G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters 17, 412 (1966).

[9](i) $\sigma_t(K^+p) = \sigma_t(K^+n)$ holds experimentally between 2 and 20 GeV and leads to $\text{Im}(K^+n \to K^0p) = 0$. (ii) The production of φ mesons in πN scattering is known to be extremely rare. (iii) The $\Lambda\rho$ and $\Lambda\omega$ production rates in K^-p scattering are roughly equal at 4.1 and 5.5 BeV [J. Mott et al., Phys. Rev. Letters 18, 355 (1967)]. At 2.24 BeV only the forward ρ and ω cross sections are equal, and the angular distributions are completely different [G. W. London et al., Phys. Rev. 143, 1034 (1966)].

MODEL FOR THE POMERANCHUK TERM*

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A model of the Pomeranchuk term as a sum over two-body intermediate states is constructed. Experimental tests involving resonance production are suggested.

The Pomeranchuk singularity (P) that drives elastic high-energy scattering is the least understood of all complex angular-momentum plane singularities. The nature of lower lying singularities is reasonably clear. A simple picture in terms of straight-line Regge trajectories is found to explain well both their effects in high-energy scattering and the observed particle spectrum. Particularly relevant for us are the two approximately exchange-degenerate nonets of trajectories (which we shall jointly label by R)[1] on which the vector and tensor mesons are located. Is there any chance of understanding the nature of P starting from these "usual" R trajectories? It

has been suggested[2] that the P contribution to an elastic process $AB \to AB$ is generated by the sequence $AB \to AB +$ [secondaries produced by multiperipheral R exchange (multi-Regge exchange)] $\to AB$, as shown in Fig. 1(a). The difficulty involved in this approach is that of mathematical complexity in treating many-particle intermediate states. We want to show that the model of Fig. 1(a) can be simplified to one including only two-body intermediate states. Using duality[3] one can successively "reduce" the many secondary lines in Fig. 1(a) while at the same time including higher and higher excitations of the "elastic lines" [Fig. 1(b)]. For a large class of diagrams