netic well and small shear in the inner part of the plasma column. Fluctuations have in fact been observed through ion cyclotron heating experiments[14] and it has been stated that these could well be flutelike with phase velocity in the ion direction. (D) A properly modified version of the modes discussed above may be utilized to explain the recent observation of a mode with $\omega < \hat{\omega}_{bi}$, phase velocity in the direction of the ion diamagnetic velocity but "ballooning" in a region of unfavorable curvature of the Livermore Levitron toroidal experiment[15] with $T_i > T_e$.

This paper has been stimulated by a recent work[9] of Jukes, where the importance of ordering $\hat{\omega}_{Di}^2/\omega^2 \sim \overline{\omega}_{Di}/\omega$ is pointed out in connection with the stability of the "electron" ($\omega_{bi} < \omega < \omega_{be}$) flute mode and the fluctuations reported in Ref. 11.[2] The author is also indebted to M. Rosenbluth and P. Rutherford for valuable discussions and suggestions, and to K. Allen and S. Yoshikawa for private communications on their experiments.

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MUTUAL FRICTION IN He II NEAR THE SUPERFLUID TRANSITION

Guenter Ahlers
Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey
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Measurements of supercritical heat flow in He II are consistent with a mutal friction constant which diverges near T_λ. They also explain the apparent depression of T_λ observed by Erben and Pobell, and the "critical heat flux" near T_λ reported by Bhagat and Winer.

Recently, Erben and Pobell[1] (EP) reported a depression by a heat current of the superfluid transition temperature T_λ in He^4, and Bhagat and Winer[2] (BW) measured a "critical heat flux" in He II by observing the formation of bubbles at a heated wire. In the present communication, measurements of the heat flux q in a long cylinder of He II, one end of which is at T_λ and the other end at $T < T_\lambda$, are reported. All three experiments, as well as the results of Keller and Hammel (KH) near T_λ,[3] can be interpreted[4] in terms of a Gorter-Mellink mutual friction constant[5] which diverges near T_λ. Therefore, it is not necessary to postulate a depression of T_λ due to q as proposed by EP,[1] and the BW results need not be explained on the basis of a new critical velocity.[2,6]

The present measurements were made in a stainless steel capillary, 2×10^{-2} cm i.d. and 4×10^{-2} cm o.d., suspended in a vacuum surrounded by liquid helium at 1.2°K. Attached to one end of the capillary and otherwise thermally isolated was a reservoir filled with He II which served as a heat sink at a temperature $T < T_\lambda$. Here, $T_\lambda - T$ was measured to $\pm 10^{-6}$ °K,[7,8] and the heat flow q through the helium in the capillary was determined from the rate of heating and the known heat capacity of the reservoir. The temperature T' of the capillary 8 cm above the reservoir could be regulated at $T' - T_\lambda \cong 10^{-3}$ °K. Because of the relatively small thermal conductivity of He I, the capillary was at T_λ immediately below this point, and the effective length of the superfluid column was 8 cm. This was verified by establishing experimentally that q was independent of T' provided $T' > T_\lambda$. The results are shown in Fig. 1 for 1.5 and 0.5 bar pressure.

Sufficiently close to T_λ, the data can be represented by

$$q = b(T_\lambda - T)^x, \tag{1}$$

where $x = 1.077 \pm 0.014$, and $b = 39.8 \pm 4.0$ at 1.5 bar and 37.0 ± 4 at 0.5 bar, for $T_\lambda - T \leq 10^{-3}$ °K [q is in W cm^{-1} (°K)$^{-1}$]. Thus, by extrapolation it is expected that at saturated vapor pressure $b \cong 36 \pm 4$. At a temperature $T_c = T_\lambda - 1.0 \times 10^{-3}$ °K there is a sharp transition to a different temperature dependence, and $x = 0.821$ and $b = 6.56$ accurately describe the data for 1.0×10^{-3} °K $\leq T_\lambda - T \leq 1.5 \times 10^{-2}$ °K at 1.5 bar.

The existence of two well-defined regions with different temperature dependences of q was observed also by BW,[2] whose results are indicated in Fig. 1 by the shaded area. In the region near T_λ, BW obtained the value $x = 1.14 \pm 0.04$, which is similar to the value obtained here. However, the transition to the second region occurred at a lower temperature ($T_\lambda - T_c = 7 \times 10^{-3}$ °K) and for $T < T_c$, BW found $x \cong 0.5$. The results of KH[3] are shown as well in Fig. 1. They, too, are consistent with the existence of two regions, one near T_λ with $x \cong 1.08$ and consistent with the present measurement, and one further below T_λ, this time with $x \cong 0.35$. The transition occurs at $T_\lambda - T_c \cong 10^{-1}$ °K.[9] The combination of these various results strongly suggests that near T_λ where x appears geometry independent, all three experiments measure an intrinsic property of He II, and that outside this region there are geometry dependent effects which do not permit a simple interpretation.[10]

In order to explain the intrinsic behavior above T_c, we consider the phenomenological approach

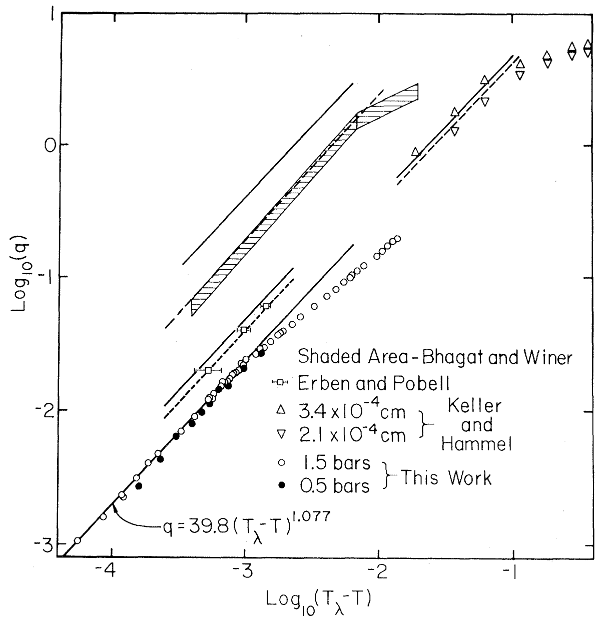

FIG. 1. The heat flux q in W cm^{-2} as a function of $T_\lambda - T$ in °K on logarithmic scales, for Bhagat and Winer (Ref. 2), Erben and Pobell (Ref. 1), Keller and Hammel (Ref. 3), and this work. The solid and dashed lines are obtained by scaling the present results according to $q \propto L^{-1/m}$, with $m = 3$ and 4, respectively.

first suggested by Gorter and Mellink.[5] They found that $q \propto (\text{grad}\, T)^{1/m}$, $m = 3$, and suggested that this relation can be understood on the basis of a mutual friction force between the normal fluid and the superfluid proportional to $(v_s - v_n)^m$, where v_s and v_n are the super- and normal-fluid velocities, respectively. Extensive investigations by others[3,11-16] have confirmed and extended the earlier ideas, and the relation[17]

$$q = \rho_s S T (S/\rho_n)^{1/m} A^{-1/m} (\text{grad}\, T)^{1/m}, \tag{2}$$

seems firmly established provided viscosity effects are negligible. However, values of m between 3[5,11] and 4[14,15,18] have been obtained, and the Gorter-Mellink constant A is found to be temperature dependent and to increase upon approaching T_λ.[11,18] Therefore, A will be written as

$$A = a(T_\lambda - T)^\alpha, \tag{3}$$

and m will be kept arbitrary. It is known[19-21] that

$$\rho_s = k(T_\lambda - T)^{2/3}. \tag{4}$$

Near T_λ, where S, T, and ρ_n are essentially constant, Eq. (2) can be integrated, and yields, with the aid of Eqs. (3) and (4), for heat flow between T_λ and T in a tube of uniform cross section,

$$q = kST[S/(\rho_n aLy)]^{1/m}(T_\lambda - T)^{y/m}, \quad (5)$$

where

$$y = 1 - \alpha + 2m/3, \quad (6)$$

and where L is the tube length. The same relation results for radial heat flow into an unbounded volume of He II in a cylindrical geometry where $T = T_\lambda$ at a radial distance R from the axis. Here, however, $L = R/(m-1)$, and q is the power density at R. Equation (5) implies that, at constant T, $q \propto L^{-1/m}$. The present results were scaled to the lengths appropriate for the other experiments [$L_{BW} = 2.5\times10^{-3}/(m-1)$ cm, $L_{EP} = 0.7$ cm, $L_{KH} = 1.9$ cm], and yield for $m = 3$ and 4 the solid and dotted lines, respectively, in Fig. 1. Although L differs by a factor of about 10^4 between the present and the BW experiment, good agreement with all the data is obtained, especially for $m = 4$.

For the Gorter-Mellink mutual friction constant[5] A [Eq. (3)] the present data yield $\alpha = -0.23 \pm 0.04$, $a = 31 \pm 10$ if $m = 3$, and $\alpha = -0.64 \pm 0.04$, $a = 0.45 \pm 0.18$ if $m = 4$. The combination of available evidence seems to indicate that m varies from about 3 at low temperatures to about 4 at T_λ. It is evident that comparison of A as determined here with previous data is possible only if the same value of m is used. Such a comparison with the data of Vinen[11] ($m = 3$) indicates that the present data are lower by a factor of about 3 near T_λ. However, Vinen's estimates are based upon $q \propto (\mathrm{grad}\,T)^{1/3}$ at large q.[18] His results at small q and 2.111°K yield a value of A smaller than that reported by him by a factor of about 2.5, and thus are in agreement with the present result. The strong dependence of A upon m makes it difficult to interpret A in a physically significant manner, unless insight is gained into the behavior of m, especially as a function of T.[22]

It is concluded that the results of Bhagat and Winer,[1] Erben and Pobell,[2] Keller and Hammel[3] near T_λ, and the present work are all consistent with a diverging mutual friction constant. However, the detailed behavior of the mutual friction force[5] cannot be characterized because information on the dependence of q on grad T is inadequate at present.

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