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DELAY TIMES IN FLUID SPIN-UP; CONTRAST TO LIQUID HELIUM*

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Measurement of the spin-up from rest of ordinary liquids in cylindrical containers shows that the delay times measured by Pellam in liquid helium-II are not classical and that the normal-fluid spin-up is suppressed.

In the controversy[1-3] over the "Pellam anomaly" (i.e., the peculiar temperature dependence of the steady-state Rayleigh disk deflection in rotating liquid helium), it has been thought that the rest of Pellam's results are consistent with ordinary-fluid spin-up from rest.[4] We report here delay-time measurements in ordinary liquids (water, ethylene glycol, and silicone fluid) which show that Pellam's delay-time results are not classical and that the spin-up of the liquid-helium normal component is suppressed.

The delay time t (defined as the time from the impulsive start of rotation of the container to the first response of the fluid a distance D from the container wall) was measured in open-topped cylindrical containers as a function of cylinder radius a (4.73 to 7.36 cm), fluid kinematic viscosity ν $[(0.90\text{-}53.0)\times10^{-2}\ \text{cm}^2/\text{sec}]$, fluid depth h (2.5 to 14.5 cm), angular speed of the container Ω (0.0375 to 2.77 rad/sec), and distance D from the detector edge to the container wall (0.51 to 5.24 cm). The detector, similar to that used by Craig,[5] consisted of two flat plates (4.0×7.5 mm^2) mounted vertically at the ends of a horizontal rod (0.74-mm diam). The midpoint of the horizontal rod was clamped to the end of a vertical support rod, which was suspended from a quartz torsion fiber.

Figure 1 shows the delay-time results for Reynolds numbers ($a^2\Omega/\nu$) between 100 and 15 000, which can be expressed by the following empirical equation:

$$t = 0.1068h^{1/2}(\nu^3\Omega)^{-1/4}D^{1.4}a^{-0.4}. \qquad (1)$$

In contrast, Pellam[1] found at 2.0°K, with $\Omega = 0.21$ rad/sec, $a = 2.45$ cm, and $h \approx 10$ cm,

$$t = 342D \text{ (cgs units)}, \qquad (2)$$

where D is the distance from the wall to the nearest edge of the Rayleigh disk. Furthermore, he found t to be independent of fluid depth[6] and relatively insensitive to temperature[7] (or possibly increasing with decreasing temperature[6]). Neither Eq. (1) nor Eq. (2) agrees with viscous diffusion theory,[8] which predicts $t \propto D^2/\nu$, or with the high-Reynolds-number theory of Wedemeyer,[9] which predicts $t = 1.128h(\nu\Omega)^{-1/2}\ln[a/(a-D)]$.

Since both the second-sound velocity[10] and the fountain pressure[11] are known to be unchanged by rotation, one would expect that the appropriate kinematic viscosity for normal-fluid spin-up would be η_n/ρ_n (normal-fluid viscosity and density). However, when Pellam's data at 2.0°K are plotted in Fig. 1 with $\nu = \eta_n/\rho_n$ (1.806×10^{-4} cm^2/sec) the results are strikingly different from those in ordinary liquids. Even when $\nu = \eta_n/\rho$ is used (ρ = total density = 0.1457 g/cm^3), the liquid-

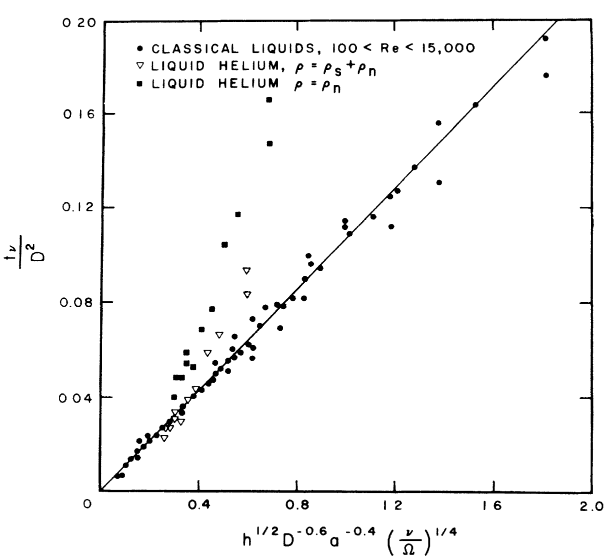

FIG. 1. Delay-time results for ordinary liquids compared with Pellam's results for liquid helium II at 2.0°K. The solid line is Eq. (1).

helium results are significantly different. The linear dependence on D in Eq. (2) suggests on dimensional grounds that in liquid helium, $t \propto (\nu\Omega)^{-1/2}$. Using this relation in Eq. (2) we find at 1.0°K with $\nu = \eta_n/\rho_n = 3.70\times10^{-2}$ cm^2/sec and with $D = 1$ cm, the delay time is 24 sec. This is an order of magnitude less than the observed delay times (~350 sec). (With $\nu = \eta_n/\rho = 2.65\times10^{-4}$ cm^2/sec, $t = 212$ sec at 1.0°K.) Since the process with the lowest delay time should dominate,[12] we conclude that normal-fluid spin-up is suppressed, and that the spin-up process in liquid helium-II is governed by an effective kinematic viscosity (~0.02×10^{-2} cm^2/sec) which is approximately independent of temperature. If Eq. (2) with $t \propto (\nu\Omega)^{-1/2}$ is found to hold in liquid helium, then delay-time measurements can be used to determine the effective ν. Furthermore, since the detector is in quiescent fluid until the last instant, delay-time measurements are not sensitive to wake formation or vortex shedding at the detector. Indeed, using small spheres and plates of different size, we have found that in ordinary liquids the delay time is independent of detector size and shape.

The results presented in Fig. 1 are very sensitive to the choice of D. We have chosen D to be the distance to the detector edge nearest to the wall. The data presented by Pellam[1] involve the distance to the center of the Rayleigh disk, but as Pellam himself points out, his results, extrapolated to $t = 0$, indicate that the disk responds when the moving liquid reaches the detector edge. For a detector of the type we have used, one would also expect the distance to the detector edge to be the appropriate one; to check this point, we have made measurements with fixed D (1.05 cm) and different plate widths (4 and 10 mm) and have found, as expected, no difference in the delay times.

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