other matters, makes much the same point discussed here.

*Work supported by the Air Force Office of Scientific Research, U. S. Air Force, under Contract No. AF49(638)-1545.

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HIGH-ENERGY pp SCATTERING AND PROTON SUBSTRUCTURE

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(Received 30 January 1969)

A proton substructure model proposed earlier is used to analyze present data on high-energy pp angular distributions. The model satisfactorily describes differential cross sections over the entire momentum range 5-21 GeV/c and over the whole angular interval 0-90°, thus indicating existence of two hadronic core distributions of the proton.

We proposed a model of high-energy large-angle pp elastic scattering some time ago.[1] The model was further developed[2] and applied to analyze the pp elastic scattering data of Allaby et al.[3] and the proton form-factor data of Coward et al.[4] Recently more data have been reported by the CERN group.[5] These data together with the small-angle data of the Brookhaven group[6] provide fairly complete pp angular distributions at a large number of energies. We therefore decided to investigate whether our proposed model could consistently describe this large amount of data and to this end carried out an extensive analysis of the pp elastic scattering data. The purpose of this note is to present briefly the main results.

The first problem one faces in a program of this nature is determining what kind of data plots test most stringently the special features of the model. To this end we examined the asymptotic form of our model.[7] This is given by

$$f(s,\Delta)=\frac{ik\sigma_T^{\,d}}{4\pi}e^{\frac{1}{8}R^2\Delta^2}-\left(1-\frac{\sigma_T^{\,d}}{\pi R^2}\right)\sum_{i=1}^{2}g_i(s)\beta_i(\tau^2+\mu_i^{\,2})^{1/2}K_1(\beta_i(\tau^2+\mu_i^{\,2})^{1/2})(\Delta^2+\mu_i^{\,2})^{-1}, \tag{1}$$

where $\Delta=(-t)^{1/2}=2k\sin\frac{1}{2}\theta$, $\tau=\Delta(1-\Delta^2/s)^{1/2}\approx k_\perp$, and $k_\perp=k\sin\theta$. The first term on the right-hand side in Eq. (1) is due to diffraction scattering. The second ($i=1$) and the third ($i=2$) terms are associated with two hadronic distributions of the proton which interact via complex energy-dependent optical potentials. These optical potentials are supposed to arise from exchanges of vector mesons ω and ω'.[2] Let us now examine what kind of differential cross section Eq. (1) will predict as a function of τ for fixed s. In the small τ region, where $\tau\approx\Delta$, the diffraction amplitude can dominate and give the characteristic Gaussian shape to the differential cross section. However, as τ increases the $i=1$ term can take over since as a function of τ this term falls off much more slowly than the Gaussian. If τ is at the same time sufficiently large, then in a plot of $\ln d\sigma/d\Omega$ vs τ the differential cross section should look almost linearly falling in τ.[8] At still larger values of τ, the $i=2$ term must dominate since it falls off less rapidly than the $i=1$ term.[9] However, the region where it dominates should have a slope less than the region where the $i=1$ term dominates. The three regions can further be identified by looking at their energy dependence. The diffraction amplitude increases as k so that in a plot of $\ln d\sigma/d\Omega$ vs τ, the diffraction peak should show a slow logarithmic increase with s. Earlier analysis has indicated that $g_1(s)$ does not vary appreciably with energy.[2] Thus, in the region where the $i=1$ amplitude dominates, the energy dependence of the differential cross section should also be small. Finally, in the near-90° region where the $i=2$ amplitude dominates the differential cross section should fall off rapidly with energy. There are two reasons for this: First, as the energy increases the near-90° scattering corresponds to larger values of τ. Since the $i=2$ term in Eq. (1) falls off exponentially in τ, this will produce a rapid drop. Second, $g_2(s)$ itself decreases rapidly with increase in energy as indicated by the earlier analysis. We thus expect to see three different regions in the angular distribution characterized by their τ dependence and energy dependence.

With these arguments in mind, let us look at the actual fits of our model to the experimental pp differential cross sections as shown in Fig. 1.[10] Complete angular distributions are given for the momenta 19.3, 12.1, 7.1, and 5.0 GeV/c together with experimental data. For other momenta only the large-angle cross sections and fits are given. The scattering amplitude has been properly symmetrized to take into account the identity of the two protons. Concentrating on one fixed-energy curve, say the 19.3-GeV/c curve, we notice that for $\tau<1.0$ its shape indicates the dominance by diffraction scattering. For $1.4<\tau<2.2$ there is indeed a linear fall in τ. Finally, for $\tau>2.6$ we find that the curve has a slope less than that in the middle region. Let us tentatively identify these three regions with the three regions discussed earlier and examine their energy dependence by comparing with the 12.1-GeV/c curve. We notice that indeed in the small-τ region the angular distribution has shown a slight increase, in the middle region the two angular distributions are quite close together, while the tail parts of the two curves show considerable fall with energy. Thus the existence of three regions in a complete angular distribution is strongly indicated. Further examination of the large-angle fits at

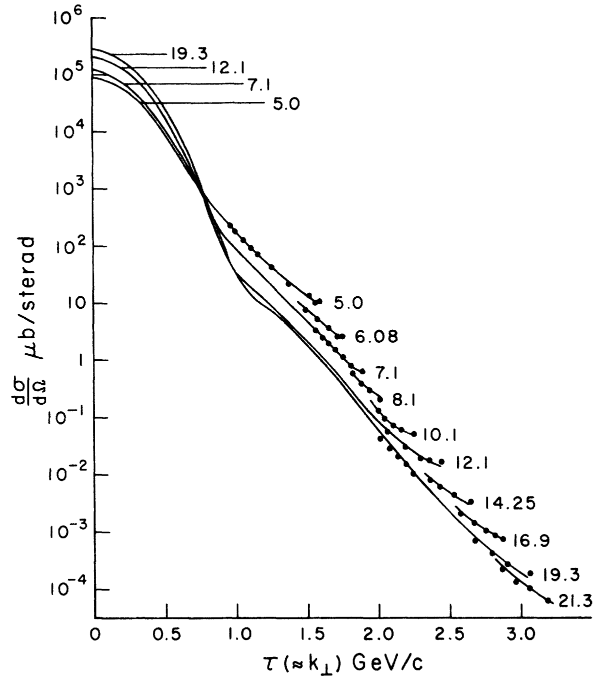

FIG. 1. Solid curves represent fits to the experimental data. Only a selected number of large-angle experimental cross sections are given. The data shown are from Allaby et al., Refs. 3 and 5; A. R. Clyde, University of California Lawrence Radiation Laboratory Report No. UCRL 16275, 1966 (unpublished); and C. M. Ankenbrandt et al., Phys. Rev. 170, 1223 (1968).

other momenta very distinctly indicate the rapid fall with energy of the near-90° scattering. In Fig. 2(a) the separate contributions of the three different amplitudes to the angular distribution at 19.3 GeV/c are given. It shows quantitatively how these amplitudes dominate the three different regions. A similar curve for $p_{\rm lab}=5$ GeV/c is shown in Fig. 2(b). Here we notice that large interference terms between the amplitudes exist; thus the characteristic of any particular amplitude does not show up.

A few remarks on some of the special features of the angular distributions can be made at this point. We notice the appearance of dip shoulder structure around $\tau=1.1$ with increase in energy. Experimentally, this is now well established.[5] In our model this is connected with the transition from the diffraction region to the region where the $i=1$ amplitude dominates. Fig. 2(a) clearly indicates this. Close examination of the fixed-energy angular distributions in Fig. 1 shows that the slopes of the middle region and the end region

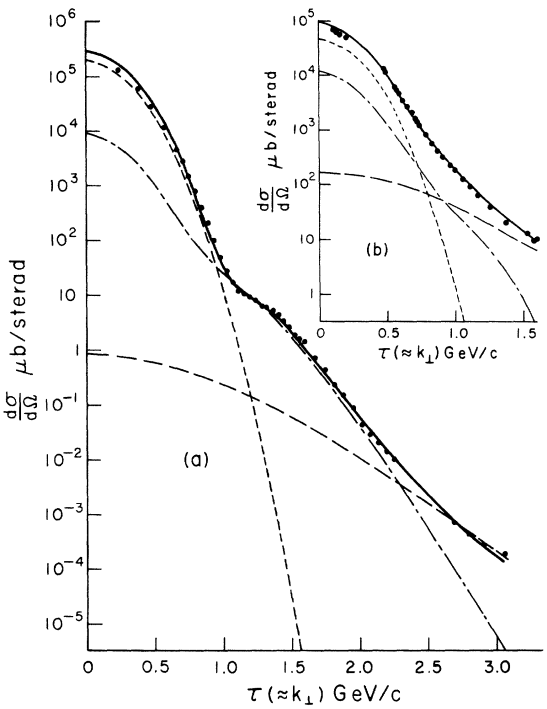

FIG. 2. Solid curves represent fits to the experimental data. The short-dashed curve is the contribution of the diffraction amplitude, the dot-dashed curve that of the $i=1$ amplitude, and the long-dashed curve that of the $i=2$ amplitude. (a) $p_{\rm lab}=19.3$ GeV/c; experimental cross sections are from Allaby et al., Ref. 5. (b) $p_{\rm lab}=5.0$ GeV/c; experimental cross sections are from A. R. Clyde, University of California Lawrence Radiation Laboratory Report No. UCRL 16275, 1966 (unpublished).

are different, which has also been observed experimentally.[3] The clustering of the differential cross sections in the middle region[5] is explained in our model by the slow variation of $g_1(s)$ with s. We have also examined the effect of identity of the two protons by comparing the unsymmetrized differential cross section with the properly symmetrized one. The effect is to decrease the latter relative to the former at large angles.

The validity of our model will imply that the proton has substructure[11]; namely, two hadronic core distributions interacting via exchange of vector mesons ω and ω'. The corresponding vector-meson–nucleon form factors provide an asymptotic connection between high-energy nucleon-nucleon scattering and proton electromagnetic form factor.[12] This is to be contrasted with other models giving similar connections, such as the Chou-Yang model[13] and the Abarbanel-Drell-Gillman model,[14] where knowledge of the electromagnetic form factor is assumed. Recently models with an infinite number of Gaussian terms, which have asymptotically exponential fall in $k_\perp$, have been proposed by Fleming, Giovannini, and Predazzi[15] and by Cocconi.[16] If the modified Bessel functions in Eq. (1) are written as integrals over Gaussian integrands,[17] then our model becomes essentially parallel to these models. However, our model predicts the existence of two slopes in an Orear-type plot for each energy, while these models do not.

Details of the present analysis will be reported elsewhere. We would like to thank Mr. Bradford J. Kirby for assistance in programming.

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*Research supported in part by the U. S. Atomic Energy Commission.

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[7]By asymptotic form we mean the $-t\to\infty$ limit for the distorted-wave Born-approximation amplitudes.

[8]This follows from the asymptotic behavior of the modified Bessel function: $K_1(\beta(\tau^2+\mu^2)^{\frac{1}{2}})\sim e^{-\beta\tau}$ for $\tau\to\infty$.

[9]Note that $\beta_1>\beta_2$.

[10]These fits are obtained using Eq. (10) of Ref. 2 and not its asymptotic form given by Eq. (1) here. Except for the energy-dependent parameters $g_1(s)$ and $g_2(s)$ all other parameters are the same as in Ref. 2.

[11]A model with proton substructure has also been proproposed by Krisch. However, present experimental evidence indicates that this model is wrong; see J. G. Asbury et al., Phys. Rev. Letters 21, 1097 (1968).

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ERRATUM

VARIATION IN CHARGE-CARRIER RELAXATION TIME IN BISMUTH WITH MAGNETIC FIELD. R. T. Isaacson and G. A. Williams [Phys. Rev. Letters 22, 26 (1969)].

Equation (4) should read

$$r_{12} = -r_{21} = \frac{N-1}{N+1}$$

instead of

$$r_{12} = -r_{21} \frac{N-1}{N+1}.$$