ASYMPTOTIC PROPERTIES OF ELECTROPRODUCTION STRUCTURE FUNCTIONS*

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The joint implications of Bjorken and Regge asymptotics are discussed for the structure function of electroproduction. It is suggested that for $q^2\to\infty$, the function νW_2 approaches a constant for large values of $\rho\equiv m\nu/q^2$.

The structure functions W_1 and W_2 which enter into the description of inelastic electron-nucleon scattering also figure as absorptive parts in the amplitude for forward Compton scattering of massive photons with spacelike momentum q. As such, these structure functions carry information about electromagnetic current commutators at unequal times. They have become objects of considerable theoretical[1-3] as well as experimental interest.[4] In particular, Bjorken[2] has argued recently that the structure functions may have simple but nontrivial properties in a certain asymptotic limit, to be described below. Our purpose here is to adjoin to these conjectures the further information that comes from a different kind of asymptotic limit, that corresponding to the regime of Regge-pole dominance.

Let us first recall the definitions of the structure functions. Consider the forward elastic scattering of a massive photon of spacelike momentum q by a physical nucleon of momentum p; so $q^2>0$ and $p^2=-m^2$. The amplitude, averaged over nucleon spins, has the form

$$T_{\mu\lambda}=T_1(q^2,\nu)\left(\delta_{\mu\lambda}-\frac{q_\mu q_\lambda}{q^2}\right)+\frac{T_2(q^2,\nu)}{m^2}\left(p_\mu+\frac{m\nu}{q^2}\right)\left(p_\lambda+\frac{m\nu}{q^2}q_\lambda\right),\quad(1)$$

where $\nu=-p\cdot q/m$ is the laboratory energy of the photon and μ and λ are photon polarization indices. The structure functions W_1 and W_2 are related to the absorptive parts of T_1 and T_2:

$$W_i=\pi^{-1}\operatorname{Im}T_i.\quad(2)$$

We may also recall how these functions enter into the process of inelastic electron scattering by a nucleon. Let E and E' be, respectively, the initial and final electron energies, as viewed in the laboratory frame, and let θ be the electron scattering angle. The differential cross section, averaged over target nucleon spin and summed over all final-state variables other than θ and E', is given by[5]

$$\frac{\partial^2\sigma}{\partial\Omega\partial E'}=\frac{\alpha^2}{4E^2\sin^4\frac12\theta}[W_2(q^2,\nu)\cos^2\tfrac12\theta+2W_1(q^2,\nu)\sin^2\tfrac12\theta],\quad(3)$$

where $\nu=E-E'$ and $q^2=4EE'\sin^2\frac12\theta$.

Let us now regard the functions W_i as depending on the variables q^2 and $\rho\equiv m\nu/q^2$. According to Bjorken,[2] the structure functions W_1 and q^2W_2 have simple limits as $q^2\to\infty$, ρ fixed:

$$\lim_{q^2\to\infty} W_1(q^2,\rho)\to F_1(\rho);$$

$$\lim_{q^2\to\infty} q^2W_2(q^2,\rho)\to\frac{m^2}{\rho}F_2(\rho).\quad(4)$$

The sum-rule arguments which suggest that the structure functions do not blow up in the above limit do not rule out the possibility that the limits are in fact zero. However, insofar as the limits <u>are</u> nonvanishing, the question arises whether there are any other principles available that bear on the functional dependence of F_1 and F_2 on the variable ρ.

Let us imagine that the functions $W_i(\nu,q^2)$ are expressed in terms of a Regge-Sommerfeld-Watson representation of the t-channel process $\gamma+\gamma\to N+\bar{N}$, appropriately analytically continued from the region $t>4m^2$, $q^2<0$ to the "Compton" region required for our problem, namely, $t=0$, $q^2>0$ and $\nu=-p\cdot q/m$ physical. This will consist of a background integral and Regge-pole terms, the latter having the general form

$$\beta(q^2,\alpha)\left(\frac{p_tq_t}{m^2}\right)^{\alpha(t)}P_{\alpha(t)}\left(\frac{m\nu-\frac14t}{p_tq_t}\right)\quad(5)$$

for $t\neq0$, where $p_t=(\frac14t-m^2)^{1/2}$, $q_t=(\frac14t+q^2)^{1/2}$; we have extracted a conventional threshold factor from the residue function. Near $t=0$, the argument of P_α is $\nu/\sqrt{(q^2)}$, which is very large in the Bjorken limit $\rho = m\nu/q^2$ finite as $q^2 \to \infty$; thus the P_α above may be replaced by $(\nu/\sqrt{q^2})^\alpha$ in the limit. The essential point is that the ν dependence is exhibited by the Regge representation in the Bjorken limit. We elaborate on this point below.

In the case at hand, taking into account the photon spin, etc., one finds (α is the leading trajectory with natural parity and even charge conjugation evaluated at $t=0$) for $\nu \to \infty$, q^2 fixed,

$$W_1 \to \beta_1(q^2, \alpha)(\nu/m)^\alpha + \cdots,$$
$$W_2 \to \beta_2(q^2, \alpha)(\nu/m)^{\alpha-2} + \cdots, \tag{6}$$

where the dots mean lower lying trajectories and background integrals. Assuming that Bjorken's limit exists and is different from zero, we conjecture that in the large q^2 limit these leading Regge terms survive; so that as $q^2 \to \infty$,

$$\beta_1(q^2, \alpha) \to \lambda_1(\alpha)(m^2/q^2)^\alpha,$$
$$\beta_2(q^2, \alpha) \to \lambda_2(\alpha)(m^2/q^2)^{\alpha-1}; \tag{7}$$

and the functions F_1 and F_2 become, as $\rho \to \infty$,

$$F_1(\rho) \to \lambda_1\rho^\alpha + \cdots,$$
$$F_2(\rho) \to \lambda_2\rho^{\alpha-1} + \cdots, \tag{8}$$

where again the dots mean lower trajectories and background.

It should be stressed that our conjectures do not follow inexorably from the assumption that both the Bjorken and Regge limits are correct in their respective domains. What could happen is that the residue of the leading pole falls off more rapidly than we have assumed; so that the behavior of the F_i might be dictated by lower trajectories or conceivably even by the background. Fortunately this is a question which can be studied experimentally since for large ρ, whatever the leading singularity in the angular momentum is will be revealed in the asymptotic dependence on ρ of the F_i.

The theoretical question of the behavior of a residue function in its dependence on an external mass (imaginary, at that), of a "particle" of fixed spin, is not one that can be posed by an S-matrix purist. It is natural to turn to the only reasonably tractable model which is known to exhibit Regge behavior, namely the set of all simple ladder graphs, to test the attractive hypothesis that the Regge-pole terms indeed fall off with q^2 in precisely the correct manner that would enable them to yield the behavior of the $F_i(\rho)$ for large ρ (rather than the background, say).

We have considered the example of the forward Compton scattering of scalar photons on scalar nucleons obtained by simply affixing the photons to the end of a ladder, the other terminal of which is the scalar nucleon. For fixed $q^2 > 0$ one finds[6] for the absorptive part of the amplitude $A(\nu, q^2) \to \beta(q^2)\nu^\alpha$ as $\nu \to \infty$, where α depends on the coupling constant. But more interestingly, one finds that the residue in fact falls off for large q^2 in a correlated way, $\beta \to (q^2)^{-\alpha-1}$. Indeed for $m\nu, q^2 \gg m^2$, the absorptive amplitude behaves like

$$A(\nu, q^2) \sim (1/q^2)(m\nu/q^2)^\alpha \tag{9}$$

with no condition that $m\nu/q^2 (=\rho)$ be large.

In this model, strictly speaking, the Bjorken limit ($q^2 \to \infty$, ρ fixed) is zero and therefore trivial. However, it is encouraging that the q^2 and ν dependence are correlated in a way which is so nearly what we are requiring. The extra factor of $(q^2)^{-1}$ in this model does not depend on the coupling constant and we are strongly inclined to regard it as "kinematic": a reflection of the spinlessness of the photon in this model. Such factors can easily be different when spin effects are taken into account, so that the relevant quantity could turn out to be q^2A for which the Bjorken limit should apply.

We are led from our study of models to believe that if the Bjorken limit is nontrivial, the limit functions $F_i(\rho)$ are likely to be governed at large ρ by the leading Regge trajectory. If we take for this the Pomeranchuk trajectory with $\alpha = 1$ our predictions are

$$F_1(\rho) \to \text{const} \times \rho,$$
$$F_2(\rho) \to \text{const}, \tag{10}$$

as $\rho \to \infty$. Present experimental indications[4] are encouraging, especially regarding the more readily measurable structure function W_2. For the larger values of ν and q^2 (though the latter quantity is still not very big) it appears that $\nu W_2(\nu, q^2)$ has become relatively insensitive to both variables, ν and q^2. This accords with the existence of a nontrivial Bjorken limit function $F_2(\rho)$ and implies, for large ρ, that this function is in turn insensitive to ρ, in agreement with our prediction.

After this work was completed, we received a preprint from H. Harari which, along with many other matters, makes much the same point discussed here.

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[2]J. D. Bjorken, Stanford Linear Accelerator Center Report No. SLAC-PUB-510, 1968 (unpublished).

[3]C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969), and 21, 311 (1968).

[4]For a good summary of the experimental picture, one may consult the review talk by W. K. H. Panofsky, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968).

[5]S. D. Drell and J. D. Walecka, Ann. Phys. (N.Y.) 28, 18 (1968).

[6]The literature on the use of ladder graphs in deducing Regge behavior is quite extensive. It may be traced from G. Tiktopoulos and S. B. Treiman, Phys. Rev. 135, B711 (1964), and 136, B1217 (1964). Strictly speaking, in these references upper and lower bounds of the form of Eq. (9) are given for massless ladder rungs and an upper bound of this form is demonstrated for massive rungs. For massless rungs exact results have been obtained by N. Nakanishi, Phys. Rev. 135, B1430 (1964).

HIGH-ENERGY pp SCATTERING AND PROTON SUBSTRUCTURE

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A proton substructure model proposed earlier is used to analyze present data on high-energy pp angular distributions. The model satisfactorily describes differential cross sections over the entire momentum range 5-21 GeV/c and over the whole angular interval 0-90°, thus indicating existence of two hadronic core distributions of the proton.

We proposed a model of high-energy large-angle pp elastic scattering some time ago.[1] The model was further developed[2] and applied to analyze the pp elastic scattering data of Allaby et al.[3] and the proton form-factor data of Coward et al.[4] Recently more data have been reported by the CERN group.[5] These data together with the small-angle data of the Brookhaven group[6] provide fairly complete pp angular distributions at a large number of energies. We therefore decided to investigate whether our proposed model could consistently describe this large amount of data and to this end carried out an extensive analysis of the pp elastic scattering data. The purpose of this note is to present briefly the main results.

The first problem one faces in a program of this nature is determining what kind of data plots test most stringently the special features of the model. To this end we examined the asymptotic form of our model.[7] This is given by

$$f(s,\Delta)=\frac{ik\sigma_T^d}{4\pi}e^{\frac{1}{8}R^2\Delta^2}-\left(1-\frac{\sigma_T^d}{\pi R^2}\right)\sum_{i=1}^{2}g_i(s)\beta_i(\tau^2+\mu_i^2)^{1/2}K_1(\beta_i(\tau^2+\mu_i^2)^{1/2})(\Delta^2+\mu_i^2)^{-1}, \quad (1)$$

where $\Delta=(-t)^{1/2}=2k\sin\frac{1}{2}\theta$, $\tau=\Delta(1-\Delta^2/s)^{1/2}\approx k_\perp$, and $k_\perp=k\sin\theta$. The first term on the right-hand side in Eq. (1) is due to diffraction scattering. The second ($i=1$) and the third ($i=2$) terms are associated with two hadronic distributions of the proton which interact via complex energy-dependent optical potentials. These optical potentials are supposed to arise from exchanges of vector mesons ω and ω'.[2] Let us now examine what kind of differential cross section Eq. (1) will predict as a function of τ for fixed s. In the small τ region, where $\tau\approx\Delta$, the diffraction amplitude can dominate and give the characteristic Gaussian shape to the differential cross section. However, as τ increases the $i=1$ term can take over since as a function of τ this term falls off much more slowly than the Gaussian. If τ is at the same time sufficiently large, then in a plot of $\ln d\sigma/d\Omega$ vs τ the differential cross section should