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FLUCTUATIONS IN MULTIPOLE CONFINED PLASMAS*

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We propose an explanation for different types of fluctuations observed in multipole experiments based on a new formal solution of the general integral equation that yields, besides the known hydromagnetic mode types, modes with frequency smaller than the average ion slow-excursion frequency. The relevant modes are of flute type, have frequency range and phase velocity in agreement with observation, and become almost purely growing and unlikely to be observed as fluctuations in conditions where these have not been detected.

A considerable number of experiments of plasma confinement have been recently carried out in multipole configurations.[1] These are generally axisymmetric and have current conductors inside the plasma that generate a stable magnetic configuration, from the hydromagnetic point of view. The observations have shown in all cases the existence of anomalous particle losses, i.e., not due to simple collisional effects, and in the case of quadrupole configurations, the appearance of fluctuations which are extended over the entire length of the magnetic field lines and have different phase velocity for different experimental situations. In the following analysis we will find a set of modes that allow an explanation for most of these observations. The relationship of the obtained results to the previously known ones[2-9] is given and their applications to experiments other than multipoles indicated.

To represent the situation of a multipole without magnetic shear we consider a plane, two-dimensional equilibrium configuration for a low-β collisionless plasma,[2,5,6] where $\vec{B}=\vec{B}(l,r_\perp)$, l representing the distance along the lines of force and $r_\perp$ the direction of the density gradient. The lines of force are closed so that all particle orbits along them are periodic. We make use of the Vlasov equation $\partial f/\partial t+\vec{v}\cdot\nabla f+(e/m)(\vec{\mathcal{E}}+\vec{v}\times\vec{B})\cdot\nabla_v f=0$, where the notation is standard. The equilibrium distribution is chosen as $f_0=n_0(p_\theta)/(\pi T)^{3/2}\exp(-E/T)$, where $E=v_\parallel^2+v_\perp^2$, the total particle energy, and p_θ, the angular momentum in the direction of symmetry, are constants of the motion. Then we look for electrostatic modes such that $\mathcal{E}=-\nabla\Phi$ and $\Phi=\varphi(l,r_\perp)\exp(ikr_\theta+i\omega t)$, where r_θ represents the direction of (axi)symmetry. These modes are assumed to be localized around a point $r_\perp=r_0$ such that $k\varphi\gg\partial\varphi/\partial r_\perp$ and $\varphi(r_\perp,l)\approx\varphi(r_0,l)$; so we neglect from here on the $r_\perp$ dependence of φ. We consider very low frequencies such that $\hat{\omega}_{Di}<\omega<\hat{\omega}_{bi}<\Omega_i$, where Ω_i is the ion cyclotron frequency, $\hat{\omega}_{bi}$ is average ion excursion frequency along the lines of force, and $\hat{\omega}_{Di}=k\hat{v}_{Di}$, $\hat{v}_{Di}$ being an average ion magnetic-curvature drift frequency. In particular, $\hat{\omega}_{bi}\sim v_{\mathrm{th}\,i}/L$, L being the connection length, i.e., the typical distance between a minimum and a successive maximum of the magnetic field, and $\hat{v}_{Di}\sim a_iv_{\mathrm{th}\,i}/R_c$, where $a_i=v_{\mathrm{th}\,i}/\Omega_i$ is the ion gyroradius and R_c is a typical local radius of curvature. Now we notice that in a realistic multipole configuration[1,5] $L \sim R_c$, so that the above inequality implies $ka_i < 1$ and, in particular, $(ka_i)^2 \ll \hat{\omega}_{Di}^2/\omega^2$. Since we know that for $ka_i < 1$ the finite gyroradius contribution is of order $(ka_i)^2$, we can neglect it in comparison with that of the curvature drift that contributes at least terms of order $\hat{\omega}_{Di}^2/\omega^2$. On this basis we can simply solve the linearized perturbed Vlasov equation by integration along the guiding center orbits[5] and obtain

$$f_{lj} = -(e_j f_{0j}/T_j)\{\varphi(l) - i(\omega - \omega_{*j})\int_{-\infty}^{t} dt'\, \varphi(l')\exp[i\omega(t'-t) + i\int_{t'}^{t}\omega_{Dj}d\hat{t}]\},$$

where $j = e$ or i, $l' = l(t')$ is the guiding-center trajectory as a function of t', $\omega_{*j} = -(kcT_j/e_jBn)(dn/dr_\perp)_0$ is the "diamagnetic" frequency, $\omega_D = kv_D$, and $v_D = v_D(l(t'))$ is the curvature drift velocity. Now since $\omega < \hat{\omega}_{bi} < \hat{\omega}_{be}$, we expand φ and v_D in harmonics of the orbit periodicity[7] so that, for instance, $\varphi(l) = \sum \Phi^{(n)}(E,\mu)\exp(2\pi i n t'/\tau)$, where $\Phi^{(n)}(E,\mu) = (1/\tau)\int_0^\tau dt'\, \varphi(l(t'))\exp(-2\pi i n t'/\tau)$, $t' = \int^{l'} dl/v_\parallel$, $\tau = \oint dl/v_\parallel$, $\omega_D(l) = \sum \omega_D^{(n)}(E,\mu)\exp(in\omega_b t')$, and $\omega_b(E,\mu) \equiv 2\pi/\tau$. Then we have to lowest order[5] $f_{ji} = -(e_j f_{0j}/T_j)\{\varphi(l) - (\omega - \omega_{*j})\varphi^{(0)}(E,\mu)/[\omega + \omega_{Dj}^{(0)}(E,\mu)]\}$. The dispersion relation is derived by the quasineutrality condition $n_{e1} = n_{i1}$ and is given by the integral equation

$$\varphi(l) - \left\langle \frac{\omega^2 - \omega_{*e}\omega_{Di}^{(0)} + \omega_{Di}^{(0)}\omega(1 - T_e/T_i)}{\omega^2 + \omega\omega_{Di}^{(0)}(1 - T_e/T_i) - \omega_{Di}^{(0)2}T_e/T_i}\Phi^{(0)} \right\rangle = 0,$$

where $\langle x\rangle = n_0^{-1}\int d^3v f_0 x = (1/2\pi n_0)\iint dE d\mu f_0 x B/|v_\parallel|$, with the convention that contributions from positive and negative values of $v_\parallel$ are to be added, $|v_\parallel| = (E - \mu B)^{1/2}$, and $\mu = v_\perp^2/B$, the magnetic moment. In the limit $\omega > \omega_{Di}$ the above equation reduces to

$$\varphi(l) - \langle\Phi^{(0)}\rangle = \frac{T_e}{T_i}\left\langle \left[\frac{\omega_{*i}\omega_{Di}^{(0)}}{\omega^2} + \frac{\omega_{Di}^{(0)2}}{\omega^2}\left(1 - \frac{\omega_{*i}}{\omega} + \frac{T_e}{T_i}\frac{\omega_{*i}}{\omega}\right)\right]\Phi^{(0)}\right\rangle = 0. \quad (1)$$

Now indicating by $(\varphi)_{\text{av}}$ the space-averaging operation $(\oint dl\varphi)/(B\oint dl/B)$ we split φ into $\bar{\varphi} + \tilde{\varphi}$ so that $\tilde{\varphi}$ has zero average over $\oint dl/B$. Then separating the average and periodic parts of Eq. (1), we have

$$\left[\bar{\omega}_{Di}\frac{\omega_{*i}}{\omega^2} + \frac{(\langle\omega_{Di}^{(0)2}\rangle)_{\text{av}}}{\omega^2}\left(1 - \frac{\omega_{*i}}{\omega} + \frac{T_e}{T_i}\frac{\omega_{*i}}{\omega}\right)\right]\bar{\varphi} + \frac{\omega_{*i}}{\omega^2}(\langle\omega_{Di}^{(0)}\tilde{\Phi}^{(0)}\rangle)_{\text{av}} = 0, \quad (2)$$

$$\tilde{\varphi}(l) - \langle\tilde{\Phi}^{(0)}\rangle = \frac{T_e}{T_i}\frac{\omega_{*i}}{\omega^2}[\langle(\omega_{Di}^{(0)} - \bar{\omega}_{Di})\bar{\varphi} + \omega_{Di}^{(0)}\tilde{\Phi}^{(0)} - (\langle\omega_{Di}^{(0)}\tilde{\Phi}^{(0)}\rangle)_{\text{av}}\rangle]. \quad (3)$$

We have considered that[6]

$$\bar{\omega}_{Di} \equiv (\langle\omega_{Di}^{(0)}\rangle)_{\text{av}} = (\langle\omega_{Di}\rangle)_{\text{av}}$$

since

$$\oint\frac{dl}{B}\iint dEd\mu\frac{B\omega_{Di}^{(0)}}{|v_\parallel|} = \oint\frac{dl}{B}\iint dEd\mu\frac{\omega_{Di}B}{|v_\parallel|},$$

and recall that the positive sign of

$$\omega_{*i}\bar{\omega}_{Di} = \omega_{*i}\frac{kM_i c}{e}\frac{\partial}{\partial\psi}\oint\frac{d\chi}{B^2}$$

determines the condition for hydromagnetic stability against interchange modes. Here ψ is a proper coordinate replacing $r_\perp$ and measuring the magnetic flux through a surface perpendicular to the lines of force and having a unit length along the θ direction, so that $d\psi = Bdr_\perp$, χ is the magnetic potential, $\vec{B} = \nabla\chi$, and $dl = d\chi/B$. Now we consider the case where $\bar{\omega}_{Di}^2 < (\langle\omega_{Di}^{(0)2}\rangle)_{\text{av}}$ as being particularly relevant to configurations with shallow magnetic wells such as quadrupoles, and the limit $\tilde{\varphi} < \overline{\varphi}$, which implies $T_e\omega_{*i}\omega_{Di}/T_i\omega^2 < 1$ according to Eq. (3). Then the right-hand side of Eq. (3) reduces to $T_e\omega_{*i} \times \langle\omega_{Di}{}^{(0)}\rangle/T_i\omega^2\overline{\varphi}$ in lowest order and its solution can be formally expressed as

$$\frac{\tilde{\varphi}}{\overline{\varphi}} = \frac{T_e}{T_i}\frac{\omega_{*i}\hat{\omega}_{Di}}{\omega^2}G(l),$$

where we define $(\hat{\omega}_{Di})^2 \equiv (\langle(\omega_{Di}{}^{(0)})^2\rangle)_{\text{av}}$ and $G(l)$ is real a finite function of l that depends on the detailed geometry of the system. On the other hand, we can write the last term on the left-hand side of Eq. (2) as being proportional to

$$\oint\frac{dl}{B}\iint dE d\mu f_0\frac{B}{|v_\parallel|}\omega_{Di}{}^{(0)}\frac{\oint dl\tilde{\varphi}/|v_\parallel|}{\oint dl/v_\parallel} = \oint\frac{dl\tilde{\varphi}(l)}{B}\iint dEd\mu\frac{f_0 B\omega_{Di}{}^{(0)}}{v_\parallel} = \oint\frac{dl\tilde{\varphi}(l)}{B}\langle\omega_{Di}{}^{(0)}\rangle$$

$$= \frac{\omega^2 T_i}{T_e\omega_{*i}\overline{\varphi}}\oint\frac{dl}{B}[\tilde{\varphi}(l)^2 - \tilde{\varphi}(l)\langle\Phi^{(0)}\rangle]$$

after seeing that $\langle\omega_{Di}{}^{(0)}\rangle$ represents to lowest order the right-hand side of Eq. (4). Therefore Eq. (3) becomes

$$0 = \omega_{*i}\overline{\omega}_{Di} + \hat{\omega}_{Di}{}^2\left[1 - \frac{\omega_{*i}}{\omega(1-T_e/T_i)}\right] + \frac{\omega^2 T_i}{T_e\langle(\tilde{\varphi}/\overline{\varphi})^2 - ([\Phi^{(0)}/\overline{\varphi}]^2)\rangle_{\text{av}}},$$

and we arrive at the dispersion relation

$$\epsilon_0 \equiv -\frac{\omega_{*i}\overline{\omega}_{Di}}{\omega^2} - \frac{\omega_{*i}{}^2\hat{\omega}_{Di}{}^2}{\omega^4}\frac{T_e}{T_i}\Gamma - \frac{\omega_{Di}{}^2}{\omega^2}\left(1 - \frac{\omega_{*i}}{\omega} + \frac{T_e}{T_i}\omega_{*i}\right) = 0, \qquad (4)$$

where $\Gamma \equiv (G(l)^2 - \langle[G^{(0)}]^2\rangle)_{\text{av}}$ is positive definite for the Schwarz inequality and finite for B having finite variation along l, and ϵ_0 is the dielectric constant. The finite solutions of Eq. (4) are

$$\omega_0 = \tfrac{1}{2}\omega_{*i}\frac{1 - T_e/T_i \pm D}{1+w},$$

where $w \equiv \overline{\omega}_{Di}\omega_{*i}/\hat{\omega}_{Di}{}^2$ and $D^2 \equiv (1-T_e/T_i)^2 - 4\Gamma \times (T_e/T_i)(1+w)$. For order-of-magnitude estimates we can write $w = (1/R_c)_{\text{av}}R_c{}^2/2r_n$, where r_n is the density-gradient scale distance, $(1/R_c)_{\text{av}}$ the average magnetic curvature, and R_c a typical local radius of curvature. Therefore:

(a) If $\Gamma(w+1) > (1-T_e/T_i)^2T_i(4T_e)^{-1}$, an algebraically unstable mode is obtained, independently of wave-particle resonance effects. Notice that the real part of the frequency gives a phase velocity in the direction of the electron diamagnetic velocity for $T_e > T_i$ and, a phase velocity in the ion direction for $T_i > T_e$.

In regions of fairly deep well depth this mode becomes almost purely growing with rate $\gamma \approx \hat{\omega}_{Di} \times (\Gamma\omega_{*i}T_e/\langle\hat{\omega}_{Di}\rangle T_i)^{1/2}$ that is decreasing as the well depth increases. These conditions are likely to be realized in an octopole configuration[11] for most of the region between the separatrix (ψ_S) and the critical surface (ψ_{crit}) which limits the region of hydromagnetic stability,[1] and are likely to be realized sufficiently close to ψ_S for a quadrupole which has a shallower magnetic well.[9] Notice that the condition $\tilde{\varphi}/\overline{\varphi} \approx \omega_{*i}\hat{\omega}_{Di}T_eG/\omega^2T_i < 1$ is verified for $\overline{\omega}_{Di} < G\hat{\omega}_{Di}$, i.e., $(1/R_c)_{\text{av}} < G/R_c$. Moreover, a mode of this kind is unlikely to be observed as a fluctuation, a fact that agrees with the lack of detected fluctuations in octopoles and in the inner regions of quadrupoles. Mathematically, this mode identifies with the known trapped-particle mode,[3] losing its fluted feature and becoming more and more localized,[5] as one proceeds[10] further toward ψ_S.

(b) If $(T_i/4T_e)(1-T_e/T_i)^2 > (w+1)\Gamma$, a condition that can be realized in regions of shallow magnetic well and for relatively large values of T_e/T_i or T_i/T_e, Eq. (4) gives two purely oscillating modes. One can see that they have phase velocity in the direction of the ion diamagnetic velocity or to the contrary, depending on whether $T_i/T_e > 1$ or $T_e/T_i > 1$. It is important to notice that the two modes are of opposite energy, i.e., positive and negative, so that we expect that, adding to the present theory the proper[1,6] ion Landau-damping effects, one of the two modes be unstable. In fact, the wave energy is proportional to $\omega\partial\epsilon_0/\partial\omega \propto (\omega/\omega_{*i})[1-T_e/T_i-2(\omega+1)\omega/\omega_{*i}] \propto \mp(1-T_e/T_i)-D$ and the dielectric constant, including the effects of resonance between the wave and the ions with orbit frequency $\omega_b(E,\mu)=\omega_0$, is (see Ref. 10) $\epsilon_1=\epsilon_0-i(\omega-\omega_{*i})\Delta/|\omega|$, where Δ is a positive definite quantity of order $(\omega_{*e}\hat{\omega}_{Di}G)^2[|\omega|\overline{\omega}_{bi}^3(1+T_e/T_i)]^{-1}$. In this case the dispersion relation can be solved by perturbation obtaining

$$\omega=\omega_0+\delta\omega$$

and

$$\delta\omega=-i[D\mp(2w+1+T_e/T_i)]\omega_0^4\Delta \times[2\hat{\omega}_{Di}^2|\omega_0|(1+w)]^{-1}.$$

From all this it is reasonable to expect the observation of flutelike fluctuations in quadrupole configurations where the magnetic well depth is shallow over a sufficiently large region and when T_i/T_e or T_e/T_i are sufficiently larger than 1.

(c) The significance of the two large roots of Eq. (4) can be understood by assuming that $R_c^2 > L^2$ so that $(ka_i)^2 \sim \hat{\omega}_{Di}^2/\omega^2$. Then following the treatment of Ref. 10, to include the finite ion gyroradius and finite Debye-length effects we arrive at the complete dispersion relation

$$\epsilon_2 \equiv k^2\lambda_D^2+b\left(1-\frac{\omega_{*i}}{\omega}\right)-\frac{\hat{\omega}_{Di}^2}{\omega^2}\left(1-\frac{\omega_{*i}}{\omega}+\frac{T_e}{T_i}\frac{\omega_{*i}}{\omega}\right)-\frac{\omega_{*i}\overline{\omega}_{Di}}{\omega^2}-\frac{\omega_{*i}^2\hat{\omega}_{Di}^2}{\omega^4}\frac{T_e}{T_i}\Gamma-i(\omega-\omega_{*i})\frac{\Delta}{|\omega|}=0, \quad (5)$$

where $b\approx k_\perp^2a_i^2$ and λ_D is the ion Debye length. Now we can see that the two mentioned roots are essentially the hydromagnetic flute modes that come from balancing the second against the fourth term in the expression for ϵ_2. The dispersion relation Eq. (5) has several interesting properties which we will not discuss here. In particular, if we consider the limit $T_e/T_i \ll 1$ and the low-density regime where the Debye-length effects are important, we find that if typically $k\lambda_D > [r_n\langle 1/R_c\rangle]^{1/2}$, a residual instability associated with the term Δ is obtained.[4,8] Unlike the case for the modes considered under points (a) and (b), the periodic part $\tilde{\varphi}$ of the eigenmode does not affect its stability and can be neglected, since $T_e/T_i \ll 1$ and ω is independent of T_e. So now Δ is determined[10] by the periodicity of $\exp(i\int_t^{t'}\omega_{Di}d\hat{t})$ along $\int dl/B$ and is of the order $|\omega/\hat{\omega}_{bi}|^3\hat{\omega}_{Di}^2\hat{\omega}_{bi}^{-2}$. One can also obtain a fluidlike, nonresidual instability with $\omega^3=-\hat{\omega}_{Di}^2\omega_{*i}/(k\lambda_D)^2$ by balancing the first and third terms of Eq. (5) in the appropriate asymptotic limit. All this is consistent with the observation of an ion flute mode in both quadrupole and octopole configurations at very low densities.[11,12]

In addition, the modes discussed in (a) and (b) can explain the following: (1) The appearance of a fluctuation in the General Atomic toroidal quadrupole configuration[11] whereas none was observed for equal plasma conditions in the octopole. In particular, the observed fluctuation is of flute type, has $\omega<\hat{\omega}_{bi}$, for $T_i>T_e$ has phase velocity in the direction of the ion diamagnetic velocity, and exists in conditions where the finite-Debye-length effects mentions under point (c) are not important. (2) The observation of fluctuations in the Princeton linear quadrupole[12] configuration which are of flute type, have $\omega<\hat{\omega}_{bi}$, exist for $T_e<T_i$ and have phase velocity in the direction of the electron diamagnetic velocity, and disappear for $T_e\sim T_i$. (3) The observation of fluctuations which are extended to the entire length of the lines of force in the Culham quadrupole[13] configuration operating with $T_i>T_e$. Although no definite result is yet available on the phase velocity and frequency of oscillation, preliminary indications appear to be consistent with the present theory.

We also notice that (A) since $L\sim R_c$ in a typical multipole configuration, a finite temperature anisotropy[8] is unlikely to have an influence on the appearance of an ion flute mode. In fact, the relevant effect is represented by a term of order $(ka_i)^2$ which, as we have seen, is negligible in comparison with terms of order $\hat{\omega}_{Di}^2/\omega^2$ in the dispersion relation. (B) In those conditions where the modes we have discussed become almost purely growing, they may achieve a considerable amplitude and give rise to sizable anomalous particle losses as observed in quadrupole regions or octopoles where fluctuations are not detected. (C) A case of application of the theory given above may be presented by the Model-C stellarator configuration that has a shallow magnetic well and small shear in the inner part of the plasma column. Fluctuations have in fact been observed through ion cyclotron heating experiments[14] and it has been stated that these could well be flutelike with phase velocity in the ion direction. (D) A properly modified version of the modes discussed above may be utilized to explain the recent observation of a mode with $\omega < \hat{\omega}_{bi}$, phase velocity in the direction of the ion diamagnetic velocity but "ballooning" in a region of unfavorable curvature of the Livermore Levitron toroidal experiment[15] with $T_i > T_e$.

This paper has been stimulated by a recent work[9] of Jukes, where the importance of ordering $\hat{\omega}_{Di}^2/\omega^2 \sim \overline{\omega}_{Di}/\omega$ is pointed out in connection with the stability of the "electron" ($\omega_{bi} < \omega < \omega_{be}$) flute mode and the fluctuations reported in Ref. 11.[2] The author is also indebted to M. Rosenbluth and P. Rutherford for valuable discussions and suggestions, and to K. Allen and S. Yoshikawa for private communications on their experiments.

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MUTUAL FRICTION IN He II NEAR THE SUPERFLUID TRANSITION

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Measurements of supercritical heat flow in He II are consistent with a mutal friction constant which diverges near T_λ. They also explain the apparent depression of T_λ observed by Erben and Pobell, and the "critical heat flux" near T_λ reported by Bhagat and Winer.

Recently, Erben and Pobell[1] (EP) reported a depression by a heat current of the superfluid transition temperature T_λ in He^4, and Bhagat and Winer[2] (BW) measured a "critical heat flux" in He II by observing the formation of bubbles at a heated wire. In the present communication, measurements of the heat flux q in a long cylinder of He II, one end of which is at T_λ and the other end at $T < T_\lambda$, are reported. All three experiments, as well as the results of Keller and Hammel (KH) near T_λ,[3] can be interpreted[4] in terms of a Gorter-Mellink mutual friction constant[5] which diverges near T_λ. Therefore, it is not necessary to postulate a depression of T_λ due to q as proposed by EP,[1] and the BW results need not be explained on the basis of a new critical velocity.[2,6]

The present measurements were made in a stainless steel capillary, 2×10^{-2} cm i.d. and 4 $\times10^{-2}$ cm o.d., suspended in a vacuum surround-