of the form $\Gamma[m-\alpha(s)]\Gamma[n-\alpha(t)]/\Gamma[m+n-\alpha(s)-\alpha(t)]$, m and/or $n>1$} because (a) the lower order terms are more important at low energies which the zero-moment sum rule emphasizes, and (b) B_{su} is more sensitive to these corrections than is B_{st}. In fact, for $s\to\infty$ with $\alpha(s)+\alpha(t)$ fixed, the so-called "lower order" corrections to B_{su} diverge faster than the leading beta function. It is necessary to sum an infinite number of them to get proper Regge asymptotic behavior. See Stanley Mandelstam, Phys. Rev. Letters 21, 1724 (1968).

[11]The tu box is just the real Regge term in nonresonant channels, discussed by H. Harari, Phys. Rev. Letters 20, 1395 (1968).

[12]Presumably nonbox amplitudes would be required when unitarity corrections would be incorporated into the model, since the vacuum "trajectory" contribution does not have the quark loop structure. Nonbox amplitudes might also be required when the nonet symmetry is broken to SU(2). Even in the SU(2) limit the finite-energy summable part of the amplitude must have the quark loop structure (with broken-symmetry quarks); however, there could be appreciable nonloop corrections (necessarily superconvergent) which shift resonance positions and residues from their nonet symmetry values.

SPONTANEOUS BREAKDOWN OF CHIRAL SU(3) ⊗ SU(3) SYMMETRY

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We show that the Hamiltonian suggested by Gell-Mann, Oakes, and Renner can be obtained as a spontaneous breakdown of chiral SU(3) ⊗ SU(3) symmetry.

It has been recently suggested[1-3] that the physics of strongly interacting particles can be better understood as a breaking of chiral SU(3) ⊗ SU(3) rather than SU(3) symmetry. A specific model of this breaking has been analyzed by Gell-Mann, Oakes, and Renner.[2] The basic assumption is to write the strong-interaction Hamiltonian in the following form:

$$H=H_0+\epsilon(u_0-\sqrt{2}\,u_8), \tag{1}$$

where H_0 is invariant under SU(3) ⊗ SU(3), and the scalar u_i ($i=0,\cdots,8$), together with the corresponding pseudoscalar partners v_i ($i=0,\cdots,8$), belong to the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ of SU(3) ⊗ SU(3).

In this note we want to point out that Eq. (1) may be understood as a dynamical breaking. More specifically, we will show that SU(3) ⊗ SU(3) symmetry breaks spontaneously along the direction $u_0-\sqrt{2}\,u_8$, and that an effective Lagrangian function can be deduced containing a breaking term which belongs to the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ of SU(3) ⊗ SU(3).

Let us denote by $\mathcal{L}$ a function which describes the system of strongly interacting particles. It may be the S matrix, or the Lagrangian, or the Hamiltonian. Just for definiteness we shall call it the Lagrangian. Let us assume that $\mathcal{L}$ is fully invariant under SU(3) ⊗ SU(3). The occurrence of a spontaneous breaking is equivalent to the existence of stationary points of $\mathcal{L}$ other than the origin.[4] In order to show that, we have to express $\mathcal{L}$ as a function of the "fields." To this purpose, we introduce the "elementary" fields u_i, v_i ($i=0,\cdots,8$) which transform according to the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ of SU(3) ⊗ SU(3) symmetry. They may be regarded as mathematical objects in terms of which one may construct the representations of SU(3) ⊗ SU(3) without necessarily implying a physical interpretation for them.

It is convenient to introduce the notation

$$W_\beta^{\pm\alpha}=U_\beta^{\ \alpha}\pm iV_\beta^{\ \alpha}, \tag{2}$$

where

$$U_\beta^{\ \alpha}=\frac{1}{\sqrt{2}}\sum_{i=0}^{8}(\lambda_iu_i)_\beta^{\ \alpha},\quad V_\beta^{\ \alpha}=\frac{1}{\sqrt{2}}\sum_{i=0}^{8}(\lambda_iv_i)_\beta^{\ \alpha},$$

and all other notation is as in Ref. 2. Then $W_\beta{}^{+\alpha}$ transforms according to the representation $(\underline{3}, \underline{3}^*)$ and $W_\beta{}^{-\alpha}$ according to $(\underline{3}^*, \underline{3})$.

The most general $\mathcal{L}$ depending on these fields and invariant under SU(3) ⊗ SU(3) will be a function of the invariants which can be formed by means of $W_\beta{}^{\pm\alpha}$. It is not difficult to see that there are only four independent invariants. We will choose the following ones:

$$I_2 = W_\beta{}^{+\alpha} W_\alpha{}^{-\beta} = \mathrm{Tr}\, U^2 + \mathrm{Tr}\, V^2,$$

$$I_4 = W_\beta{}^{+\alpha} W_\gamma{}^{-\beta} W_\delta{}^{+\gamma} W_\alpha{}^{-\delta} = \mathrm{Tr}\, U^4 + \mathrm{Tr}\, V^4 + 4\,\mathrm{Tr}\, U^2 V^2 - 2\,\mathrm{Tr}\, UVUV;$$

$$I_3{}^{+} = \tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\epsilon^{\lambda\mu\nu}(W_\lambda{}^{+\alpha} W_\mu{}^{+\beta} W_\nu{}^{+\gamma} + W_\lambda{}^{-\alpha} W_\mu{}^{-\beta} W_\nu{}^{-\gamma}) = \epsilon_{\alpha\beta\gamma}\epsilon^{\lambda\mu\nu}(U_\lambda{}^{\alpha} U_\mu{}^{\beta} U_\nu{}^{\gamma} - 3 V_\lambda{}^{\alpha} V_\mu{}^{\beta} U_\nu{}^{\gamma});$$

$$I_3{}^{-} = \tfrac{1}{2}i\epsilon_{\alpha\beta\gamma}\epsilon^{\lambda\mu\nu}(W_\lambda{}^{+\alpha} W_\mu{}^{+\beta} W_\nu{}^{+\gamma} - W_\lambda{}^{-\alpha} W_\mu{}^{-\beta} W_\nu{}^{-\gamma}) = \epsilon_{\alpha\beta\gamma}\epsilon^{\lambda\mu\nu}(V_\lambda{}^{\alpha} V_\mu{}^{\beta} V_\nu{}^{\gamma} - 3 U_\lambda{}^{\alpha} U_\mu{}^{\beta} V_\nu{}^{\gamma}). \quad (3)$$

Now we have to see whether there are stability points $\tilde{W}^{\pm}$ for $\mathcal{L}$ other than the trivial one $\tilde{W}^{\pm} = 0$, which does not break the symmetry. Therefore we will look for a solution of the equations

$$\partial\mathcal{L}/\partial U_\alpha{}^{\beta} = 0; \quad \partial\mathcal{L}/\partial V_\alpha{}^{\beta} = 0. \quad (4)$$

In order to have a stability point (which plays the role of the vacuum) invariant under parity, and consequently a parity-invariant theory, we will look for solutions of (4) such that

$$\tilde{V}_\beta{}^{\alpha} = 0.$$

In this case we have

$$(\partial\mathcal{L}/\partial I_3{}^{-})_{\tilde{V}} = 0,$$

as by conservation of parity $\mathcal{L}$ must be an even function of $I_3{}^{-}$.

Hence the first of Eqs. (4) takes the following form:

$$2\frac{\partial\mathcal{L}}{\partial I_2} U_\alpha{}^{\beta} + 4\frac{\partial\mathcal{L}}{\partial I_4} U_\gamma{}^{\beta} U_\delta{}^{\gamma} U_\alpha{}^{\delta} + 3\frac{\partial\mathcal{L}}{\partial I_3{}^{+}}\epsilon_{\alpha\sigma\tau}\epsilon^{\beta\mu\nu} U_\mu{}^{\sigma} U_\nu{}^{\tau} = 0, \quad (5)$$

the second one being identically satisfied.

A solution is

$$\tilde{U}_3{}^{3} = \eta \neq 0, \quad \text{the remaining } \tilde{U}_\beta{}^{\alpha} = 0;$$

$$\tilde{V}_\beta{}^{\alpha} = 0. \quad (6)$$

Clearly, together with (6), all the points which can be obtained from this by SU(3) ⊗ SU(3) symmetry transformations [i.e., the points of the "orbit" of (6)] are equivalent solutions. The original Lagrangian being invariant under SU(3) ⊗ SU(3) cannot fix the coordinates in SU(3) ⊗ SU(3) space. We will thus choose them in such a way that the stability point may be given in the form of (6).

The other solutions not belonging to the orbit of (6) will be discussed later.

The result that SU(3) ⊗ SU(3) symmetry breaks spontaneously according to the direction $\tilde{U} = U_3{}^3 \sim u_0 - \sqrt{2}u_8$ has some interesting consequences. It fixes the residual invariance of the theory as the chiral isotopic symmetry SU(2) ⊗ SU(2). This is the analog to the result that SU(3) breaks spontaneously in the direction of the hypercharge.[4]

In order to get further information about this breakdown, we must face the general problem of describing a spontaneously broken symmetry by means of an effective Lagrangian which explicitly exhibits the symmetry breaking. A simple and plausible prescription is to write

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_0 + g\mathcal{L}',$$

where $\mathcal{L}'$ breaks SU(3) ⊗ SU(3) symmetry but still preserves the residual isotopic SU(2) ⊗ SU(2) invariance. However, this does not seem to us to be a complete answer to the problem as it still leaves undetermined the representation to which $\mathcal{L}'$ belongs.

Therefore we will adopt the following prescription: We will expand the original Lagrangian in the neighborhood of the stability point (vacuum) up to second order. This will provide us with an effective Lagrangian with known transformation properties. By introducing

$$\mathfrak{u}=U-\tilde{U},\quad \mathfrak{v}=V-\tilde{V}=V,$$

we get

$$\mathcal{L}\approx\mathcal{L}(\tilde{U},\tilde{V})+\frac{\partial\mathcal{L}}{\partial I_2}(\mathfrak{u}_\beta{}^\alpha\mathfrak{u}_\alpha{}^\beta+\mathfrak{v}_\beta{}^\alpha\mathfrak{v}_\alpha{}^\beta)+4\eta^2\frac{\partial\mathcal{L}}{\partial I_4}(\mathfrak{u}_\alpha{}^3\mathfrak{u}_3{}^\alpha+\mathfrak{v}_\alpha{}^3\mathfrak{v}_3{}^\alpha)+2\eta^2\frac{\partial\mathcal{L}}{\partial I_4}[(\mathfrak{u}_3{}^3)^2-(\mathfrak{v}_3{}^3)^2]$$
$$+2\eta^2\left(\frac{\partial^2\mathcal{L}}{\partial I_2{}^2}+4\eta^2\frac{\partial^2\mathcal{L}}{\partial I_2\partial I_4}+4\eta^4\frac{\partial^2\mathcal{L}}{\partial I_4{}^2}\right)(\mathfrak{u}_3{}^3)^2+3\eta\frac{\partial\mathcal{L}}{\partial I_3{}^+}\epsilon_{\alpha\sigma3}\epsilon^{\beta\mu3}(\mathfrak{u}_\beta{}^\alpha\mathfrak{u}_\mu{}^\sigma-\mathfrak{v}_\beta{}^\alpha\mathfrak{v}_\mu{}^\sigma)\equiv\mathcal{L}_{\text{eff}},\quad(7)$$

where the derivatives are calculated at the stationary point given by Eqs. (6).

The new Lagrangian $\mathcal{L}_{\text{eff}}$ consists of an invariant term, plus the following noninvariant terms: (1) $\mathcal{L}_1\sim\mathfrak{u}_\alpha{}^3\mathfrak{u}_3{}^\alpha+\mathfrak{v}_\alpha{}^3\mathfrak{v}_3{}^\alpha$, which transforms[5] as an element of the representation $(\underline{1}\oplus\underline{8},\underline{1})\oplus(\underline{1},\underline{8}\oplus\underline{1})$, (2) $\mathcal{L}_2\sim(\mathfrak{u}_3{}^3)^2$ and $\mathcal{L}_3\sim(\mathfrak{v}_3{}^3)^2$, which transform as elements of the representation $(\underline{1}\oplus\underline{8},\underline{1}\oplus\underline{8})\oplus(\underline{6},\underline{6}^*)\oplus(\underline{6}^*,\underline{6})$, and finally a term (3) $\mathcal{L}_4\sim\epsilon_{\alpha\sigma3}\epsilon^{\beta\mu3}(\mathfrak{u}_\beta{}^\alpha\mathfrak{u}_\sigma{}^\mu-\mathfrak{v}_\beta{}^\alpha\mathfrak{v}_\sigma{}^\mu)$, which behaves just as the component $U_3{}^3\sim u_0-\sqrt{2}u_8$ of the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$.

It is interesting to note that $\mathcal{L}_4$ occurs in $\mathcal{L}_{\text{eff}}$ with a coefficient which is completely independent of the coefficients of the other terms. Therefore, depending on the form of the original $\mathcal{L}$, one could make the terms $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$ very small (even zero) without affecting $\mathcal{L}_4$. Up to now, it is not very clear[2] whether the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ is dominant with respect to the representation $(\underline{1},\underline{8})\oplus(\underline{8},\underline{1})$, mainly because the effect of a term transforming like the representation $(\underline{1},\underline{8})\oplus(\underline{8},\underline{1})$ does not affect the presently available experimental data sensibly. It is interesting to note that in the spontaneous-breaking scheme, the representation $(\underline{1},\underline{8})\oplus(\underline{8},\underline{1})$ occurs with the same weight as the representation $(\underline{6},\underline{6}^*)\oplus(\underline{6}^*,\underline{6})$. This last would give rise to currents[2] with $T=2$ and $Y=2$. In order to avoid this, the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ must be dominant with respect to the others, or equivalently, $\partial\mathcal{L}/\partial I_3{}^+$ must be different from zero and larger than the other coefficients. In this case, the only surviving solution of Eq. (5), besides the one considered up to now [Eqs. (6)], is $u_0\neq 0$ (and the remaining $u_i, v_i=0$), corresponding to the usual SU(3) invariance.

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[1]S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[2]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[3]R. Gatto, G. Sartori, and M. Tonin, Phys. Letters 28B, 128 (1968); N. Cabibbo and L. Maiani, Phys. Letters 28B, 131 (1968).

[4]N. Cabibbo, in Proceedings of the International School of Physics "Ettore Majorana," Erice, Italy, 1967, edited by A. Zichichi (Academic Press, Inc., New York, 1968); P. de Mottoni and E. Fabri, Nuovo Cimento 54A, 42 (1968); L. Michel and L. A. Radicati, in Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January, 1968 (W. A. Benjamin, Inc., New York, 1968).

[5]This can be immediately seen in terms of the $W^\pm$.