(1968).

[5]J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960); N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).

[6]F. Bulos et al., "Photoproduction of Rho Mesons at 9 BeV," in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (unpublished).

[7]M. Beniston, "On Line Analysis of Wire Spark Chamber Data," in Fifteenth IEEE Nuclear Science Symposium, Montreal, Canada, October, 1968 (to be published); R. Russell, "On-Line Wire Spark Chamber Data Acquisition System," in Fifteenth IEEE Nuclear Science Symposium, Montreal, Canada, October, 1968 (to be published); F. Bulos et al., in Proceedings of the International Symposium on Nuclear Electronics and Institute for High Energy Physics, Versailles, France, September, 1968 (to be published).

[8]J. Ballam et al., Stanford Linear Accelerator Center Report No. SLAC-PUB-530, 1968 (unpublished).

[9]German Bubble Chamber Collaboration, Nuovo Cimento 46A, 795 (1966), and 48A, 262 (1967); Cambridge Bubble Chamber Group, Phys. Rev. 146, 994 (1966); Brown–Harvard–Massachusetts Institute of Technology–Padova–Weizmann Institute Bubble Chamber Group, Phys. Rev. 155, 1468 (1967).

[10]S. D. Drell, Phys. Rev. Letters 5, 278 (1960.

[11]P. Söding, Phys. Rev. Letters 19, 702 (1966).

[12]This normalization constant is

$$\left\{\tfrac{1}{2}\pi-\tan^{-1}\left[\frac{4\mu^2-M_0^2}{\Gamma_0 M_0}\right]\right\}^{-1}=\frac{1.07}{\pi}.$$

In the limit $\Gamma_0/M_0 \ll 1$, this becomes $1/\pi$, the normalization used by McClellan et al., Ref. 2.

[13]M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966).

[14]R. Hofstadter, Ann. Rev. Nucl. Sci. 4, 231 (1957); H. R. Collard, L. R. B. Elton, R. Hofstadter, and H. Schoper, in Landolt-Börnstein, Numerical Data and Functional Relationships in Science and Technology, edited by K.-H. Hellwege (Springer-Verlag, Berlin, 1967), New Series, Group I, Vol. 2.

[15]We define the photon-rho coupling constant to be $em_\rho^2/2\gamma_\rho$.

[16]S. D. Drell and J. S. Trefil, Phys. Rev. Letters 16, 552 (1966); K. S. Kölbig and B. Margolis, Nucl. Phys. B6, 85 (1968).

[17]R. G. Glauber and G. Matthiae, Istituto Superiore di Sanità, Rome, Italy, Report No. ISS 67/16, 1967 (unpublished).

[18]In the photon energy range of 2.7 to 9 BeV the extrapolation factor on Pb changes from 19 to 1.3, and for Be, from 2 to 1.1.

[19]Particle Data Group, University of California Radiation Laboratory Report No. UCRL-8030 Revised, 1968 (unpublished).

[20]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[21]Orsay Storage Ring Group, Linear Accelerator Laboratory, Orsay, France, Report No. LAL-1204, 1968 (unpublished); S. C. C. Ting, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968); V. L. Auslender et al., to be published.

[22]R. Diebold and J. A. Poirier, Phys. Rev. Letters 22, 255 (1969).

AN EXTENSION OF THE VENEZIANO REPRESENTATION

Donald E. Neville
Temple University, Philadelphia, Pennsylvania 19122
(Received 16 January 1969)

This paper extends Veneziano's representation from $\pi\pi\to\pi\omega$ to $\Pi\Pi\to\Pi V$, where Π (V) is a pseudoscalar (vector) SU(3) octuplet or singlet.

This note extends Veneziano's representation for the $\pi\pi\to\pi\omega$ scattering amplitude to $\Pi\Pi\to\Pi V$, where Π is a pseudoscalar octuplet or singlet and V is a vector octuplet or singlet.[1-3] The SU(3) structure proposed here for $\Pi\Pi\to\Pi V$ can be justified by arguments largely independent of the details of $\Pi\Pi\to\Pi V$ scattering (or of the Veneziano representation for that matter). It is likely, therefore, that this same structure applies to all meson-meson scattering processes.

Because we choose to work with external particles having the same spin and parity as Veneziano's, our amplitude will have the same Lorentz dependence as his; only the SU(3) dependence will differ:

$$T=\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu}P_1^{\ \nu}P_2^{\ \rho}P_3^{\ \sigma}A(s,t,u). \tag{1}$$

A is a scalar in Lorentz space and a tensor with four subscripts in SU(3) space. We set

$$A=(\bar\beta/\pi)[(M_{st}-\bar M_{st})B_{st} -(M_{su}-\bar M_{su})B_{su}-(M_{tu}-\bar M_{tu})B_{tu}], \tag{2}$$

where the SU(3) subscripts (suppressed, for clarity) are now on the M_{ij} and $\bar M_{ij}$, $\bar\beta=\text{const}$, and the B_{ij} are the Euler beta functions chosen by

Veneziano:

$$B_{st}=\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}. \tag{3}$$

We choose the M_{ij} to have the same SU(3) structure as the quark-loop Feynman diagrams shown in Fig. 1. (The $\bar{M}_{ij}$ are identical to the M_{ij} except that the internal quark loops are counterclockwise.[4]) This choice is motivated by the quark model, since each external meson dissociates into a $Q\bar{Q}$ (quark-antiquark) pair [SU(3) quarks, ($\underline{3}\,\underline{3}^*$)]. The "sides" of quark boxes M_{ij}, $\bar{M}_{ij}$ face toward channels i and j. This insures that there are no $\underline{10}$, $\underline{10}^*$, and $\underline{27}$ resonances (or Regge poles) in the theory: For example, the two beta functions with s-channel poles, B_{st} and B_{su}, are multiplied by tensors $M_{st}, \bar{M}_{st}, M_{su}$, $\bar{M}_{su}$, all of which contain an intermediate $\underline{3}\,\underline{3}^*$ state if cut in two by a horizontal line; hence none of these contribute to SU(3) channels $\underline{10}$, $\underline{10}^*$, and $\underline{27}$.

M_{ij} and $\bar{M}_{ij}$ may be calculated by straightforward Feynman theory applied to an underlying $M \to Q\bar{Q}$ vertex (M = meson), or one may use SU(3) tensors, e.g.,[5]

$$M_{st}=\Pi_{1a}{}^{b}\Pi_{3c}{}^{a}V_{d}{}^{c}\Pi_{2b}{}^{d}. \tag{4}$$

$\bar{M}_{st}$ is identical to M_{st} except for $\Pi_3 \leftrightarrow \Pi_2$. The various plus and minus signs in Eq. (2) are determined by the Bose symmetry requirement of total antisymmetry of A under interchange of any two pseudoscalar mesons.[6]

It is straightforward to check that the asymptotic energy, signature, duality, and superconvergence behavior of the original Veneziano representation have been preserved. For instance, to verify signature, one can manipulate Eq. (2) to the form[7]

$$A=\beta\frac{\Gamma(\alpha(s)+\alpha(t)-1)}{\Gamma(\alpha(s))\Gamma(\alpha(t))}\sum_{\mu_i}\left\{\frac{[1+e^{i\pi\alpha(s)}\sigma_{12}]}{\sin\pi\alpha(s)}(\bar{C}_{us}-C_{us})P(\mu_s)+\frac{[1+e^{-i\pi\alpha(t)}\sigma_{13}]}{\sin\pi\alpha(s)}(\bar{C}_{ut}-C_{ut})P(\mu t)\right\}. \tag{5}$$

In Eq. (5) the SU(3) tensors $P(\mu_i)$ project out pure SU(3) state μ_i (= octuplet or singlet) in channel i; and in going from Eq. (2) to Eq. (5) we have expanded the tensors $M_{ij}, \bar{M}_{ij}$ in series which define the scalar coefficients C_{ij} and $\bar{C}_{ij}$[8] as follows:

$$M_{st}=\sum_{\mu_t}C_{st}(\mu_t)P(\mu_t),$$

$$\bar{M}_{st}=\sum_{\mu_s}\bar{C}_{ts}(\mu_s)P(\mu_s), \text{ etc.} \tag{6}$$

$\sigma_{12}=\sigma_{12}(\mu_s)$ is the parity of $P(\mu_s)$ under the interchange $\Pi_1 \leftrightarrow \Pi_2$; i.e., $\sigma_{12}=+1$ for $\mu_s=\underline{1}$ or $\underline{8}_D$; $\sigma_{12}=-1$ for $\mu_s=\underline{8}_F$. Similarly for $\sigma_{13}=\sigma_{13}(\mu_t)$. The σ_{ij} enter in because we have used the relations

$$\sigma_{12}C_{us}=C_{ts},\ \sigma_{13}C_{ut}=C_{st}, \text{ etc.,} \tag{7}$$

which follow because the tensors M_{su}, M_{ts} differ by $\Pi_1 \leftrightarrow \Pi_2$, etc. Equation (5) is the generalization to $\Pi\Pi \to \Pi V$ of Veneziano's Eq. (10). In the $\pi\pi \to \pi\omega$ case $\sigma_{ij}=-1$ only, since $\pi\pi\to\pi\omega$ admits only the ρ trajectory. The present case admits the even-signature A_2 trajectory as well.

The verification of superconvergence is likewise straightforward. In fact, one can show that

$$\int_0^{\bar{\nu}}\mathrm{Im}B_{st}(\nu',t)\nu'd\nu' = \int_0^{\bar{\nu}}\mathrm{Im}B_{st}^{(\mathrm{R})}(\nu',t)\nu'd\nu', \tag{8}$$

where $B_{st}^{(\mathrm{R})}$ (R for Regge) is that part of the $P(\mu t)$ term in Eq. (5) which comes from B_{st}.[7,9] Equation (8) means that the B_{st} term is finite-energy summable by itself: I.e., the summability does not come from a conspiracy between B_{st} and B_{su} or B_{tu}. This is crucial, of course, because now the B_{st}, B_{su}, and B_{tu} terms have markedly different SU(3) structure and conspiracies are therefore impossible.

Similarly, the B_{su} and B_{tu} terms must each be superconvergent or finite-energy summable by

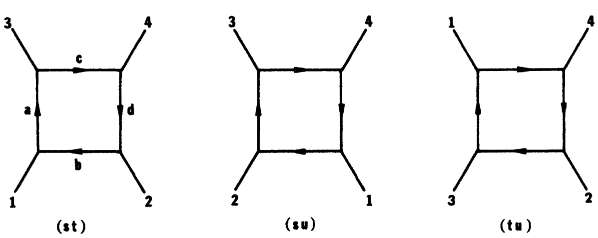

FIG. 1 SU(3) structure of M_{st}, M_{su}, and M_{tu}. All internal lines are SU(3) quarks. External lines 1234 $=\Pi\Pi\Pi V$.

themselves. B_{tu} contributes neither to ImA nor to Im$A^{(R)}$, while B_{su} is superconvergent in the sense that

$$\int_0^{\bar{\nu}} \mathrm{Im}B_{su}(\nu', t)\nu' d\nu' = (\mathrm{const})\bar{\nu}^{\alpha(t)}; \qquad (9)$$

i.e., the B_{su} sum rule diverges one power more slowly than one would expect from the asymptotic behavior $B_{su} \sim \nu^{\alpha(t)-1}$.[10] Note that B_{su} must be superconvergent near $t=0$ and not merely finite-energy summable there because the factor $M_{su} - \bar{M}_{su}$ multiplying B_{su} contributes to t channels which contain no Regge poles.

We emphasize again that most of the above conclusions are independent of the details of the Veneziano representation or the reaction $\Pi\Pi \rightarrow \Pi V$. If part of the s-channel resonant amplitude is merely finite-energy summable at $t \simeq 0$ rather than superconvergent, then it must cross to only $\underline{1}$ plus $\underline{8}$ in the t channel, besides being only $\underline{1}$ plus $\underline{8}$ in the s channel, and the st box structure of Fig. 1 is inevitable for this part of the amplitude. Crossing symmetry (in the present case) or the dynamical similarity of s, t, and u channels (in other cases) then demand (superconvergent, not just finite-energy summable) su and tu terms in addition to the st one.[11] The simplest assumption to make about the remaining nonbox part of the amplitude at least for the present is that it is identically zero. An SU(3) amplitude of the form (2) is then the simplest possibility.[12]

In the exact SU(3) limit the present formalism demands nonet symmetry in a dynamically well-motivated way, since deletion of either $\underline{1}$ or $\underline{8}$ in any channel would destroy the simple crossing properties of the model.

For production amplitudes with $5, 6, \cdots$ external particles, presumably the quark square would generalize to a quark pentagon, hexagon, $\cdots$.

The author is grateful to Dr. Jerrold Franklin for several discussions and to Dr. Gabriel Veneziano for introducing him to several of the references mentioned in the footnotes.

[1]G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[2]Joel Shapiro and Joel Yellin, to be published. These authors consider $\pi\pi \rightarrow \pi\pi$ scattering and the methods of the present paper could be used to extend their results to $\Pi\Pi \rightarrow \Pi\Pi$.

[3]C. Lovelace, to be published.

[4]The reader who wishes to see that clockwise and counterclockwise loops are not identical should consider a process such as $K^-\pi^+ \rightarrow K^-\rho^+$: The clockwise loop M_{st} contributes to this process but the counterclockwise loop $\bar{M}_{st}$ cannot.

[5]The indices a, b, etc. in Eq. (4) refer to the corresponding quark lines a, b, etc. in Fig. 1. Thus if $a = \rho, b = \lambda$, then Π_1 is an incoming K^+ or outgoing K^-, etc. For external singlets, replace the appropriate Π or V by a Kronecker delta.

[6]As it stands, Eq. (2) is correct only if all three pions (pseudoscalar mesons) are identical, all singlets or all octuplets. Otherwise, Eq. (2) should be written as a sum of six terms multiplied by six arbitrary constants; after Bose symmetry is imposed, three of these constants ($\bar{\beta}$ included) remain undetermined if the only external singlet is a pion; and two constants remain undetermined if there are two external singlets. [Amplitudes involving three external singlets vanish because of SU(3) selection rules. The amplitude for four external singlets vanishes because the quark loops cancel one another; with two or more external singlets, clockwise and counterclockwise loops are no longer distinct.] Thus in some respects amplitudes involving singlets are actually more complex than that for the "complicated" case of four octuplets. In all cases, however, these amplitudes have the correct high-energy behavior, satisfy the proper superconvergence relations, etc., just as they should.

[7]The reader may verify, after a half-dozen lines of algebra, that the "one" terms in the first and second square brackets of Eq. (5) are just the B_{su} and B_{tu} terms, respectively, of Eq. (2), while the σ_{12} and σ_{13} terms in these brackets add up to give the B_{st} term. Thus the imaginary part of the Regge term comes from the same function which produces the poles (duality of the imaginary part; the real part is not dual, nor does it need to be). The $\pi\pi \rightarrow \pi\omega$ amplitude can be manipulated further into a product of s-, t-, and u-channel signature factors, but this extreme duality is not possible in the general case because the three B_{ij} terms are multiplied by SU(3) tensors which are too different in structure simply to "factor out."

[8]The $P(\mu_i)$ are constructed explicitly in D. E. Neville, Phys. Rev. 132, 844 (1963).

[9]In checking Eqs. (8) and (9), one must evaluate a certain sum over even-J resonances. One needs the identity

$$\sum_{n=0}^{\bar{n}} \Gamma(\alpha+2n+1)(4n+\alpha+2)/\Gamma(2n+2)$$
$$= \Gamma(\alpha+2\bar{n}+3)/\Gamma(2\bar{n}+2)(\alpha+1).$$

Eq. (16) of Ref. 1 gives the sum over odd-J resonances.

[10]Strictly speaking, we should investigate the zero-moment sum rules too because the t channel now contains both even- and odd-signature amplitudes. The zero-moment sum rule for B_{st} can be evaluated using Eq. (0.151.1) of I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (Academic Press, Inc., New York, 1965), and is well obeyed; however, the superconvergence relation for B_{su} does not converge as it should for $\alpha(t) < 1$. This relation might be better obeyed if "lower order" beta functions were added to the simple form (3) {i.e., beta functions

of the form $\Gamma[m-\alpha(s)]\Gamma[n-\alpha(t)]/\Gamma[m+n-\alpha(s)-\alpha(t)]$, m and/or $n>1\}$ because (a) the lower order terms are more important at low energies which the zero-moment sum rule emphasizes, and (b) B_{su} is more sensitive to these corrections than is B_{st}. In fact, for $s\to\infty$ with $\alpha(s)+\alpha(t)$ fixed, the so-called "lower order" corrections to B_{su} diverge faster than the leading beta function. It is necessary to sum an infinite number of them to get proper Regge asymptotic behavior. See Stanley Mandelstam, Phys. Rev. Letters 21, 1724 (1968).

[11]The tu box is just the real Regge term in nonresonant channels, discussed by H. Harari, Phys. Rev. Letters 20, 1395 (1968).

[12]Presumably nonbox amplitudes would be required when unitarity corrections would be incorporated into the model, since the vacuum "trajectory" contribution does not have the quark loop structure. Nonbox amplitudes might also be required when the nonet symmetry is broken to SU(2). Even in the SU(2) limit the finite-energy summable part of the amplitude must have the quark loop structure (with broken-symmetry quarks); however, there could be appreciable nonloop corrections (necessarily superconvergent) which shift resonance positions and residues from their nonet symmetry values.

SPONTANEOUS BREAKDOWN OF CHIRAL SU(3) ⊗ SU(3) SYMMETRY

G. Cicogna
Istituto di Fisica dell'Università, Pisa, Italy

and

F. Strocchi
Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Pisa, Italy, and Scuola Normale Superiore, Pisa, Italy

and

R. Vergara Caffarelli
Scuola Normale Superiore, Pisa, Italy

(Received 29 January 1969)

We show that the Hamiltonian suggested by Gell-Mann, Oakes, and Renner can be obtained as a spontaneous breakdown of chiral SU(3) ⊗ SU(3) symmetry.

It has been recently suggested[1-3] that the physics of strongly interacting particles can be better understood as a breaking of chiral SU(3) ⊗ SU(3) rather than SU(3) symmetry. A specific model of this breaking has been analyzed by Gell-Mann, Oakes, and Renner.[2] The basic assumption is to write the strong-interaction Hamiltonian in the following form:

$$H=H_0+\epsilon(u_0-\sqrt{2}\,u_8), \tag{1}$$

where H_0 is invariant under SU(3) ⊗ SU(3), and the scalar u_i ($i=0,\cdots,8$), together with the corresponding pseudoscalar partners v_i ($i=0,\cdots,8$), belong to the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ of SU(3) ⊗ SU(3).

In this note we want to point out that Eq. (1) may be understood as a dynamical breaking. More specifically, we will show that SU(3) ⊗ SU(3) symmetry breaks spontaneously along the direction $u_0-\sqrt{2}\,u_8$, and that an effective Lagrangian function can be deduced containing a breaking term which belongs to the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ of SU(3) ⊗ SU(3).

Let us denote by $\mathcal{L}$ a function which describes the system of strongly interacting particles. It may be the S matrix, or the Lagrangian, or the Hamiltonian. Just for definiteness we shall call it the Lagrangian. Let us assume that $\mathcal{L}$ is fully invariant under SU(3) ⊗ SU(3). The occurrence of a spontaneous breaking is equivalent to the existence of stationary points of $\mathcal{L}$ other than the origin.[4] In order to show that, we have to express $\mathcal{L}$ as a function of the "fields." To this purpose, we introduce the "elementary" fields u_i, v_i ($i=0,\cdots,8$) which transform according to the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ of SU(3) ⊗ SU(3) symmetry. They may be regarded as mathematical objects in terms of which one may construct the representations of SU(3) ⊗ SU(3) without necessarily implying a physical interpretation for them.

It is convenient to introduce the notation

$$W_\beta^{\ \pm\alpha}=U_\beta^{\ \alpha}\pm iV_\beta^{\ \alpha}, \tag{2}$$

where

$$U_\beta^{\ \alpha}=\frac{1}{\sqrt{2}}\sum_{i=0}^{8}(\lambda_i u_i)_\beta^{\ \alpha},\quad V_\beta^{\ \alpha}=\frac{1}{\sqrt{2}}\sum_{i=0}^{8}(\lambda_i v_i)_\beta^{\ \alpha},$$