

## PHOTOPRODUCTION OF RHO MESONS FROM COMPLEX NUCLEI AT 9 BeV\*

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We have measured the absolute differential cross section for photoproduction of neutral rho mesons from complex nuclei at a photon energy of 8.8 BeV. Using a two-parameter optical model, we have deduced a value for the total rho-nucleon cross section of  $30_{-4}^{+6}$  mb. Application of the vector-dominance model results in a value for the rho-photon coupling of  $\gamma_{\rho}^2/4\pi = 1.1 \pm 0.2$ .

We report here the results of a measurement of the photoproduction of pion pairs from complex nuclei at 8.8 BeV. These results are part of a systematic study of rho-meson photoproduction from a variety of targets ( $H_2$  through Pb), in the energy range of 5-16 BeV. Similar experiments performed at 2 to 6 BeV<sup>1-4</sup> have produced disparate results when interpreted within the framework of the vector-dominance model (VDM).<sup>5</sup> Since the VDM has successfully correlated many phenomena, it is of interest to study the limits of its applicability and to resolve any theoretical or experimental inconsistencies that arise. A preliminary report of these data has been presented elsewhere.<sup>6</sup>

The experiment was performed in the Stanford Linear Accelerator Center monochromatic photon beam using a wire spark-chamber spectrometer which was on-line to an IBM 1800 computer. This system is described elsewhere.<sup>6,7</sup> The properties of the beam and the measurement of the spectrum are also described elsewhere.<sup>8</sup> The energy of the monochromatic peak was  $8.85 \pm 0.02$  BeV with a full width at half-maximum of 0.52 BeV. To calibrate the absolute photon flux, the spark chamber system was periodically used as a pair spectrometer. The system had a mass acceptance of  $\sim 1000$  MeV at any given setting, with a maximum detectable mass of  $\sim 3500$  MeV. The mass resolution varied from  $\pm 6$  MeV at 700 MeV to  $\pm 15$  MeV at 3000 MeV. The momentum transfer acceptance ranged from 0 to  $0.25$  (BeV/c)<sup>2</sup>, with a resolution of  $\sim 0.0005$  (BeV/c)<sup>2</sup> for small  $t$  increasing to  $0.002$  (BeV/c)<sup>2</sup> for large  $t$ . The apparatus detected meson decays over 80% of the

decay solid angle at 9 BeV.

Data were taken using targets of Be, C, Al, Cu, Ag, and Pb, ranging in thickness from 0.1 radiation length for Be to 0.3 radiation length for Pb. The target position was set to maximize the spectrometer acceptance for pion pairs with energy comparable with the energy of the monochromatic photons and mass in the vicinity of the rho. Electromagnetic attenuation of the photons and absorption of pion pairs were measured by varying the thickness of the Be target; the results were in agreement with calculations and these calculated corrections (12%) were applied to the data from all other elements. Other corrections, measured and calculated, include target-out yield (2%), track-finding inefficiency (8%), pion decay (2%), pions grazing the edges of the beam stopper (2%), and dead time (<1%). The total correction factor varies from  $1.26 \pm 0.06$  for Be to  $1.35 \pm 0.09$  for Pb.

The energy spectrum of the pion pairs from Be is shown in Fig. 1(a). We have verified that the pion-pair yield is independent of the size of the energy cut for cuts less than  $\pm 0.75$  BeV centered around 8.8 BeV. This, together with the observation that the pion-pair spectrum and photon spectrum<sup>8</sup> are very similar, is a strong indication that we have a negligible contribution from inelastic events. The events included in the cross-section determinations were those with an energy between 8.4 and 9.5 BeV. The observed di-pion mass spectrum from Be, corrected for the spectrometer acceptance [Fig. 1(b)], is similar to that observed in other photoproduction experiments.<sup>1-4,9</sup> The observed distributions for each

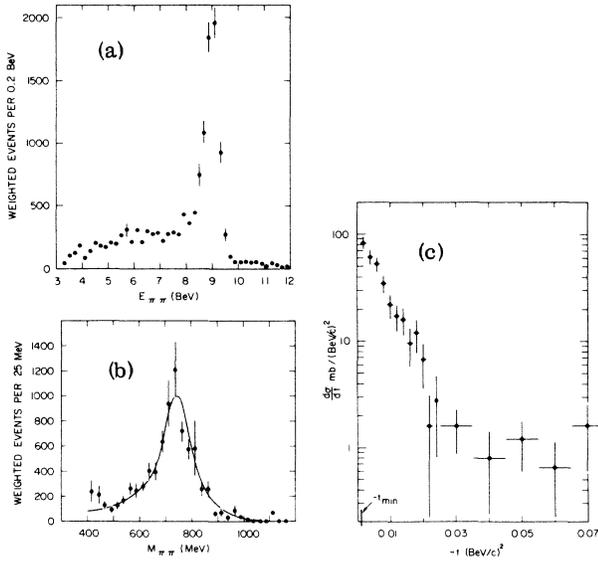


FIG. 1. (a) The energy spectrum of  $\pi^+\pi^-$  pairs produced from Be detected by the wire chamber spectrometer, showing the monochromatic peak at 8.8 BeV. (b) The mass distribution of pion pairs produced from Be by photons from the monochromatic peak. The solid line is the best fit to the data using a coherent mixture of resonant and diffractive background amplitudes (see text). (c) The differential cross section of the rho events from Cu.

element were modified to include the fact that the minimum momentum transfer,  $t_{\min} = -(M_{\pi\pi}^2/2k)^2$ , tends to suppress large masses, particularly for the heavy elements which have rapidly falling form factors. This correction is small at 8.8 BeV but is very serious for lower photon energies. The number of events  $N$  per unit mass  $M_{\pi\pi}$  was fitted with

$$\frac{dN}{dM_{\pi\pi}} = C_0 M_\rho \left[ \frac{M_{\pi\pi} \Gamma}{(M_\rho^2 - M_{\pi\pi}^2)^2 + M_\rho^2 \Gamma^2} + C_1 \frac{M_\rho^2 - M_{\pi\pi}^2}{(M_\rho^2 - M_{\pi\pi}^2)^2 + M_\rho^2 \Gamma^2} + C_2 \right] \quad (1)$$

with

$$\Gamma = \Gamma_\rho \frac{M_\rho}{M_{\pi\pi}} \left( \frac{M_{\pi\pi}^2 - 4\mu^2}{M_\rho^2 - 4\mu^2} \right)^{3/2},$$

where  $\mu = 0.139$  BeV, and  $C_0, C_1, C_2, M_\rho, \Gamma_\rho$  were allowed to vary. The first term in (1) is the ordinary  $p$ -wave Breit-Wigner, and the second term represents the interference with a coherent back-

ground which might arise from a process<sup>10</sup> in which one of the pions diffractively scatters off the nucleus. This interference has been proposed by Söding<sup>11</sup> as a possible explanation of the observed differences in rho production by pions and photons. We find our best fit ( $\chi^2$  per degree of freedom 1.26) to be  $M_\rho = 767 \pm 4$  MeV and  $\Gamma_\rho = 142 \pm 7$  MeV. According to our fit the background diffractive amplitude is so small that only the interference term is appreciable. The subsequent analysis includes only those events with masses in the range 650 to 850 MeV. This mass region accounts for 63% of the rhos contained in a  $p$ -wave Breit-Wigner normalized to unity with the  $s$ -wave normalization constant.<sup>12</sup> We have also fitted the data to a Breit-Wigner multiplied by  $(M_\rho/M_{\pi\pi})^4$ , as proposed by Ross and Stodolsky,<sup>13</sup> plus a polynomial background. We find an acceptable fit with  $M_\rho = 761 \pm 3$  MeV and  $\Gamma = 131 \pm 8$  MeV. The determination of the cross section is insensitive, within our quoted accuracy, to the type of fit used.

The momentum-transfer distribution of the events surviving the energy and mass cut, and a cut in the decay angle  $\theta_d$  given by  $|\cos\theta_d| \leq 0.5$ , is shown in Fig. 1(c). These data clearly show the rapidly falling contribution from coherent production and an underlying incoherent contribution. In order to determine the forward ( $t = t_{\min}$ ) value of the differential cross section, the data in the diffraction peak were fitted to an optical model, which included both a coherent contribution and an incoherent part that vanished in the forward direction; a hard-sphere nuclear density of radius  $R = r_0 A^{1/3}$  was used, and  $r_0$  was allowed to vary for each element. The values of  $r_0$  obtained in this way are listed in Table I. They show a definite decrease with increasing  $A$ . This  $A$  dependence is a consequence of the well-known inadequacy of the hard-sphere approximation to the nuclear density; in fact, our own values of  $r_0$  are consistent with the electromagnetic "equivalent hard-sphere" radii.<sup>14</sup> We emphasize that the hard-sphere model was only used to smooth the data and to obtain the forward cross section. The corrected forward cross sections, listed in Table I, may now be used to determine the total rho-nucleon cross section,  $\sigma_{\rho N}$ , and the photon-rho coupling constant for zero-mass rho mesons.<sup>15</sup>

The photoproduction of rho mesons from nuclei is related to the photoproduction from single nucleons and to the nuclear absorption of rhos as they emerge from the nuclear matter (i.e.,  $\sigma_{\rho N}$ ). The explicit relation<sup>16</sup> based on an optical model

Table I. The forward cross sections, the extrapolated  $t=0$  cross sections (using  $\sigma_{\rho N}=30$  mb and  $C_0=1.18$ ), the equivalent "hard-sphere" radii, and the photon-rho coupling constant determined in this experiment.

Nuclide	Events (unweighted)	$d\sigma/dt _{t_{\min}}$ [mb/(BeV/c) <sup>2</sup> ]	$d\sigma/dt _{t=0}$ [mb/(BeV/c) <sup>2</sup> ]	Radius $r_0=RA^{-1/3}$	$\gamma_\rho^2/4\pi$ for	
					$C_0=1.08$	$C_0=1.18$
Be <sup>9</sup>	549	3.48 ± 0.27	3.73 ± 0.29	1.35 ± 0.09	1.08 ± 0.08	1.37 ± 0.1
C <sup>12</sup>	229	6.85 ± 0.74	7.39 ± 0.80	1.40 ± 0.14	0.89 ± 0.10	1.14 ± 0.12
Al <sup>27</sup>	316	26.7 ± 2.7	29.6 ± 3.0	1.35 ± 0.08	0.87 ± 0.09	1.11 ± 0.11
Cu <sup>64</sup>	278	90.4 ± 9.2	104.7 ± 10.2	1.17 ± 0.05	0.98 ± 0.10	1.26 ± 0.12
Ag <sup>108</sup>	82	147 ± 29	178 ± 35	1.10 ± 0.12	1.30 ± 0.26	1.68 ± 0.33
Pb <sup>208</sup>	123	525 ± 69	677 ± 89	1.16 ± 0.08	0.91 ± 0.12 (0.97 ± 0.04)	1.18 ± 0.15 (1.23 ± 0.05)

of the nucleus is

$$\frac{d\sigma}{dt}(\gamma A - \rho^0 A) = \frac{d\sigma}{dt}(\gamma N - \rho N) f(\sigma_{\rho N}, \rho(r), t), \quad (2)$$

where

$$f(\sigma_{\rho N}, \rho(r), t) = |2\pi \int_0^\infty \int_{-\infty}^\infty b db dz J_0(q_\perp b) \rho(b, z) \exp[i(-t_{\min})^{1/2} z - \frac{1}{2} \sigma_{\rho N} \int_z^\infty \rho(b, z') dz']|^2, \quad (3)$$

where  $b$  and  $z$  are nuclear coordinates,  $q_\perp$  is the transverse momentum transfer, and  $\rho(r)$  is the nucleon density. We use the Woods-Saxon form for  $\rho(r)$ ,

$$\rho(r) = \rho_0 (1 + e^{(r-c)/a})^{-1}, \quad (4)$$

because it successfully describes electron-nuclear scattering<sup>14</sup> over a wide range of  $A$  with only two parameters; a half-density radius  $C = C_0 A^{1/3} = 1.08 A^{1/3}$  F; and a surface parameter  $a = 0.535$  F. We have performed our analysis using  $C_0 = 1.08$  and 1.18 F. These values represent our estimates for the limits on the nuclear radii in hadronic interactions and include the "hard-sphere" radii determined in this experiment (see Table I). We will discuss the results obtained with  $C_0 = 1.18$  F; results based on the electromagnetic radii are shown in Table I and Fig. 2(c). The value of  $C_0$  may, however, be a function of  $A$ , changing ~15% between Pb and Be.<sup>17</sup> This would increase our estimate of  $\sigma_{\rho N}$  by ~15%. This effect is included in our estimate of the systematic errors.

The  $A$  dependence of the forward ( $t=t_{\min}$ ) cross sections depend only on  $\sigma_{\rho N}$  for a given nuclear density. The measured forward cross sections, relative to Cu, are shown in Fig. 2(a). Taking the relative cross sections avoids, in this part of the analysis, assumptions about the details of the  $\rho$  photoproduction amplitude and utilizes only the  $A$  dependence predicted by the optical model. The solid line is the best fit to our

data using Eq. (2), from which we deduce that  $\sigma_{\rho N} = 31 \pm 4$  mb. Our measured relative  $A$  dependence is in good agreement with that of McClellan *et al.*<sup>4</sup>; in fact, a reanalysis of their data, using the method described above, yields  $\sigma_{\rho N} = 30$  mb. The difference between this and their quoted value of  $\sigma_{\rho N} = 38.5 \pm 4.5$  is due to the different nuclear densities used. We are unable to make a similar comparison with data of Asbury *et al.*<sup>2</sup> because the calculations are so sensitive<sup>18</sup> to the model at low energies.

Having deduced the value of  $\sigma_{\rho N}$ , we can use the VDM of photon-hadron interactions to determine the value of the coupling constant. The VDM together with the optical theorem leads to

$$\begin{aligned} \frac{d\sigma}{dt} \Big|_{t_{\min}} (\gamma A - \rho A) \\ = \frac{\alpha}{4} \left( \frac{4\pi}{\gamma^2} \right) \frac{\sigma_{\rho N}^2}{16\pi} f(\sigma_{\rho N}, t_{\min}) \end{aligned} \quad (5)$$

assuming that  $\rho N$  scattering amplitude is imaginary.

The value of  $\gamma_\rho^2/4\pi$  was calculated for each target nucleus using our data and our  $\sigma_{\rho N}$ . The results are listed in Table I and shown in Fig. 2(b). The solid line is the best  $A$ -independent fit to our data, and gives  $\gamma_\rho^2/4\pi = 1.21 \pm 0.05$  (statistical error). This is in strong disagreement with the

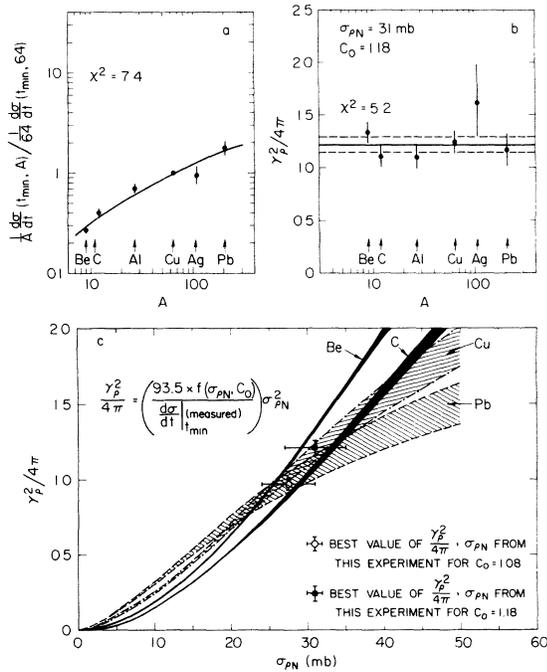


FIG. 2. (a) The forward cross section as a function of  $A$ , relative to Cu. The solid line is the best fit using the optical model described in the text with  $C_0 = 1.18$  F; the  $\sigma_{\rho N}$  deduced from this fit is  $31 \pm 4$  mb (statistical error). (b) The photon-rho coupling constant as determined from our forward cross sections and  $\sigma_{\rho N}$ . The solid line is the best  $A$ -independent fit to the data and gives  $\gamma_\rho^2/4\pi = 1.21 \pm 0.05$  (statistical error). (c) The dependence of  $\gamma_\rho^2/4\pi$  on  $\sigma_{\rho N}$  for our measured forward cross sections for several nuclei. In the relation  $\sigma_{\rho N}$  is in barns and  $d\sigma/dt$  in  $\text{mb}/(\text{BeV}/c)^2$ . The two points are our best value of  $\gamma_\rho^2/4\pi$  and  $\sigma_{\rho N}$  determined as illustrated in (a) and (b).

value quoted by Asbury *et al.*,  $0.45 \pm 0.1$ ; the difference (a factor of 2.5) is due either to the absolute normalization of the cross-section data or to the difficulty of extrapolation at low energies.<sup>18</sup> On the other hand, our value appears to be in very good agreement with that of McClellan *et al.*<sup>4</sup>; the precision of the agreement is due to the somewhat fortuitous cancellation of the different absolute normalization (20%) and different  $\sigma_{\rho N}$  (~15%). However, it must be emphasized that these two experiments do agree to within 20% on a value of  $\gamma_\rho^2/4\pi$  about 1.0 and are incompatible with a value of 0.5.

To display more graphically the implicit relation of  $\gamma_\rho^2/4\pi$  and  $\sigma_{\rho N}$  and to demonstrate the sensitivity of these deductions, we plot in Fig. 2(c) this relation, (5), for several complex nuclei using our measured cross sections. The two

points located in the overlap region are our best solutions for the limiting radii. The widths of the bands represent the uncertainty in  $\gamma_\rho^2/4\pi$  due to lack of knowledge of nuclear radii. The figure also dramatically demonstrates that the determination of  $\gamma_\rho^2/4\pi$  becomes less sensitive to  $\sigma_{\rho N}$  as  $A$  increases and nuclei become more opaque; in fact, for Pb, a  $\pm 5$ -mb change in a 30-mb  $\sigma_{\rho N}$  leads to a  $\pm 14\%$  change in  $\gamma_\rho^2/4\pi$ . In contrast, a precise measurement of the forward cross section for hydrogen does not result in a precise determination of  $\gamma_\rho^2/4\pi$  because of the intrinsic difficulties in the measurement of  $\sigma_{\rho N}$ , caused by uncertainties in the nuclear density.

Taking the average of our values of  $\sigma_{\rho N}$  and  $\gamma_\rho^2/4\pi$  based on the limiting radii, we obtain  $\sigma_{\rho N} = 30_{-4}^{+6}$  mb and  $\gamma_\rho^2/4\pi = 1.1 \pm 0.2$ , where the errors include contributions from statistics, normalization of photon flux, and an uncertainty attributed to the optical model and nuclear density.

We observe that (1) there are now two determinations of  $\gamma_\rho^2/4\pi \approx 1.0$  from the reaction  $\gamma + A \rightarrow \rho^0 + A$ ; (2) a proper evaluation of  $\gamma_\rho^2/4\pi$  using the measured branching ratio<sup>19</sup> for  $\omega \rightarrow \pi^0 + \gamma$ <sup>20</sup> and  $\Gamma_\rho = 111 \pm 6$  MeV<sup>21</sup> yields  $\gamma_\rho^2/4\pi = 0.9_{-0.12}^{+0.06}$ ; (3) there is a large disparity observed in the comparison<sup>22</sup> of polarized photoproduction of pions with rho production by pions; (4) nucleon isovector form factors are not compatible with a single rho pole; (5) the determinations of  $\gamma_\rho^2/4\pi$  from a study of  $e^+e^- \rightarrow \pi^+\pi^-$ <sup>21</sup> and leptonic decays<sup>21</sup> (i.e., photons on the rho mass shell) yield a value of  $0.52 \pm 0.03$ . These observations directly confront the validity of simple rho dominance and may imply a  $q^2$  dependence to the  $\gamma\rho$  coupling and/or contributions from higher vector mesons. A more thorough comparison of theory and experiment is in progress.

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<sup>12</sup>This normalization constant is

$$\left\{ \frac{1}{2}\pi - \tan^{-1} \left[ \frac{4\mu^2 - M_0^2}{\Gamma_0 M_0} \right] \right\}^{-1} = \frac{1.07}{\pi}.$$

In the limit  $\Gamma_0/M_0 \ll 1$ , this becomes  $1/\pi$ , the normal-

ization used by McClellan *et al.*, Ref. 2.

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<sup>15</sup>We define the photon-rho coupling constant to be  $em_\rho^2/2\gamma_\rho$ .

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<sup>18</sup>In the photon energy range of 2.7 to 9 BeV the extrapolation factor on Pb changes from 19 to 1.3, and for Be, from 2 to 1.1.

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## AN EXTENSION OF THE VENEZIANO REPRESENTATION

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This paper extends Veneziano's representation from  $\pi\pi \rightarrow \pi\omega$  to  $\Pi\Pi \rightarrow \Pi V$ , where  $\Pi$  ( $V$ ) is a pseudoscalar (vector) SU(3) octuplet or singlet.

This note extends Veneziano's representation for the  $\pi\pi \rightarrow \pi\omega$  scattering amplitude to  $\Pi\Pi \rightarrow \Pi V$ , where  $\Pi$  is a pseudoscalar octuplet or singlet and  $V$  is a vector octuplet or singlet.<sup>1-3</sup> The SU(3) structure proposed here for  $\Pi\Pi \rightarrow \Pi V$  can be justified by arguments largely independent of the details of  $\Pi\Pi \rightarrow \Pi V$  scattering (or of the Veneziano representation for that matter). It is likely, therefore, that this same structure applies to all meson-meson scattering processes.

Because we choose to work with external particles having the same spin and parity as Veneziano's, our amplitude will have the same Lorentz dependence as his; only the SU(3) dependence

will differ:

$$T = \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu P_1^\nu P_2^\rho P_3^\sigma A(s, t, u). \quad (1)$$

$A$  is a scalar in Lorentz space and a tensor with four subscripts in SU(3) space. We set

$$A = (\bar{\beta}/\pi) \left[ (M_{st} \bar{M}_{st})_{st} B_{st} - (M_{su} \bar{M}_{su})_{su} B_{su} - (M_{tu} \bar{M}_{tu})_{tu} B_{tu} \right], \quad (2)$$

where the SU(3) subscripts (suppressed, for clarity) are now on the  $M_{ij}$  and  $\bar{M}_{ij}$ ,  $\bar{\beta} = \text{const}$ , and the  $B_{ij}$  are the Euler beta functions chosen by