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ADIABATIC COMPRESSIONAL COOLING OF He^3†

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By adiabatic compression of a two-phase mixture of liquid and solid He^3, temperatures below 2.5 mK have been obtained. These are in the range expected for nuclear ordering in solid He^3.

By adiabatically compressing a two-phase mixture of liquid and solid He^3, we have cooled it to temperatures in the range of 2 to 3 mK from initial temperatures in excess of 24 mK. Independent evidence suggests that these low temperatures are near the ordering temperature in the solid He^3. The possibility for such cooling was suggested by Pomeranchuk[1] and qualitatively demonstrated by Anufriyev,[2] who obtained a low temperature below 20 mK with a starting temperature of 50 mK. The present low temperature is the lowest ever obtained by purely mechanical means.

Only a few remarks on experimental method will be given here. The He^3 cell consists of two interconnected parts: a flexible-walled metal cell of special design[3] which has elastic characteristics such that it can support pressure differences in excess of P_{min}, the minimum melting pressure of 28.9 atm, and a rigid-walled epoxy cell. We have 0.5 g of powdered cerium magnesium nitrate (in a right circular cylinder with diameter equal to height) for thermometry[4] at the bottom of the rigid section and in the He^3. Outside the flexible section is pure liquid He^4, which may be pressurized to as much as 25 atm (367 lb/in.^{2}), to compress the He^3. For precooling, the above device is sealed into a specially designed mixing chamber for our dilution refrigerator.[5] Owing to high thermal resistances, mainly boundary resistance, precooling is slow but thermal isolation is correspondingly good once compressional cooling has begun.

In practice, because of plug slippage problems in the He^3 filling line, the He^3 is precooled to about 100 mK and then, with zero He^4 pressure, the plug is formed. After further precooling the He^4 is pressurized, usually at a rate of about 1 lb/in.^{2} min. Results for two runs are shown in Fig. 1. Starting at the lowest He^4 pressure indicated for a given run, there is a slight cooling with increasing He^4 pressure due to the negative expansion coefficient[6] of liquid He^3. There is a sudden increase in the cooling rate when solid He^3 begins to form. The temperature then falls steadily into the low millikelvin range. Particu-

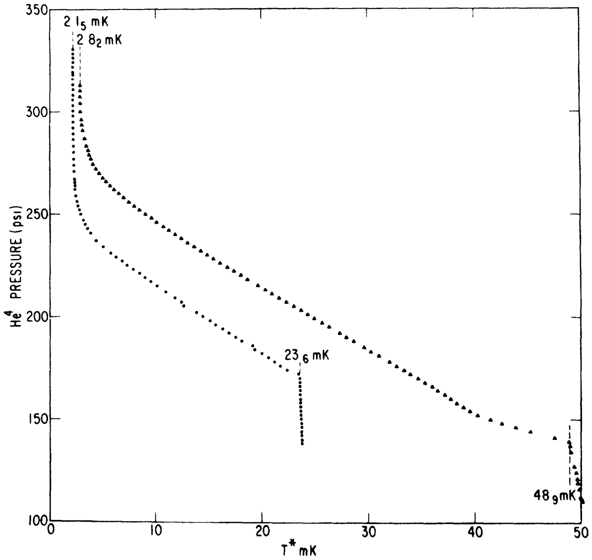

FIG. 1. Dependence of He^3 temperature, as indicated magnetically with cerium magnesium nitrate, on the pressure of liquid He^4 used to compress the He^3.

larly for the run starting around 24 mK, the final temperature is quite constant as He^4 pressure increases. Ultimately, not shown in Fig. 1, the temperature rises sharply with further increases in He^4 pressure, probably from "crunching" of solid He^3. The lowest magnetic temperature of 2.1_5 mK, indicated on Fig. 1, is significantly greater than the minimum magnetic temperature measured with similar thermometers, typically 1.7 to 1.9 mK.[7] According to Black's analysis[8] of data on the critical field of tungsten, magnetic temperatures of 2.1_5 and 2.8_2 mdeg are, respectively, 2.40 ± 0.05 and 2.87 ± 0.05 mK on the Kelvin scale.

The above temperatures are in the range where nuclear ordering is expected in solid He^3. According to preliminary work[9] with the present device, the solid is characterized by a transverse relaxation time $\tau_2 = 275 \pm 10$ msec and spin diffusion coefficient $D_z = (12 \pm 1) \times 10^{-8}$ cm^2/sec (where the errors estimate precision only; the accuracy is not known) in a temperature region high enough[9] that these quantities are temperature independent. These may be used[10,11] to estimate values for the effective exchange interaction parameter J in the assumed Hamiltonian $\mathcal{H} = -\hbar J \vec{I}_1 \cdot \vec{I}_2$. For this purpose we used a molar volume[12-14] of 24.2 cm^3/mole for the solid, in equilibrium with liquid, near $T = 0$. Assuming that bcc He^3 has antiferromagnetic ordering, we find from Baker, Gilbert, Eve, and Rushbrooke[15] that the transition is expected at $T_N = 1.37\hbar|J|/k$, or at 1.8_7 mK from τ_2 and 2.7_6 mK from D_z. Measurements of $(\partial P/\partial T)_V$ by Panczyk, Scribner, Straty, and Adams[16] give $T_N = 1.9_8$ mK while Meyer's[10] average $|J|$, from measurements of τ_1 and τ_2, gives T_N to be 1.7_5 mK. Measurements by Anderson, Reese, and Wheatley[17] of the solid susceptibility at 35.0 atm fit the Curie-Weiss equation, $\chi = C/T + \theta)$, with $\theta = +3.5 \pm 0.5$ mK. Using the susceptibility expansion for a bcc Heisenberg antiferromagnet given by Rushbrooke and Wood[18] this corresponds to $T_N = 2.4 \pm 0.3$ mK. All of these estimates are very close to the final low temperatures attained in our experiment.

For the two runs shown in Fig. 1, the liquid molar entropies at the beginning of solidification[12,19] are $0.109R$, final $T = 2.40 \pm 0.05$ mK (see above), and $0.210R$, final $T = 2.87 \pm 0.05$ mK. If the compression were isentropic and all solid were obtained, then the average solid specific heat would be $\overline{T}\Delta S/\Delta T \approx 0.6R$. This value is sufficiently large to support the contention that the solid entropy is falling rapidly at our lowest temperatures. Actually it is unlikely that our compressions are isentropic. If heat is added to hold T constant while the two-phase mixture is compressed, the ratio of heat added to mechanical work done is $-(T/P)(dP/dT)_m \cong T/(8 \times 10^2$ mK) at low (but not the lowest) temperatures. Frictional heating can thus be extremely damaging, though one might speculate that for our geometry the cerium magnesium nitrate is least affected by the frictional heating problem.

Although the above remarks make it plausible that spin ordering in the solid is limiting the low temperatures obtained in these experiments, this has not been proved. Measurements of dP/dT_m will determine the solid entropy using ΔV_m data of Ref. 14, while measurements of susceptibility and D_z will indicate the type of ordering and the effect of ordering on transport, respectively. Using adiabatic compressional cooling of ^3He, both types of measurements are in progress.

We note in conclusion that if the He^3 is cooled by compression in a sufficiently large magnetic field the resultant solid He^3 has a high nuclear spin polarization.

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STABILITY OF STRAIGHT MULTIPOLES WITH SHEAR*

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Every scalar-pressure, perfectly conducting, two-dimensional equilibrium with three magnetic field components and an arbitrary pressure profile but no current in the ignorable direction is magnetohydrodynamically stable. The stability of these equilibria bears no visible relation to any magnetic well or average magnetic well criterion and is only partly ascribable to shear.

Consider a perfectly conducting, scalar-pressure magnetohydrodynamic equilibrium, subject to $\nabla p = \vec{J}\times\vec{B}$. The second variation of total energy, $\int(\frac{1}{2}B^2+\frac{3}{2}p)dV$, can be written in several equivalent forms:

$$2\delta W_1 = \int\{Q^2+\vec{\xi}\cdot\vec{Q}\times\vec{J}+(5/3)p(\operatorname{div}\vec{\xi})^2+(\vec{\xi}\cdot\nabla p)(\operatorname{div}\vec{\xi})\}dV+\oint[\partial p_*/\partial n]\xi_n{}^2 dS, \tag{1}$$

$$2\delta W_2 = \int\{(\vec{B}\cdot\nabla\vec{\xi}-\vec{B}\operatorname{div}\vec{\xi})^2+\nabla p_*\cdot(\vec{\xi}\operatorname{div}\vec{\xi}-\vec{\xi}\cdot\nabla\vec{\xi})+(5/3)p(\operatorname{div}\vec{\xi})^2\}dV, \tag{2}$$

$$2\delta W_3 = \int\{(\vec{B}\cdot\nabla\vec{\xi}-\vec{B}\operatorname{div}\vec{\xi})^2+p_*(\partial\xi_i/\partial x_j)(\partial\xi_j/\partial x_i)+[(5/3)p-p_*](\operatorname{div}\vec{\xi})^2\}dV+\oint p_*[\vec{\xi}\operatorname{div}\vec{\xi}-\vec{\xi}\cdot\nabla\vec{\xi}]dS, \tag{3}$$

where

$$\vec{Q}=\operatorname{curl}(\vec{\xi}\times\vec{B}), \tag{4}$$

$$p_*=p+\tfrac{1}{2}B^2. \tag{5}$$

The surface integrals are kept in order to allow discontinuities in $\vec{B}$ and p at selected flux surfaces; equilibrium requires that p_* be continuous across these surfaces,

$$[p_*]=0. \tag{6}$$

The same notation is used in (1), where $[\partial p_*/\partial n]$ signifies the jump in $\partial p_*/\partial n$ across an interface, and similarly in (3).

The first form, δW_1, explicitly exhibits the stability of a vacuum field for which $\delta W_1=\frac{1}{2}\int Q^2 dV$ [this elementary fact is quite hidden in (2) and (3)]. The neutral variations in a vacuum field are interchanges, $\vec{Q}=\operatorname{curl}(\vec{\xi}\times\vec{B})=0$.

The second form, δW_2, can be obtained from (1) by innumerable integrations by parts, or more directly, via a Lagrangian (rather than Eulerian) evaluation of the energy variation.[1] This form is notable in that there is no explicit contribution at an interface. It also explicitly exhibits the stability of the elementary configuration in which $\vec{B}$ is unidirectional [B_z and p depend on (x,y) and $p_*=\text{const}$]. This elementary result is not apparent from δW_1. The neutral perturbations in this case are flutes, $\operatorname{div}\vec{\xi}=0$ and $\vec{B}\cdot\nabla\vec{\xi}=0$. We can contrast flutes which are incompressible in physical space with interchanges which are incompressible in flux coordinates.

The third form, δW_3, can be obtained from (2) by a single integration by parts. We shall find this form particularly useful because no derivatives of the equilibrium quantities $\vec{B}$ and p appear explicitly.

In two-dimensional equilibrium with z as an ignorable coordinate, the field can be written

$$\vec{B}=\vec{B}_0+\vec{B}_z=\vec{n}\times\nabla\psi+\vec{n}B_z, \tag{7}$$

$$\vec{J}=\vec{n}\Delta\psi-\vec{n}\times\nabla B_z, \tag{8}$$

where $\vec{n}$ is the unit vector in the z direction. The most general equilibrium with this symmetry requires p and B_z to be constant on ψ lines, and ψ is governed by the nonlinear elliptic equation

$$\Delta\psi=-p'(\psi)-f'(\psi), \tag{9}$$

where $p(\psi)$ and

$$f(\psi)=\tfrac{1}{2}B_z{}^2 \tag{10}$$

are considered to be arbitrarily given functions.[2]