FIELD-THEORETICAL NUCLEON-NUCLEON POTENTIAL

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An unambiguous formulation is presented for the derivation of the two-pion as well as one-boson exchange contributions to the nucleon-nucleon potential. It contains no internal momentum expansions, and depends only on experimentally determined coupling and mass parameters. The calculated result, containing terms linear in the momentum operator, is remarkably close to the Hamada-Johnston potential for distances $r \gtrsim 0.7$ F.

For many applications, it is useful to have a potential which, when used in the Schrödinger equation, generates the field-theoretical S matrix to some required accuracy. In the case of the two-nucleon system, the potential respresentation is important in many-body considerations. Fortunately, there are strong indications that a potential representation based on the boson-exchange hypothesis is valid.[1,2] The present work is motivated by the belief that a suitably accurate representation of the effect of the one- and two-pion exchanges amended by the known multipion resonances and rescattering corrections may account for the nuclear interaction for relative distances $r \gtrsim 0.5\hbar/\mu c$ (μ the pion mass), up to the pion threshold energy.

The early attempts to derive the two-pion exchange potential (TPEP) led to a variety of schemes which differed importantly in their consequences.[3,4] The best known of these are the Taketani-Machida-Ohnuma (TMO)[5] and Brueckner-Watson (BW)[6] potentials. An unsatisfactory aspect of these works is the so-called static approximation which has been eliminated in the more recent formulation of Charap and Fubini, and Charap and Tausner (CFT).[7] This formulation, however, suffers from a divergence, which, when made finite by subtraction, leads to unacceptable results.[8] Our procedure is based on the observation that the iteration of the one-pion exchange kernel (OPEK) involved in the derivation of TPEP is very sensitive to the choice of OPEK off as well as on the energy shell. Indeed, the divergence in CFT can be traced to the iteration of their "adiabatic" approximation of OPEK. The TMO-BW difference can also be attributed to the different off-shell choices of OPEK as well as to the static approximation.[3] A satisfactory treatment must give a well-defined derivation of OPEK accurate over the entire range of iteration and not only the low-energy region.

In the following we shall describe a method based on the Blankenbecler-Sugar[9] reduction of the Bethe-Salpeter equation and a unitarity-preserving definition of the corresponding Lippmann-Schwinger (LS) amplitude. We will further show that the resulting potential is identical on the energy shell with one obtained from a modified CFT method that employs an unapproximated OPEK defined according to the same unitarity requirement.

The Bethe-Salpeter equation for the (elastic) scattering of two nucleons in the center-of-momentum system from a state of relative four-momentum p to one of p' is

$$\mathfrak{M}(p',p|W)=K(p',p|W)+\int d^4R\,K(p',R|W)G(R|W)\mathfrak{M}(R,p|W), \tag{1}$$

where $\mathfrak{M}$ is the invariant amplitude, W half the total momentum, K the interaction kernel consisting of all irreducible diagrams, and G the two-particle (free) propagator,

$$G(R|W)=\frac{i}{2\pi}\left[\frac{1}{\not W+\not R-m}\right]^{(1)}\left[\frac{1}{\not W-\not R-m}\right]^{(2)}, \tag{2}$$

where m is the nucleon mass. The superscripts (1) and (2) refer to the two nucleons, and the spin and isospin indices have been suppressed.[10] The equivalent "equal-times" equation of Blankenbecler and Sugar is obtained by replacing G by a propagator g that produces a two-particle cut only in the physical region where its discontinuity is equal to that of G. The propagator g is readily constructed, and the resulting equation is

$$\mathfrak{M}(p',p|W)=U(p',p|W)+\int d^4R\,U(p',R|W)\frac{\delta(R^0)[\gamma^0E(k)-\vec\gamma\cdot\vec{\mathrm{k}}+m]^{(1)}[\gamma^0E(k)+\vec\gamma\cdot\vec{\mathrm{k}}+m]^{(2)}}{4E(k)[W^2-E^2(k)+i\epsilon]}\mathfrak{M}(R,p|W), \tag{3}$$

where $E(k)=[\vec{k}^2+m^2]^{1/2}$ and U is the equivalent interaction given by the integral equation

$$U=K+K(G-g)U. \tag{4}$$

We note the appearance of the δ function and of the positive energy projection operators in Eq. (3). The former serves to reduce (3) to a three-dimensional equation, while the latter restrict $\mathfrak{M}$ to the positive energy spinors. Passing to the Pauli (two-component) spinor representation, and integrating over R^0, we get

$$\tilde{\mathfrak{M}}(\vec{p}',\vec{p}|W)=\tilde{U}(\vec{p}',\vec{p}|W)+\int d^3k\,\tilde{U}(\vec{p}',\vec{k}|W)\frac{1}{W^2/2m-E^2(k)/2m+i\epsilon}\frac{m}{E(k)}\tilde{\mathfrak{M}}(\vec{k},\vec{p}|W), \tag{5}$$

where tilde signifies the restricted quantities. The appearance of the factor $m/E(k)$ may seem undesirable.[9] However, $\tilde{\mathfrak{M}}$ (being essentially the invariant amplitude) is not the LS amplitude (T) and must not be thus identified. The correct identification is easily ascertained by requiring that the relativistic unitarity relation on $\mathfrak{M}$ reduce to the nonrelativistic one on T. This requirement immediately leads to

$$T(\vec{p}',\vec{p}|W)=\left[\frac{m}{E(p')}\right]^{1/2}\tilde{\mathfrak{M}}(\vec{p}',\vec{p}|W)\left[\frac{m}{E(p)}\right]^{1/2}. \tag{6}$$

The above factors connecting $\tilde{\mathfrak{M}}$ and T evidently account for the difference between relativistic and nonrelativistic phase-space factors. They therefore ensure the equality of the physical observables (e.g., cross section) in the two formulations. Equations (5) and (6) now yield the final result in our reduction:

$$T(\vec{p}',\vec{p}|W)=V(\vec{p}',\vec{p}|W)+\int d^3k\,V(\vec{p}',\vec{k}|W)\frac{1}{W^2/2m-E^2(k)/2m+i\epsilon}T(\vec{k},\vec{p}|W), \tag{7}$$

where the potential V is related to the interaction kernel by

$$V(\vec{p}',\vec{p}|W)=\left[\frac{m}{E(p')}\right]^{1/2}\tilde{U}(\vec{p}',\vec{p}|W)\left[\frac{m}{E(p)}\right]^{1/2}. \tag{8}$$

Note that the factors of m/E make a correction to the standard OPEP (as well as other potentials) equivalent to a potential of a range of the order of the nucleon Compton wavelength. From Eq. (4), we can write down the various orders of the interaction U, of which we will record OPEK and TPEK (two-pion exchange kernel):

$$U^{(2)}=K^{(2)},$$
$$U^{(4)}=K^{(4)}+K^{(2)}(G-g)K^{(2)}. \tag{9}$$

We now assert that if the CFT method is modified so as to conform to the identification of Eq. (8) (implying the use of the exact OPEK for iteration), it will give rise to a potential identical on the energy shell to that defined by (8) and (9).[11] This circumstance must be regarded as a consistency check, since in both derivations the requirement is that the potential, when used in the Schrödinger equation, produces the physical equivalent of the field-theoretical amplitude in each order of the perturbation expansion.

The procedure discussed above yields an energy-dependent potential in momentum space, giving rise to a nonlocal as well as energy-dependent (W) potential in configuration space. The general characteristics of the energy dependences involved have been studied by Hoshizaki and Machida[12] in momentum space and by CFT using dispersion theoretical arguments. The usual practice of converting energy dependences into differential operators in configuration space is an asymptotic procedure valid for distances large compared with the nucleon Compton wavelength. Such asymptotic expansions are satisfactory at distances of the order of the pion Compton wavelength; at half that distance they are not quite so satisfactory, and indeed the potential becomes extremely energy-dependent at or inside this distance. The same conclusion follows from the observation that the expansion will fail when the strength of the potential is comparable with the reduced mass of the system $\frac{1}{2}m$. This fact together with the importance of many-meson exchanges sets a natural boundary for the potential representation at about half the pion Compton wavelength.[2] We therefore adopt the view that the potential representation is a fairly accurate

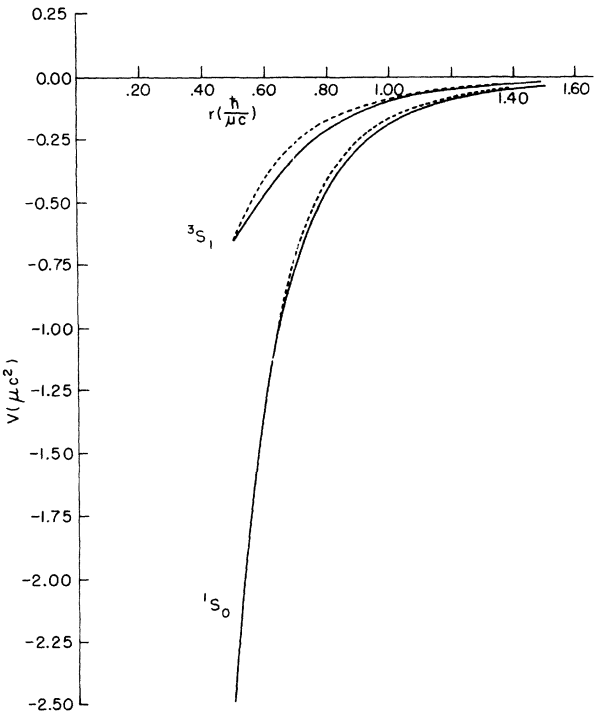

FIG. 1. Complete S-state potentials. The solid curves are those of the present calculation. The dashed curves are those of Ref. 15. With respect to pion mass differences, the isospin $T=1$ potentials represent the P-P case.

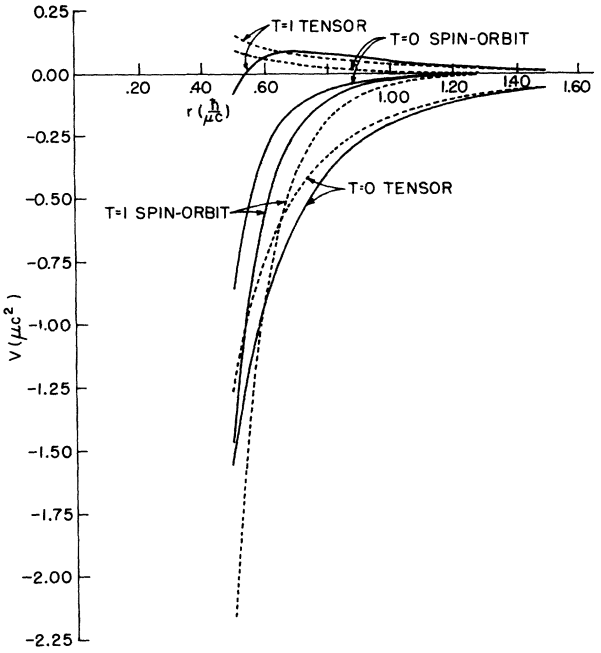

FIG. 2. Complete tensor and spin-orbit potentials. The solid and dashed lines have the same meaning as in Fig. 1.

representation up to the boundary region, inside which it must be amended by some suitable representation of its extreme nonlocality such as the boundary-condition model.[2,13]

On the basis of the above considerations, we expand our potential in powers of the momentum operator (in configuration space) and retain terms up to the first order, thereby including the central, spin-orbit, tensor, and spin-spin potentials. It is worth emphasizing here that nowhere have we used the static approximation involving the socalled μ/m expansion and the nonrelativistic limit on internal momenta. This procedure is in accord with the CFT treatment, and it goes further by avoiding their adiabatic approximation on OPEK.

The potentials, given by multiple integrals, have been calculated numerically. The complete potential includes OPEP and TPEP (with PS coupling), and η, ρ, and ω exchanges. The masses used in the potential have been taken from the Rosenfeld tables, and the coupling parameters have been taken from experiments.[2,14] They are $g_\pi^2 = 14.4$, $g_\eta^2 = 1$, $g_\rho^2 = 0.53$, $g_\rho/f_\rho = 1.83$, $g_\omega^2 = 12 g_\rho^2$, $g_\omega/f_\omega = -0.06$, where all coupling parameters are rationalized and dimensionless. Our definition of the coupling parameters for the vector mesons are specified by the following interaction Hamiltonian for ρ:

$$H_\rho = [4\pi]^{\frac{1}{2}} \bar{\Psi} \times [g_\rho \gamma^\mu \vec{\varphi}_\mu + (f_\rho/2m)\sigma^{\mu\nu}(\partial_\nu \vec{\varphi}_\mu - \partial_\mu \vec{\varphi}_\nu)] \cdot \vec{\tau} \Psi.$$

The mass of the charged pions is used as the unit of mass. We have incorporated the pion mass differences in our potentials.

Figures 1 and 2 compare our complete potential with that of Hamada and Johnston.[15] Figure 1 compares the well-determined $^3S_1$ and $^1S_0$ potentials, and Fig. 2 compares the tensor and spin-orbit potentials in the two isospin states. The agreement is very good with the exception of the isosinglet spin-orbit potential (which is not determined very well phenomenologically). The above comparison is not intended to establish the validity of our potential on the basis of agreement with a phenomenological one. Indeed, the Hamada-Johnston potential includes quadratic spin-orbit terms which have no counterpart here. However, among the cases exhibited here, the singlet and triplet S-wave potentials are deter-

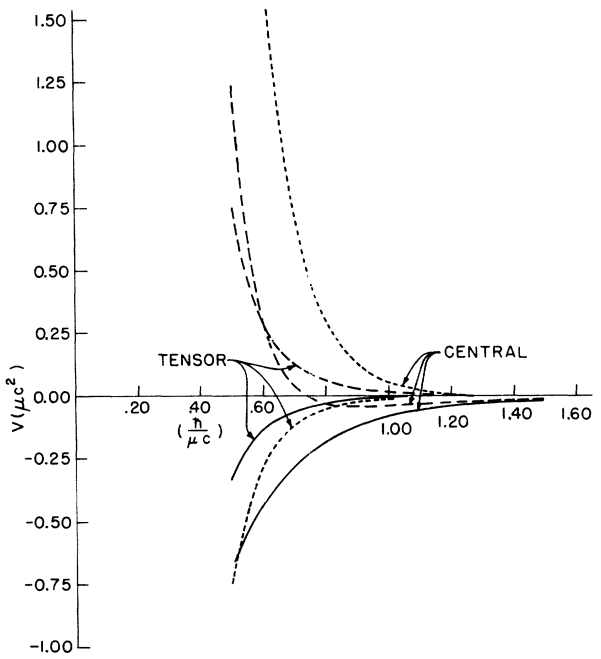

FIG. 3. Central and tensor $T=0$ TPEP potentials. The solid curves are those of the present calculation, the short-dashed are TMO (Ref. 5), and the long-dashed are BW (Ref. 6).

mined experimentally with small ambiguity for $r>0.5\hbar/\mu c$, and therefore afford a meaningful comparison. Figure 3 compares our TPEP to the corresponding TMO and BW TPEP's without pair suppression. As in the two cases illustrated here, the sign of the BW alteration to TMO is always substantiated, as expected theoretically.[16] We believe that the somewhat large deviation of our TPEP from both TMO and BW in the central potentials is due to the static approximation used in the latter pair.

In conclusion, the procedure presented above seems to be a well-defined and unambiguous prescription for deriving potentials from field-theoretical models. The principal contributions to be added to the present potential are the rescattering corrections; we intend to incorporate these by including the N^* resonances via a coupled-channel formalism. The 3π continuum is difficult to compute, and it is probably mainly confined to the inner region. The role of a possible S-wave two-pion resonance should also be investigated. Although the large mass of this resonance (~730 MeV) would presumably confine its effects mainly to the inner region, a consistent treatment must consider the subtraction of some of the two-pion continuum contribution on account of the apparently broad width of the resonance.[17] We expect the resulting correction to be small for $r>0.5\hbar/\mu c$. The role of pair suppression should also be investigated in this connection. In a future publication, we shall present a detailed description of the potential and discuss some of the relevant aspects omitted above.

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[11]This assertion is easily verified. It is interesting to note that the identical cut structures of G and g in the physical region is the reason for the cancellation of a strong energy dependence in U, Ref. 4. This cancellation occurs in CFT only in the nonrelativistic limit, and it is the basis of their definition of an energy-independent potential.

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