TESTS OF THE PERIPHERAL MODEL FOR A CLUSTER KNOCKOUT REACTION*†

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The reaction ^{6}Li$(\alpha, 2\alpha)d$ was studied at 50.4, 59.0, 60.5, 70.3 and 79.6 MeV. The angular distributions and energy dependence of the off-energy-shell α-α cross section were extracted and compared with those for free α-α scattering. The agreement between the two is excellent, as predicted by the peripheral model in the quasifree approximation, but the quasifree approximation is not precise enough.

In the last few years there has been increasing interest in cluster knockout reactions, which potentially can give information about correlations between groups of nucleons in the target nucleus.[1] A variety of reactions has been studied, including the (p,pd), (p,pt), $(p,p^3\mathrm{He})$, $(p,p\alpha)$, and $(\alpha, 2\alpha)$ reactions at bombarding energies up to 1 BeV, and analyzed using the "quasifree" or "peripheral" model to extract nuclear structure information. However, few tests have been made to verify the reaction mechanism and the tests so far reported have not been very demanding.

As a suitable starting point for a systematic study of the reaction mechanism we have chosen the reaction ^{6}Li$(\alpha, 2\alpha)d$, and have studied the cross section for this reaction as a function of scattering angle and of bombarding energy. The observed cross section is found to depend in a striking way on these variables. The peripheral model in the quasifree approximation gives an excellent qualitative description of these dependences, while the detailed comparisons of theory and experiment indicate the necessity for further theoretical and experimental work.

The peripheral model[2] describes the reaction $A(a, ab)C$ by the diagram of Fig. 1(a), where the lower vertex represents the virtual decay $A \to b + C$ and the upper vertex represents the elastic scattering of particle a from the virtual particle b. If we ignore the particle spins, the cross section for emission of a and b into the solid angles $d\Omega_a$ and $d\Omega_b$ with the kinetic energy of a within dE_a is given by

$$\frac{d\sigma}{d\Omega_a d\Omega_b dE_a} = |\varphi(\vec{q}_C)|^2 (d\sigma/d\Omega)_{ab} \times (\text{kinematic factor}). \quad (1)$$

The function $\varphi(\vec{q}_C)$, where $\vec{q}_C$ is the momentum of C measured in the rest frame of A, may be interpreted as a momentum-space wave function for that component of A which can be described in terms of the pair b, C. The quantity $(d\sigma/d\Omega)_{ab}$ is the elastic a-b scattering cross section at the upper vertex. Because this scattering is off the two-body energy shell, the cross section is not obtainable directly from free a-b scattering measurements and it is customary to make the quasifree approximation in which $(d\sigma/d\Omega)_{ab}$ is replaced by the measured a-b cross section at some nearby point on the energy shell.

The characteristic feature of expression (1) which results from the peripheral model is its factorizability.[3] This factorizability may be tested by seeing whether the cross section depends in the expected way upon each of the quantities $|\varphi(\vec{q}_C)|^2$ and $(d\sigma/d\Omega)_{ab}$ when the other is held constant. Unfortunately our knowledge of what to expect is limited. Since the cluster structure of nuclei is not well known enough to make accurate predictions of $\varphi(\vec{q}_C)$, tests of the dependence of the cross section on q_C are limited to qualitative observations. One such feature is the dependence of $|\varphi(\vec{q}_C)|^2$ on the relative orbital angular momentum L of the pair b, C: Since $|\varphi(\vec{q}_C)|^2$ must behave like q_C^{2L} near $q_C = 0$, the cross section should vanish at $q_C = 0$ if $L \neq 0$.[4] This feature has been observed in the cluster knockout reaction ^{7}Li$(p, pt)\alpha$ at 55 MeV.[5]

Tests of the dependence of the cross section (1) on $(d\sigma/d\Omega)_{ab}$ require a knowledge of the off-energy-shell cross section, usually obtained by making the quasifree approximation. Ruhla et al.[6] have applied a test of this type by comparing the reactions ^{6}Li$(p, p\alpha)d$ and ^{6}Li$(p, pd)\alpha$, which have the same $\varphi(\vec{q}_C)$, at 155 MeV. The ratio of

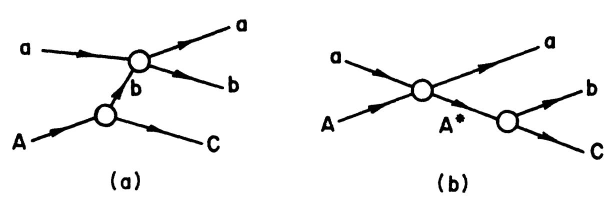

FIG. 1. (a) Peripheral diagram for a knockout reaction. (b) Sequential diagram for a reaction proceeding via an excited state A^* of the target nucleus.

the cross sections was approximately equal to the ratio of the free p-α and p-d cross sections. This comparison was, however, essentially at a single bombarding energy and scattering angle. Very recently[7] the angular dependence of several $(p, p\alpha)$ reactions has been observed to be as predicted by the quasifree approximation.

For the present work we have selected a single reaction, ^{6}Li$(\alpha, 2\alpha)d$, and studied systematically the dependence of the cross section on both scattering angle and bombarding energy. Within the framework of the peripheral model we have eliminated the dependence of the cross section on $|\varphi(\vec{q}_d)|^2$ by comparing cross sections always at the same value of $\vec{q}_d$, namely $\vec{q}_d = 0$. We could then, subject to a constant normalizing factor $|\varphi(0)|^2$, deduce the values of the off-energy-shell $(d\sigma/d\Omega)_{\alpha\alpha}$ and compare the angular and energy dependence of the off-energy-shell and free α-α scattering cross sections.

We chose the target ^{6}Li because it is known to have an important $(\alpha + d)$ component in its structure.[1] We chose the $(\alpha, 2\alpha)$ reaction because the free α-α scattering cross section, which has been extensively studied,[8,9] has a rapid and characteristic dependence on both scattering angle and bombarding energy, which present a severe test for the reaction mechanism if they are to be reproduced in the cross section for the $(\alpha, 2\alpha)$ reaction. We chose initially to compare cross sections at $\vec{q}_d = 0$ because this offers the best chance of success for the peripheral model. In addition, previous measurements of the reaction ^{6}Li$(\alpha, 2\alpha)d$ had been made at 62 MeV[10]; these showed a large peak in the cross section near $\vec{q}_d = 0$ which was interpreted as due to the knockout of $L = 0$ alpha particles.

A beam of alpha particles from the Oak Ridge isochronous cyclotron was used to bombard a self-supporting ^{6}Li target of thickness 0.6 mg cm^{-2}. A variety of bombarding energies was used, 50.4, 59.0, 60.5, 70.3, and 79.6 MeV. At each bombarding energy coincident alpha particles from the reaction ^{6}Li$(\alpha, 2\alpha)d$ were detected in ΔE-E silicon surface-barrier detectors. The two detectors were at angles θ_1 and θ_2, coplanar with, and on either side of, the beam direction. For each pair of angles, spectra of the coincident alpha particle energies E_1 and E_2 were recorded on a Victoreen 20 000-channel analyzer. From these spectra events were selected which satisfied the kinematics for the reaction ^{6}Li$(\alpha, 2\alpha)d$ and these events were displayed as a function of E_1.

Figure 2 shows the E_1 energy spectra at 70.3 MeV for $\theta_1 = 44.25°$ and six values of θ_2. The absolute cross-section scale for the experiment is uncertain within ±40%, but internal comparisons should be reliable within counting statistics. In each spectrum the arrow indicates the location at which the contribution from alpha-particle knockout is expected, namely, the value of E_1 for which the momentum $\vec{q}_d$ of the recoil deuteron is smallest. The knockout peak is largest near $\theta_2 = 44.25°$ since only at that angle can the recoil deuteron momentum be exactly zero. The other peaks in the spectra shown are due to sequential processes described by diagrams like that of Fig. 1(b). Here the intermediate state A^* is the long-lived 2.18-MeV state of ^{6}Li. Spectra similar to those of Fig. 2 were obtained at about 50 pairs of angles with special emphasis on the "quasifree angle pairs," for which $q_d = 0$ is kinematically possible.

To compare the ^{6}Li$(\alpha, 2\alpha)d$ cross section with the free α-α elastic scattering cross section, we extracted from the spectra at the "quasifree angle pairs" the value of $(d\sigma/d\Omega_1 d\Omega_2 dE_1)$ at $q_d = 0$.

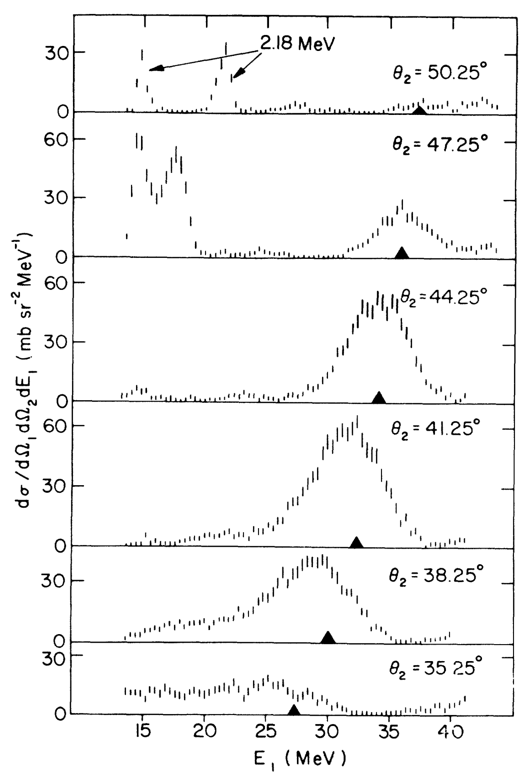

FIG. 2. Energy spectra from the reaction ^{6}Li$(\alpha, 2\alpha)d$ at 70.3 MeV and $\theta_1 = 44.25$ deg, for various values of θ_2. The arrow in each spectrum indicates the expected position of the knockout peak.

The correct location in each E_1 spectrum was given by a relativistic three-body kinematics program together with the E_1 calibration obtained from α-^{6}Li elastic scattering measurements. The cross section at this value of E_1 was inserted into Eq. (1) to obtain[11] a value for $|\Phi(0)|^2(d\sigma/d\Omega)_{\alpha\alpha}$. If the peripheral model is correct this gives, apart from the unknown constant factor $|\Phi(0)|^2$, the off-energy-shell α-α cross section for this reaction. If the quasifree approximation is used as a guide, this off-energy-shell cross section should depend on $E_{\text{c.m.}}$ and $\theta_{\text{c.m.}}$ in much the same way as the free cross section.

The deduced off-energy-shell cross sections are plotted as a function of $\theta_{\text{c.m.}}$ in Figs. 3(a)-3(c) for the three bombarding energies $E_0 = 79.6$, 70.3, and 59.0 MeV. The cross sections have been normalized to unity at $\theta_{\text{c.m.}} = 90°$ to simplify comparison with the free cross sections measured at nearby energies,[8] shown as solid curves also normalized to unity at $\theta_{\text{c.m.}} = 90°$. The α-α c.m. energy in the reaction ^{6}Li(α, 2α)d might be taken either as $E_{\text{c.m.}} = E_i$, calculated assuming that the incident particle strikes a stationary free alpha particle in the target nucleus, or as $E_{\text{c.m.}} = E_f$, the c.m. energy of the two outgoing alpha particles. E_f is less than E_i because of the 1.47-MeV binding energy of ^{6}Li, while for free scattering $E_{\text{c.m.}} = E_i = E_f$ ($\approx \frac{1}{2}E_0$ nonrelativistically). The angular dependence of the off-energy-shell cross section is clearly very similar to that of the free cross section, and a systematic variation with energy is also clearly present and similar to that of the free scattering.

To illustrate the energy dependence of the off-energy-shell cross section, we show in Fig. 3(d) the measured values of $|\varphi(0)|^2(d\sigma/d\Omega)_{\alpha\alpha}$ at $\theta_{\text{c.m.}} = 90°$ plotted as a function of E_i. All the points have been divided by $|\varphi(0)|^2$ 2.5 fm^3 sr^{-1} to obtain the best overall agreement with the free-scattering cross section which is shown as a solid line. The agreement between the energy dependence of the off-energy-shell cross section and that of the free cross section is remarkably good.

The qualitative success of the peripheral model and quasifree approximation is clearly excellent. When we try to achieve a detailed quantitative agreement we find, however, that the quasifree approximation is not adequately specified. In the quasifree approximation there is no unique prescription for the correct values of $E_{\text{c.m.}}$ and $\theta_{\text{c.m.}}$ at which to evaluate the free cross section for use in Eq. (1). For the comparison in Fig. 3(d) we assumed that $E_{\text{c.m.}} = E_i$. In Fig. 3(e) we plot the same data as a function of E_f. The fit is at least as good, but the deduced value of $|\varphi(0)|^2$ is changed by 30% to $|\varphi(0)|^2 = 1.9$ fm^3 sr^{-1}. The energy dependence at $\theta_{\text{c.m.}} = 90°$ thus does not, with the presently available data, distinguish between $E_{\text{c.m.}} = E_i$ and $E_{\text{c.m.}} = E_f$. The angular distributions in Figs. 3(a)-3(c) do however give clearer indications of the values of $E_{\text{c.m.}}$ to use in making the quasifree approximation. The choice $E_{\text{c.m.}} = E_f$ gives an excellent fit to the angular distribution at $E_0 = 70.6$ MeV. However, this choice is not so successful at 70.3 and 59.0 MeV, where still lower values of $E_{\text{c.m.}}$ fit the data better.[12]

Our analysis of the cross sections at $\vec{q}_d \neq 0$ is still at a very early stage. We have found that it is important to take into account the variations of $(d\sigma/d\Omega)_{\alpha\alpha}$ within each energy spectrum, such as those of Fig. 2, before extracting information on $|\varphi(\vec{q}_d)|^2$. In a preliminary analysis we have

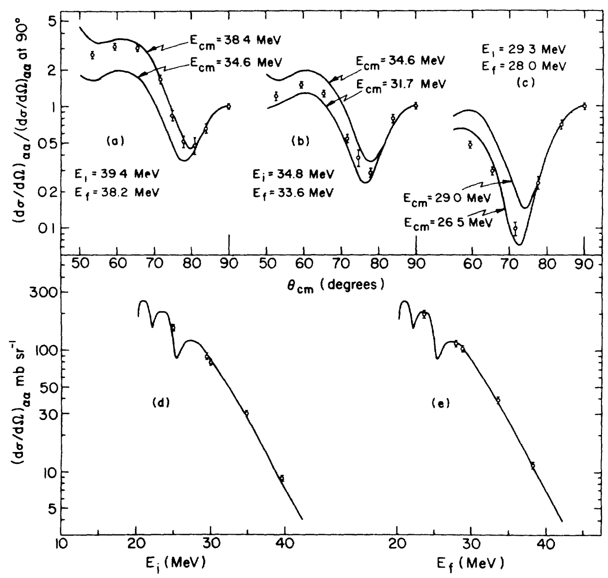

FIG. 3. (a)-(c) The off-energy-shell $(d\sigma/d\Omega)_{\alpha\alpha}$ deduced from the reaction ^{6}Li(α, 2α)d at 79.6, 70.3, and 59.0 MeV, plotted against $\theta_{\text{c.m.}}$ for the α-α collision. The solid curves are values of $(d\sigma/d\Omega)_{\alpha\alpha}$ for free scattering taken from Ref. 9. The results are normalized to unity at $\theta_{\text{c.m.}} = 90°$. (d) The off-energy-shell $(d\sigma/d\Omega)_{\alpha\alpha}$ deduced from the reaction ^{6}Li(α, 2α)d at $\theta_{\text{c.m.}} = 90°$, plotted against E_i, the α-α c.m. energy before the collision. The data points are normalized using $|\varphi(0)|^2 = 2.5$ fm^3 sr^{-1}. The solid curve is $(d\sigma/d\Omega)_{\alpha\alpha}$ for $\theta_{\text{c.m.}} = 90°$ taken from Refs. 8 and 9.
(e) The same as for (d) except that the data are plotted against E_f, the α-α c.m. energy after the collision, and the normalization is obtained using $|\varphi(0)|^2 = 1.9$ fm^3 sr^{-1}.

used a simple phenomenological $(\alpha + d)$ cluster model for ^{6}Li, derived from the α-d scattering phase shifts and the ^{6}Li binding energy. We find that the value of $|\varphi(0)|^2$ obtained from the data at $\theta_{\text{c.m.}} = 90°$ is roughly a factor of 4 lower than predicted. Furthermore the observed width of $|\varphi(\vec{q}_d)|^2$ is too small. Use of a cutoff radius for the α-d wave function improves the agreement with experiment in both these respects.

To summarize, we have studied the reaction $^6\text{Li}(\alpha, 2\alpha)d$ systematically over a wide range of angles and energies. The peripheral model in the quasifree approximation provides an excellent qualitative description of the results. However, the choice of what on-energy-shell cross sections to use in applying the quasifree approximation is not precisely enough specified, since the various ways of making the choice give predictions differing by much more than the experimental uncertainties. A better way of predicting the off-energy-shell cross sections is needed. Additional experimental data are needed, particularly in the energy region below 55 MeV, since the existence (or absence) of resonances in the $^6\text{Li}(\alpha, 2\alpha)d$ yield would give valuable information on the reaction mechanism. It would also be extremely useful to perform an experiment to test the reaction for invariance under the Treiman-Yang rotation[13,14]; this would provide a test of the peripheral nature of the reaction without requiring any detailed knowledge of the interactions at the vertices, hence avoiding the quasifree approximation altogether.

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[1]V. G. Neudachin and Y. F. Smirnov, At. Energy Rev. 3, 157 (1965); A. B. Clegg, High Energy Nuclear Reactions (Clarendon Press, Oxford, England, 1965), p. 112.

[2]I. S. Shapiro, in Proceedings of the International School of Physics "Enrico Fermi," Course XXXVIII, (Academic Press, Inc., New York, 1966), p. 210.

[3]When particle spins are included, the cross section does not in general factorize. However, Shapiro has shown that under certain conditions, expression (1) is still valid. All the reactions discussed in this Letter satisfy one or more of the following conditions, each of which is sufficient to ensure factorizability: (a) Particle b has spin $J = 0$. (b) Particle b is nonrelativistic and has spin $J = \frac{1}{2}$. (c) Particle b is nonrelativistic and the relative orbital angular momentum of the pair b-c is $L = 0$.

[4]T. Berggren, G. E. Brown, and G. Jacob, Phys. Letters 1, 88 (1962).

[5]D. L. Hendrie, M. Chabre, and H. G. Pugh, University of California Radiation Laboratory Report No. UCRL 16580, 1966 (unpublished); see also H. G. Pugh, M. Jain, M. Epstein, H. D. Holmgren, and P. G. Roos, Bull. Am. Phys. Soc. 12, 1176 (1967).

[6]C. Ruhla, M. Riou, M. Gusakow, J. C. Jacmart, M. Liu, and L. Valentin, Phys. Letters 6, 282 (1963).

[7]B. Gottschalk and S. L. Kannenberg studied the reactions $^9\text{Be}(p,p\alpha)^5\text{He}$, $^{12}\text{C}(p,p\alpha)^8\text{Be}$, and $^{16}\text{O}(p,p\alpha)^{12}\text{C}$ at 160 MeV [S. L. Kannenberg, thesis, Northeastern University, 1968 (unpublished); B. Gottschalk, private communication]. H. D. Holmgren, M. Jain, H. G. Kim, and P. Roos studied the reaction $^6\text{Li}(p,p\alpha)d$ at 61 MeV (to be published).

[8]P. Darriulat, G. Igo, H. G. Pugh, and H. D. Holmgren, Phys. Rev. 137, B315 (1965).

[9]H. E. Conzett, P. Darriulat, H. G. Pugh, E. Shield, and R. J. Slobodrian, University of California Radiation Laboratory Report No. UCRL 16580, 1966 (unpublished).

[10]M. Jain, M. Epstein, H. D. Holmgren, H. G. Pugh, and P. G. Roos, Bull. Am. Phys. Soc. 12, 1176 (1967). Similar results have been found at 56 MeV by J. R. Pizzi, R. Bouche, M. Gaillard, P. Gaillard, A. Guichard, M. Gusakow, J. L. Leonhardt, and C. Ruhla, Phys. Letters 28B, 32 (1968).

[11]The kinematic factor used in this analysis was

$$\frac{4m_\alpha k_1 k_2}{\hbar^2 k_0}\left(3 + 2\frac{k_1}{k_2}\cos\theta_{12} - 2\frac{k_0}{k_2}\cos\theta_2\right)^{-1}.$$

This expression was first derived by Gottschalk and Kannenberg [S. L. Kannenberg, thesis, Northeastern University, 1968 (unpublished)] by transforming the reaction into the c.m. frame of the incident and struck α particles. The differential cross section $(d\sigma/d\Omega)_{\alpha\alpha}$ is then, in the quasifree approximation, the c.m. system cross section for free elastic α-α scattering calculated at appropriate values of $E_{\text{c.m.}}$ and $\theta_{\text{c.m.}}$. The quantities k_0, k_1, and k_2 are the wave numbers of the incident and two emitted α particles and θ_{12} is the angle between the two emitted α particles, all evaluated in the laboratory system. The values of $\theta_{\text{c.m.}}$ were, throughout our analysis, calculated in the c.m. system of the final two α particles.

[12]We assume here that the free cross section varies smoothly between the energies at which it has been measured. This assumption is probably fairly reliable in the region of energies between $E_{\text{c.m.}} = 26.5$ MeV and $E_{\text{c.m.}} = 38.4$ MeV since the data of Ref. 8 vary very systematically in that region. The possibility of pre-

viously unobserved resonances should not, however, be excluded.

[13]S. B. Treiman and C. N. Yang, Phys. Rev. Letters 8, 140 (1962).

[14]I. S. Shapiro, V. M. Kolybasov, and G. R. August, Nucl. Phys. 61, 353 (1965).

COSMIC-RAY NEGATRON AND POSITRON SPECTRA BETWEEN 12 AND 220 MeV*

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The interplanetary negatron and positron spectra from 12 to 220 MeV have been determined with a balloon-borne magnetic spectrometer. The observed charge ratio $e^+/(e^+ + e^-) \approx 0.3$ indicates that the flux most likely consists of a mixture of "primary" negatrons and interstellar "secondary" negatrons and positrons. We deduce an absolute solar modulation of the interstellar positron flux which decreases with decreasing magnetic rigidity below about 80 MV.

Measurements of the shape and the charge composition of the interplanetary electron[1] spectrum are important to studies of physical phenomena in the interstellar and interplanetary media. At present, two source mechanisms are considered major potential contributors to the equilibrium cosmic-ray electron flux in the galaxy. "Primary" electrons are directly accelerated in "hot" loci in the galaxy, e.g., supernovae, and "secondary" electrons are produced in collisions of high-energy cosmic-ray nuclei with interstellar matter. Positrons in significant numbers are expected only from the collision source, and their spectrum can be calculated.[2,3] Comparison of the calculated spectra with our measured results, which include information on both the energy spectrum and charge composition of the electron flux between 12 and 220 MeV, allows us to determine the relative proportions of interstellar primary and secondary electrons and to impose restrictions on the propagation and solar modulation of the observed cosmic-ray electron fluxes.

Our measurements were performed near the top of the atmosphere with a balloon-borne magnetic spectrometer. The detector system[4] consists of an array of scintillation counters, a 1-kG permanent magnet, wire spark chambers with magnetostrictive readout, and a Čerenkov counter. The instrument has a geometry factor equal to 3.7 cm^2 sr between 25 and 200 MeV/c, decreasing at lower momenta to 1.5 cm^2 sr at 6 MeV/c. The momentum resolution for electrons below 100 MeV/c is limited primarily by scattering and equals about 25% full width at half-maximum, independent of momentum. Above 100 MeV/c, the momentum resolution is a function of the intrinsic angular resolution and is linear with momentum, rising to 50% full width at half-maximum at 200 MeV/c.

The data presented in this paper are derived from three high-altitude ballon flights launched from Fort Churchill, Canada, on 15 July, 20 July, and 28 July 1968. Average float altitudes were 2.45, 2.40, and 2.35 g/cm^2 of residual atmosphere, respectively, with variations of ±0.15 g/cm^2. The data discussed below were measured during the nighttime period when the local geomagnetic cutoff was below our analysis threshold. Since no statistically significant temporal variations in the measured electron flux were observed among the three flights, the data have been combined for increased statistical accuracy.

A large fraction of the low-energy electrons observed at an atmospheric depth of 2.4 g/cm^2 originate in collisions of cosmic-ray nuclei with atmospheric nuclei above. The separation of cosmic-ray electrons from atmospheric secondaries is nontrivial. Our technique is to express the atmospheric depth dependence of the measured positron or negatron flux $J_i^\pm(d)$, for an energy interval i, as

$$J_i^\pm(d) = a_i^\pm s_i^\pm(d) + b_i^\pm p_i^\pm(d),$$

where d is atmospheric depth, $s^\pm(d)$ is the depth dependence of the flux of (atmospheric) secondary positrons or negatrons, $p^\pm(d)$ is the depth dependence of the flux of primary positrons or negatrons, and $a^\pm$ and $b^\pm$ are parameters which represent the relative contribution of the secondary and primary components. The parameters $a^\pm$ and $b^\pm$ were determined by a least-squares fit to seven data points from 2.4 to 42 g/cm^2 atmo-