$$\sum_{\kappa'}\theta(\kappa-\kappa')=2\pi K_\kappa+\sum_\lambda\theta(2\kappa-2\lambda), \tag{16'}$$

where $\theta(x)=2\tan^{-1}(x/c)$, and I_j, J_λ, K_κ are either integers or half-integers, coming from logarithm function.

We thus have the following representation for the real part of α:

$$\begin{aligned}\mathrm{Re}(\alpha)&=\sum_j v_j \,\mathrm{Im}(k_j)\\&=\mathrm{Im}[(1/L)\sum_j\sum_\lambda v_j\theta(2\lambda+2iv_j)]\leqslant 0.\end{aligned} \tag{17}$$

When λ is distributed with a density $\sigma(\lambda)$ and $\rho(v)=\rho(-v)$, then

$$\mathrm{Re}(\alpha)=iL\int\int\rho(v)dv\,\sigma(\lambda)d\lambda\,v\theta(2\lambda+2iv). \tag{18}$$

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COUPLING BETWEEN FERROMAGNETIC LAYERS THROUGH A SUPERCONDUCTOR

G. Deutscher and F. Meunier
Service de Physique des Solides,* Faculté des Sciences d'Orsay, 91 Orsay, France

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Three-layer systems [ferro-super-ferro (FSF)] with very thin insulating junctions between S and both F layers have been prepared. The superconducting transition temperature $T_{c\uparrow\uparrow}$ (when both F magnetizations are parallel) is lower than $T_{c\uparrow\downarrow}$.

Some years ago Sarma[1] calculated the thermodynamical critical temperature of a superconductor in the presence of a uniform exchange field h.

More recently, de Gennes[2] indicated that such an exchange field could be induced by the proximity effect of a ferromagnetic layer F on a thin and clean superconducting film S, and that in a sandwich (FSF) configuration, the value of h could be varied by varying the angle between the magnetization directions of the F films, thus giving rise to a variation of the critical temperature.

In this Letter we report experiments made on FeNi-In-Ni sandwiches giving evidence of this coupling effect.

Figure 1 shows the extreme critical temperature of a sandwich when the magnetization directions are (a) parallel ($T_{c\uparrow\uparrow}$) and (b) antiparallel ($T_{c\downarrow\uparrow}$). The rather large difference between the coercive fields of the FeNi (H_1) and Ni (H_2) layers allows us to produce both configurations.

Our samples have been made in the following way: First a 1500-Å thick FeNi layer is evaporated and oxidized for 30 sec under a 10^{-3}-Torr pressure of O_2; then a 2500-Å thick In layer is evaporated under a pressure of 10^{-6} Torr. Lastly the sample is cooled down to a temperature of about 150°K, and a 1500-Å thick Ni layer is deposited.

We thus obtain thin magnetic insulating layers on each side of the In layer. The procedure used to get the barrier at the In-Ni interface (Ni oxidized from below by O_2 or H_2O molecules absorbed on the In layer) is similar to that previously used by other authors.[3,4]

Two important theoretical conditions[2] are fulfilled in our experiments:

(a) $d_S \leq \xi_0$ and $d_S \lesssim l$, so that h can be considered as constant in S (d_S, ξ_0, l are, respectively, the thickness, coherence length, and mean free path of S).

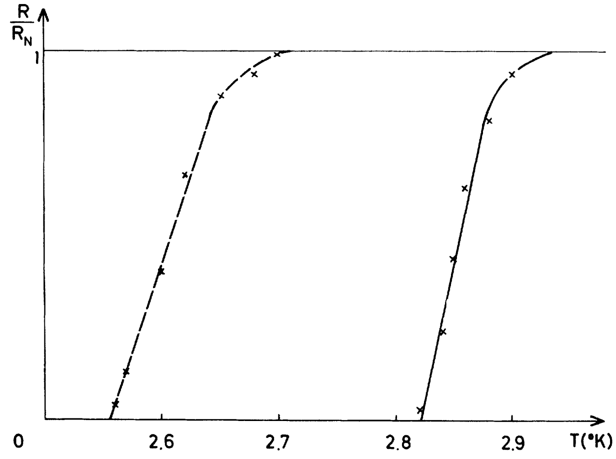

FIG. 1. Resistive measurements of the critical temperatures (R_N = resistance in the normal state) in zero field after the following: dashed line, application of 10 000 G ($T_{c\uparrow\uparrow}$) (all fields are applied parallel to the plane of the films); solid line, application of −10 000 G and subsequently +300 G to return the magnetization of the FeNi layer ($H_1 < 300\text{ G} < H_2$) ($T_{c\downarrow\uparrow}$).

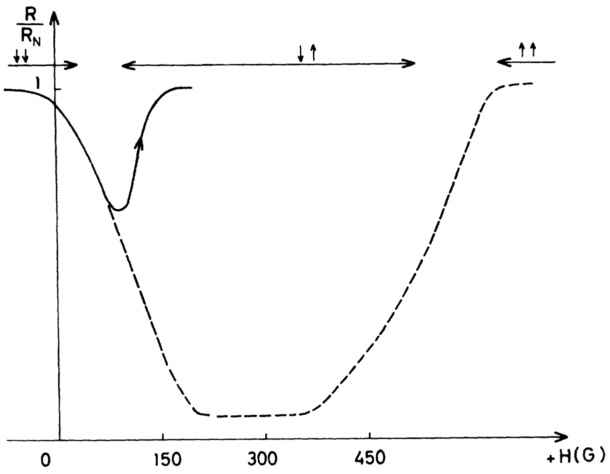

FIG. 2. Solid line: $R(+H)$ after previous application of −10 000 G. We are at a temperature T ($T_{c\uparrow\uparrow} < T < T_{c\downarrow\uparrow}$), where the critical field in the ↓↑ configuration $H_{c\downarrow\uparrow}$ is of the same order as H_1. Dashed line: resistive measurements in low field after application of −10 000 G and subsequently $+H$. The ↓↑ configuration is best realized for 180 G $< H <$ 380 G.

(b) The electrical contact between the S and F layers does correspond to a very thin insulating barrier: If we had no barrier, superconductivity could not be observed for $d_S \lesssim \xi_0$. But the junction resistance of the S-F contacts is below 10^{-3} Ω; thus the barrier is much thinner than in a conventional tunneling junction (~10 Ω).

On the other hand, the low-temperature hysteresis cycles of the F layers are far from square: Therefore the F layers are not monodomains in zero field. It follows that: (a) $\delta T_c = T_{c\downarrow\uparrow} - T_{c\uparrow\uparrow}$ is limited to a few 10^{-1}°K. (b) Following the magnetic history of the sample, intermediate critical temperatures can be obtained in the range $T_{c\downarrow\uparrow} > T_c > T_{c\uparrow\uparrow}$. The intermediate $R(H)$ values reported in Fig. 2 correspond to these intermediate critical temperatures.

We also want to point out that our results cannot be explained by a difference between the stray-fields configuration of the (↑↑) and (↑↓) states. Such effects should persist when there is no electrical contact between the S and F layers. We have made measurements on F-SiO-S-SiO-F systems, where the thickness of the SiO layers (~200 Å) is small compared with the thickness of the S layer: For such systems we observe that $T_c(\uparrow\uparrow) = T_c(\uparrow\downarrow)$.

In conclusion, our experiments show that when suitable interface conditions are realized, ferromagnetic materials can induce an exchange field in a superconductor by a proximity effect. A detailed study of the influence of this exchange field on the superconducting properties is presently under way.

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*Laboratoire associé au Centre National de la Recherche Scientifique.

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OBSERVATION OF SINGULAR POINTS IN DEFECT-INDUCED FAR-INFRARED SPECTRA OF KBr*

T. Timusk and R. W. Ward†
Department of Physics and the Institute for Materials Research,
McMaster University, Hamilton, Ontario, Canada

(Received 9 December 1968)

Far-infrared absorption spectra of KBr crystals containing Li^+, Na^+, Sm^{++}, OH^-, and F^- are shown all to have sharp discontinuities at 74.8, 75.2, 85.5, and 89.5 cm^{-1}. It is shown by comparison with dynamical models for the lattice and the defect that these frequencies correspond closely to expected van Hove singularities in the host lattice. These experiments, when extrapolated to zero defect concentration, provide accurate measurements of certain phonon frequencies in the pure lattice.

The lattice-vibration-frequency spectra of solids are dominated by sharp discontinuities of slope at points in the zone where the gradient of frequency as a function of wave vector q vanishes.

It has been recognized for some time that defect-induced optical absorption spectra should reflect in some way these "van Hove singulari-